Review of European Studies; Vol. 8, No. 2; 2016
ISSN 1918-7173 E-ISSN 1918-7181
Published by Canadian Center of Science and Education

A Modified Baumol Approach-Optimal Withdrawal and Holding of Cash Liquid Assets

L. D. Gonen[1], M. Weber[2], T. Tavor[3] & U. Spiegel[4]

[1] Department of Economics and Business Administration, Ariel University, Ariel, Israel

[2] Faculty of Business Administration, Israeland Bar-Ilan University, Ramat Gan, Israel

[3] Department of Economics and Management, The Max Stern Yezreel Valley College, Yezreel Valley, Israel

[4] Department of Management, Bar-Ilan University, Ramat Gan, Israel

Correspondence: U. Spiegel, Department of Management, Bar-Ilan University, Ramat Gan, 5290002, Israel. Tel: 972-3531-8282. E-mail: spiegeu@biu.ac.il

Received: January 26, 2016 Accepted: February 23, 2016 Online Published: March 9, 2016

doi:10.5539/res.v8n2p8 URL: http://dx.doi.org/10.5539/res.v8n2p8

Abstract

Baumol developed an equation for the transaction demand for money. It is affected positively by cost per withdrawal and transaction value, and negatively by the interest loss from holding cash.

Our objective is to modify the Baumol equation by including another factor. The demand for money is also affected by the customer concern that holding a more available liquid asset encourages more spontaneous purchases with resulting losses in their real value. We develop a new theoretical model by adding to the original Baumol cash demand equation another demand for a deposit which has positive yield and is less liquid. Holding this deposit restrains some of the spontaneous purchases. This modified Baumol equation leads to the following new results: Customers withdraw cash more often; maintain, on average, a smaller cash balance and larger amount of less liquid assets; and reduce their spontaneous and "nonrational" purchases.

Keywords: Baumol equation, liquidity, yields, transactions, spontaneous purchases

1. Introduction

Baumol (1952) developed a model of the money demand for the transaction motive. The theory relies on the tradeoff between the liquidity of holding cash to facilitate commodities transactions and the resulting loss of interest. The key variables of the demand for money are thus the (nominal) interest rate, and the level of real income with respect to the amount required for these transactions. Transaction costs related to the demand for money have continued to be an important issue for decades following Baumol (1952) and Tobin (1956). See for example Krueger (2012).

In Baumol's traditional approach, the total cost of holding money assumes two basic and contradictory effects.

The cost of each withdrawal is b dollars, including the charge per withdrawal, waiting time on line, etc. Thus, the total withdrawal cost is $b \cdot \frac{Y}{m}$ where $\frac{Y}{m}$ represents the number of withdrawals over a given period of time. Y represents the total value of commodities transactions during each period and m is the money balance that is withdrawn each time.

By holding on the average more money during a given period of time we lose interest that is r per dollar cash holding Thus, the cost is $r \cdot \frac{m}{2}$.

The sum of the two elements of the money holding cost that we want to minimize is

$$Min\, TC = b \cdot \frac{Y}{m} + r \cdot \frac{m}{2}$$

The derivative of this equation with respect to m leads to the optimal withdrawal of money m^* such that

$m^* = \sqrt{\frac{2bY}{r}}$. We refer to this as the Baumol equation.

However, according to this approach we may expect that a significant amount of money be withdrawn at each withdrawal and that relatively few withdrawals be made during a given period. However, people actually behave differently. They make many more frequent cash withdrawals than expected and on the average relatively small amounts of cash withdrawals each time.

The question that arises is whether such behavior is simply irrational and should be changed, or whether it is appropriate and based on good reason that Baumol himself did not consider. As classical economists we are possibly unaware of such solid rationales and perhaps do not capture the real motivation of people whose actions differ from our expectations. People do indeed depart from the routine equation of Baumol due to a new factor recently raised by behavioral economists. Observing the behavior of people who hold small amounts of cash in their pocket, the behavioral economists assert that people do behave very carefully. Most likely, people carefully consider certain factors when they determine how much money to withdraw and how often.

The most frequent immediate response of consumers to our question regarding cash withdrawals can be summarized as follows. The availability of more money in the consumer's pocket generates the illusion of being rich and "strong", and thus encourages him to spend more rapidly and sometimes even carelessly, impulsively and spontaneously. Moreover, according to Chang et al. (2007) an increase in the ratio of consumer purchases to cashable deposits, will increase the real cost of cash withdrawals. Thus the typical consumer realizes that he, and more often she (Note 1), may spend money more quickly and less efficiently. Whenever he holds less money in his pocket, he spends it less spontaneously and more carefully. Thus the real benefit from the same amount of annual spending with constrained availability is that it eventually generates higher satisfaction. Recent papers of Badgaiyan and Verma (2014) and Amir et al. (2014), support these "findings" with respect to impulsive behavior. Amir et al. examine it with respect to Pakistani women, in the case of women's apparel, while the former paper investigates the impulsive behavior in India and does not find significant distinctions between men and women.

The role of liquid assets in affecting personal consumption expenditure has been discussed in the literature (e.g., Zellner, 1975; Griliches et al., 1962; Modigliani & Brumberg, 1954). The theory of money reveals the importance of the balance effect in the process of transitioning towards equilibrium. Individuals, who try to reduce the number of dollars they hold, raise the flow of expenditure (Friedman, 1959).

The above literature can be illustrated by the following argument. When I am exposed to a high availability of candy that is both approachable and affordable, then I have the desire and temptation to use that availability more often, even if not in the most efficient, valuable, or rational way. Since I am aware of my human weaknesses I try to avoid the consequences by avoiding the availability, and preventing unreasonable actions. We can apply a similar analogy to holding cash. I prefer to keep my liquid assets in a checking account instead of in my pocket due to my concern that the accessibility of cash money may cause me to misuse it by spontaneous and impulsive purchasing, without appropriate self-control. This tendency is part of human nature and the reason that the Bible states, "Do not muzzle anoxwhile it is threshing."

How can one restrain himself from impulsive purchasing? Different kinds of payment measures have different restraining effects on impulsive purchasing. Therefore, if a person recognizes that the efficiency of his purchases is negatively correlated withholding cash, he will restrain himself from doing so by using various means of payments with different degrees of liquidity (such as cash and demand deposits). This will lead to the development of a new and modified version of holding cash that differs from and significantly changes the original Baumol formula. We refer to the new formula as the modified Baumol equation.

Our study includes "intermediate assets" which are liquid (but less liquid than cash) and bear a positive return, although lower than the yields of non-liquid assets. As the degree of liquidity bearing very low yields increases, so does the natural tendency to use liquid assets for spontaneous and inefficient purchasing.

In a short note presently under preparation, "Modified Baumol Approach: Optimal Holding of Money" (2015), we present a modification to the original equation of Baumol (1952) by introducing the element of undesired and non-valuable purchases resulting from liquid money availability.

In our present study we further modify the Baumol equation with a different transactions demand for money that is influenced by assets differing in degrees of liquidity yields.

Determination of different yields for different asset liquidity levels reduces the loss of real value of spontaneous purchases. Such purchases are impacted by the accessible cash held by customers. Our model allows for holding a liquid asset that yields some interest payments on one hand while it also avoids spontaneous purchases and losses on the other hand.

2. Literature Review

2.1 Demand for Money by Firms

Baumol (1952) established a model of the demand for cash based on the micro-economic idea of optimal enterprise inventory. It was also based on relatively low-risk assets similar to money that earn interest. This afforded firms a choice of whether to keep their income in cash or as interest-earning assets.

Following up on Baumol's approach, Miller and Orr (1966) found that the typical pattern of cash management by firms is somewhat complex, with a cash balance that irregularly fluctuates in both directions. As such, they developed an analytical model, which added this varying cash balance in business operations to the aspect of transfer cost in the established Baumol model. Orr (1974) further argued the merits of this model over Sprenkle's (1969) premise of the "uselessness of transaction demand models". Orr found that not only was Sprenkle's premise unsupported by results but also that his heavy reliance on compensating balances was misplaced.

Frenkel and Jovanovic (1980) developed a theory based on the work of Baumol (1952), Tobin (1956), and Miller and Orr (1966). They created a stochastic framework to analyze transactions and the precautionary demand for money. At about the same time, Tapiero and Zuckerman (1980) presented a new analytical solution to cash management problems of firms when Compound Poisson processes describe cash income and demand. By generalizing past results in cash management literature to arbitrary income and demand distribution functions, they created a system that can be used in banking.

Cash management in typical businesses was considered by Premachandra (2003) to exist in a "two-asset" setting consisting of both the firm's cash balance and other assets such as treasury bills, commercial papers, etc. From time to time, they found that firms would have to transfer money from one asset account to another by buying or selling securities. The results, they argued, showed a model superior to the Miller-Orr model.

Another alternative method to the Miller-Orr model came from da Costa Moraes and Nagano (2012). They Applied Genetic Algorithms (AGA) and Particular Swarm Optimization (PSO) to cash balance management in conjunction with the aforementioned model. They found that the application of these methods could in fact determine cash level from the lower limit, with the PSO yielding the best results.

In recent years, studies have examined how external factors have affected the demand for money by a firm. Khurana, Martin and Pereira (2006) investigated the influence of financial development on the demand for liquidity by focusing on the sensitivity of a firm's "cash holdings to their cash flows" (Khurana, Martin, & Pereira, 2006, p. 787). They found that this sensitivity decreases with financial development.

Bates, Kahle and Stulz (2009) examined whether agency conflicts for U.S. industrial firms played an important role in doubling the average cash-to-assets ratio for these firms during the years 1980 to 2006. Their research did not yield any consistent evidence showing that they did.

Amromin and Chakravorti (2009), showed that despite the introduction, acceptance and usage of cash alternatives, cash remains significant for businesses and households.

2.2 Private Demand for Money

We described, above, the demand for cash by corporations. The demand for money by private individuals and consumers is more complicated. While Baumol's approach suggests a logical approach to spending, people often seem to behave otherwise. For example, they make many cash withdrawals despite the cost of each transaction, and hold less cash in their pockets than might be expected. Researchers in this field question whether such behavior is irrational. More specifically, they ask which factors lead to such apparently "inexplicable" actions.

One such factoris the role of liquidity in consumption behavior. In this regard, Kalckereuth et al., (2014), focused on and emphasized the special characteristics of cash that although not reflected in standard transaction cost measures are nevertheless valued by consumers, "Cash contains memory—the amount spent and the remaining budget can easily be gathered by a glance into one's pocket" (p. 1779). The authors suggest that using cash is a simple device for monitoring liquidity and abudget either due to the high costs of overdraft or the need to avoid overspending.

Zellner, Huang and Chau (1965) explored in greater detail the short-run consumption function. Their study results explained the role of liquid assets in determining consumption expenditures. The data supported the hypothesis that imbalances in consumer liquid asset holdings exert a statistically and economically significant influence on consumption expenditures. This served as important evidence that monetary variables may both directly affect consumer expenditure and indirectly affect it through interest rates.

Pissarides (1978) provided further evidence for this when he proposed a model that extended the standard wealth theory of consumption to include liquidity effects, and arrived at several important conclusions. First, even an individual who is aware of the role of liquidity when formulating a plan of consumption may still be constrained by it, as well as by lifetime wealth, in the Implementation of the plan. Furthermore, Pissarides found that individuals who could purchase and sell assets with different liquidities had a chosen consumption-wealth ratio that was dependent upon the time of income payments. This remained true even with lifetime utility maximization and homothetic practices. As such, the individual's concept of wealth used in determining optimal consumption is not the objective market measure as expressed by wealth, but is dependent, instead, on subjective actions.

Liquidity was also determined to play a role in borrowing for consumption. Deaton (1991) examined the behavior of consumers who had restricted ability to borrow for consumption. Specifically, he wanted to see whether the theory that such individuals would save accounted for some of what Deaton named "stylized facts of saving behavior" (Deaton, 1991, p. 1221). He found that although many consumers perceive steady growth of incomes, and maintain smooth consumption patterns, it is unclear why they nevertheless do not borrow money early in life. He concluded that it was likely due to borrowing limitations. In a growing economy, as opposed to astationaryone, such borrowing is affected by liquidity constraints.

Similarly Patterson (1991) found significance in liquid assets as well. He applied a nonparametric approach to assess whether a data set comprising nineteen consumption goods, four liquid assets and leisure is consistent with the utility maximization model. He found that while the consumption of goods and services alone are insufficient, when liquid assets and leisure are added to the set, the data is consistent with utility maximization. Thus Patterson found that the nonparametric approach offered a range of useful techniques in addition to existing quantitative tools for applied consumption research.

In addition to these findings, Ludvigson (1999) investigated the role of consumer credit in determining real consumption growth in the aggregate, in post war United States. He presented data suggesting that growth in consumer credit is significantly related to growth in consumption. This finding was a departure from previous research and predictions, including: 1) the permanent income/life cycle hypothesis; 2) the "rule of thumb" models where some consumers simply consume their current income; and 3) the models of liquidity constraint in which individuals face a fixed borrowing limit. Given his findings, Ludvigson advocated the reinterpretation of excessive sensitivity of consumption to current resources. Moreover, he suggested a new model of liquidity constraints in which the borrowing limit varies with time and is dependent on current income.

An opposing view was presented by Carroll (2001), who argued that in many cases, the effects of precautionary saving and liquidity constraints are virtually indistinguishable. As such, Milton Friedman's (1957) original intuitive description of the Permanent Income Hypothesis still explains the modern consumption model that had emerged over the preceding fifteen years, even though the model did not include liquidity constraints. However, in a model in which impatient consumers faced serious and uninsurable labor income risks, the Friedman model could explain high marginal propensity to consume from windfalls, high discount rates on future labor income, and the importance of precautionary behavior better than subsequent perfect foresight or certainty equivalent models.

Behavioral economists (Whalen, 1966), have built and expanded on the idea of precautionary behavior. Rather than using liquidity to explain impulse buying and other irrational behavior, these economists believed that emotional reactions played an important role. Rook and Hoch (1985) outlined a psychological model of consumer impulse buying. In their analysis, they distinguished five crucial elements of impulse buying. Using interviews, they were able to identify distinctive emotional and psychological elements that characterize the prototypic impulse buying episode including spontaneous urges to consume, inner dialogues, impulse persistence and power and product emanations. According to their conclusions, impulse buying is an emotional response.

In continuation to these findings, Gross and Souleles (2000) examined how people respond to changes in credit card supply. They found that increases in credit limits generated rise in debt. This result is consistent with conventional theory. Unlike other studies, however, they found that changes in account-specific interest rates also yielded strong effects. Ultimately, the results implied that the consumer himself and more directly his behavior could play an important role in the transmission of monetary policy and what Gross and Souleles called other credit shocks.

Similarly, Saleh (2012) investigated the relationship between unplanned buying and post-purchase regret influenced by consumer-family income and gender in the Saudi market. He also explored the association of both sales promotion and credit card payment with unplanned buying in the same marketplace. In an analysis of 903 respondents, he found that there was a positive relationship between unplanned buying and post-purchase regret. This was especially true among low-income earners and male consumers. Furthermore, he found that credit card payments rather than sales promotion had a significant connection with unplanned buying. As such, Saleh hypothesized that both consumers and marketers could avoid post-purchase regret through consumer awareness of the factors that promote it and through marketer perception of these factors in setting strategies to satisfy and retain long-term customers.

Yun, Xie and Lei (2011) introduced a different conclusion. They studied the difference in "mental satisfaction" between paying with a credit card or with cash and whether it had any effect on consumer behavior. Using questionnaires, they found that consumers believe the two payment options differ in both function and payment. However, this difference did not yield significant variance in consumer price sensitivity and impulsive buying according to which payment method is used.

Consumers did prefer, however, to buy non-essential goods with credit cards and essential goods with cash.

In addition, Karbasivar and Yarahmadi (2011) show that there are more than emotional reactions involved. They analyze consumer impulse buying using window display, credit card, and promotional activities (cash discounts and free products). Using a survey format and a small sample size (n=275), they determine an essential relationship between the four external cues mentioned above and consumer impulse buying behavior. Their research suggests that sellers and marketers should have ATMs in their shops and advertise them to consumers. Furthermore, by offering complementary products and decorating their stores in modern styles with attractive lights and colors, marketers can encourage the consumer buying impulse. (For further and most recent studies on impulsive buying behavior see also Rizwan, Vishnu, & Muhammad, 2014; Harwani & Singh, 2014; Moayery, Zamani, & Vazlfehoost, 2014).

The discussion, above, presented grounds for further quantitative and qualitative research methods, given the idea that impulsive buying is strongly related to emotional reactions and behavior.

Let us return to the consumers who stand more often in the line at the bank and withdraw small amounts of money. Why do they adopt these practices and are they behaving "appropriately"? The answer may be positive due to the following considerations:

Availability of more liquid money in the hands of a consumer creates an illusion that he is wealthy and "strong". This encourages him to purchase more quickly. Rapid spending that is referred to in some earlier studies as impulsive purchasing, leads to inefficient use of money over time. Less money in his pocket leads the consumer to spend money less spontaneously and more carefully. Thus, the real benefit from the same amount of annual spending is that it eventually generates higher satisfaction for him.

The gap between a good and efficient purchase and a bad one becomes greater as the average amount of money available to the consumer increases.

3. The Expanded Baumol Approach

Each time that a consumer performs a transaction of Y dollars in nominal value, he is required to retain a certain amount of liquid assets. He also differentiates between the nominal value and the real value of actual transactions, *U*. According to the approach of behavioral economics, a gap exists between how much the consumer spends in dollar terms, *Y*, and the value he "subjectively" gains in real terms, *U*. We may define this gap as a dollar loss, *L*, due to the "spontaneous and impulsive" nature of the purchase. The loss, *L*, is positively affected by the availability of liquidity, *m*, to the consumer. If money or any other liquid asset is not immediately accessible, the gap between the dollar amount of *Y* and *U* is minimized. Thus the loss, *L*, may be diminished. We may assume further that if no money is held then there is no loss and therefore *L* approaches zero.

In our model we define γ coefficients that represent real value loss from purchase due to the availability of cash. In contrast, an individual may hold liquid assets for the sake of fast deals that require quick and immediate availability of cash in order to make a profit of δ. This advantage of holding money, δ, should be compared with the possibility that the money could be used for hasty purchases that may not be well considered. Therefore a loss could result from available liquidity. Either contradictory possibility might occur due to high liquidity.

Thus, we can define the net loss function as positively affected by the available cash m_1 as follows:

$$L = \gamma(m_1) - \delta m_1$$

The real value of a nominal transaction, *Y*, at a given period is *U*, when

$U = Y - L = Y - [\gamma(m_1) - \delta m_1]$

More liquidity is required for other regular purchases for which consumers hold cash, m_1, (the most liquid assetand thus free of yield). If the liquidity is in the form of cash, then no interest is paid and the entire interest of r_M., on m_1 is defined asthe cost of holding cash.

In addition people may hold a less liquid asset of m_2 . m_2 can be defined as a balance the consumer holds in his demand deposit account. m_2 might have a very low interest rate of r_2. This rate is lower than r_M that is acceptable for non-liquid financial assets such as long-term savings deposits. Thus, each withdrawal that the consumer makes is in the total amount of m, from non-liquid financial assets, $m = (m_1 + m_2)$. m_1 is available cash and m_2 is deposited into a checking account for the purpose of the transaction motive. The consumer has to define the total amount of m and to decide upon the optimal allocation between m_1 and m_2 ; between cash (which is the most liquid asset) consisting of coins and currency in his pocket and a less liquid asset of demand deposit. Based on the above, we write the objective function of costs minimization as follows:

$$(1)\ Min_{m_1,m_2} TC = r_M \cdot \frac{m_1}{2} + (r_M - r_2) \cdot \frac{m_2}{2} + \frac{bY}{m_1+m_2} - [\gamma(m_1) - \delta]m_1$$

The original Baumol approach minimizes the holding cost with respect to two components: (a) the cost of each liquid (cash) withdrawal; and (b) the interest payment cost of holding on an average $\frac{m}{2}$ dollars in yield free assets. In contrast, we modify the original approach with the following two additional elements that more closely reflect actual practice:

1) We allow for an intermediate liquid asset, m_2 , with some positive yield. The yield is less than would be acceptable for a non-liquid asset, r_M , but not necessarily zero as when cash is held.

2) By holding m_2 we do not lose all yield, but we also reduce the loss L, since to a certain degree m_1 is substituted with m_2 .

We want to investigate the function of $\gamma(m_1)$ that, as defined above, represents the loss of real value of a transaction due to cash availability and the waste of easy and accessible money by spontaneous purchasing. We can define γ as a constant but also negative function of m_1 such as $\gamma = \gamma_0 - \varepsilon m_1$ (as in Case I, below) or $\gamma = \gamma_0 - \varepsilon\, ln\ m_1$ (as in Case II, below).

The same can be assumed regarding $\delta\,(m_1)$. However, for simplicity we assume that δ is constant.

Case 1

First we will discuss the case in which γ is assumed to diminish as a linear function of m_1 as follows: $\gamma(m_1) = \gamma_0 - \varepsilon m_1$

Thus the objective function is as follows:

$$(1)\qquad Min_{m_1,m_2} TC = r_M \cdot \frac{m_1}{2} + (r_M - r_2) \cdot \frac{m_2}{2} + \frac{bY}{m_1+m_2} - (\gamma_0 - \varepsilon m_1)m_1 + \delta m_1$$

The First Order Conditions (F.O.C.) are:

$$(2)\quad \frac{\partial TC}{\partial m_1} = \frac{\mathrm{r_M}}{2} - \frac{b \cdot Y}{(m_1+m_2)^2} - (\gamma_0 - \delta) + 2\varepsilon m_1 = 0$$

and

$$(3)\quad \frac{\partial TC}{\partial m_2} = \frac{\mathrm{r_M} - r_2}{2} - \frac{b \cdot Y}{(m_1+m_2)^2} = 0$$

From (3) we find that since $m_1 + m_2 = m$, the total demand for liquid assets:

(4) $$m = \left(\frac{2bY}{\mathrm{r_M} - r_2} \right)^{1/2}$$

From (2) and (4) we find that

(5) $$\frac{\mathrm{r_M}}{2} - \frac{\mathrm{r_M} - r_2}{2} - (\gamma_0 - \delta) + 2\varepsilon m_1 = 0$$

Alternatively we can define $\boldsymbol{m_1}$, the optimal withdrawal amount of liquid cash that the consumer makes in each transaction cycle

(6) $$m_1 = \frac{(\gamma_0 - \delta)}{2\varepsilon} - \frac{r_2}{4\varepsilon}$$

From (4) and (6) we also find for each transaction the optimal amount that the consumer leaves in his checking account $\boldsymbol{m_2}$ that yields $\boldsymbol{r_2}$.

(7) $$m_2 = \left(\frac{2bY}{r_M - r_2} \right)^{1/2} - \frac{(\gamma_0 - \delta)}{2\varepsilon} + \frac{r_2}{4\varepsilon}$$

The discussion, above, illustrates the modified Baumol demand for total liquid assets with different degrees of liquidity. We can identify not only the total liquid assets that the consumer withdraws at each transaction, *m*, but also the allocation between a very liquid asset, i.e., cash $\boldsymbol{m_1}$, that has no return at all, and a less liquid and available means of payment (such as a checking account balance with very low positive yield of r_2), $\boldsymbol{m_2}$.

The monetary amount as a function of r_2 are presented in Table 1 for the following parameters values: δ=30, γ=32.5, ε=0.5, rm=20%, Y=1000, b=0.3.

Table 1. Case 1—Monetary amount as a function of r_2

			Case 1		
r_2	*M1*	*M2*	*M*	*M1%*	*M2%*
10.0%	2.45	75.01	77.46	3.2%	96.8%
10.5%	2.45	77.02	79.47	3.1%	96.9%
11.0%	2.45	79.20	81.65	3.0%	97.0%
11.5%	2.44	81.57	84.02	2.9%	97.1%
12.0%	2.44	84.16	86.60	2.8%	97.2%
12.5%	2.44	87.01	89.44	2.7%	97.3%
13.0%	2.44	90.15	92.58	2.6%	97.4%
13.5%	2.43	93.64	96.08	2.5%	97.5%
14.0%	2.43	97.57	100.00	2.4%	97.6%
14.5%	2.43	102.02	104.45	2.3%	97.7%
15.0%	2.43	107.12	109.54	2.2%	97.8%
15.5%	2.42	113.05	115.47	2.1%	97.9%
16.0%	2.42	120.05	122.47	2.0%	98.0%

16.5%	2.42	128.51	130.93	1.8%	98.2%
17.0%	2.42	139.01	141.42	1.7%	98.3%
17.5%	2.41	152.51	154.92	1.6%	98.4%
18.0%	2.41	170.80	173.21	1.4%	98.6%

In Figure 1 we demonstrate that the values of $\boldsymbol{m_1}$, $\boldsymbol{m_2}$ and m are influenced by r_2 .

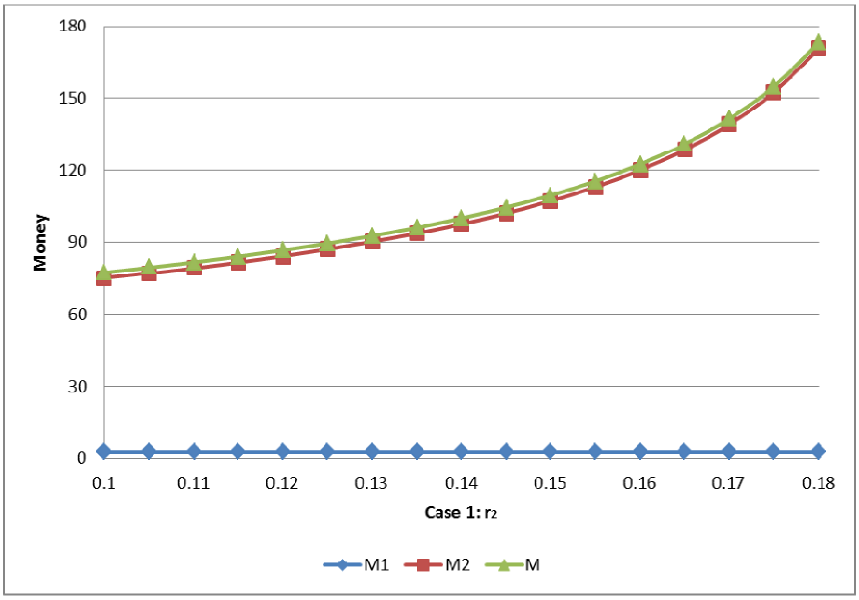

Figure 1. Case I

The next section presents another modified demand function for money with a different shape of $\boldsymbol{\gamma}$.

Case 2

In this case we introduce different shapes of a diminishing function of $\boldsymbol{\gamma}$ with respect to m_1 when $\boldsymbol{\gamma}$ is not diminishing linearly as follows:

$$\boldsymbol{\gamma = \gamma_0 - \varepsilon \ln m_1}$$

Again our goal is to minimize costs of the objective function (1) above:

$$(8)\quad \boldsymbol{Min_{m_1,m_2} TC = r_M \cdot \frac{m_1}{2} + (r_M - r_2) \cdot \frac{m_2}{2} + \frac{bY}{m_1+m_2} - (\gamma_0 - \varepsilon ln m_1) m_1 + \delta m_1}$$

In this case the F.O.C. are:

$$(9)\quad \frac{\partial TC}{\partial m_1} = \frac{r_M}{2} - \frac{b \cdot Y}{(m_1 + m_2)^2} - (\gamma_0 - \delta) + \varepsilon\left(\ln m_1 + \frac{m_1}{m_1}\right) = 0$$

and

$$(10)\quad \frac{\partial TC}{\partial m_2} = \frac{r_M - r_2}{2} - \frac{b\,Y}{(m_1 + m_2)^2} = 0$$

From (10) we determine the optimal total withdrawal amount, *m*, at each transaction:

(11) $\boldsymbol{m} = \left(\frac{2bY}{r_M - r_2}\right)^{1/2}$

That is similar to Case 1, above. (See equation (4))

From (9), (10) and (11) we conclude (12):

(12) $\frac{\mathrm{r_M}}{2} - \frac{\mathrm{r_M} - r_2}{2} - (\gamma_0 - \delta) + \varepsilon(\ln m_1 + 1) = 0$

From (12) we get the optimal cash money withdrawal by the consumer at each transaction.

(13) $\boldsymbol{m_1} = \boldsymbol{e}^{\left(\frac{\gamma_0 - \delta}{\varepsilon} - \frac{r_2}{2\varepsilon} - 1\right)}$

From (11) and (13) we get (14), the optimal amount that is held in demand deposit at each withdrawal transaction.

(14) $\boldsymbol{m_2} = \left(\frac{2bY}{r_M - r_2}\right)^{\frac{1}{2}} - \boldsymbol{e}^{\left(\frac{\gamma_0 - \delta}{\varepsilon} - \frac{r_2}{2\varepsilon} - 1\right)}$

Again, as in the previous case, from (11), (13) and (14) we conclude the optimal total amount of money withdrawn at each transaction $\boldsymbol{m_1}$ and the internal allocation between more liquid assets (cash), $\boldsymbol{m_1}$, and less liquid assets (demand deposit), $\boldsymbol{m_2}$.

The monetary amount as a function of r_2 are presented in Table 2 for the following parameters values: δ=30, γ=32.5, ε=0.5, rm=20%, Y=1000, b=0.3.

Table 2. Case 2—Monetary amount as a function of r_2

	Case 2				
r_2	*M1*	*M2*	*M*	*M1%*	*M2%*
10.0%	49.40	28.06	77.46	63.8%	36.2%
10.5%	49.16	30.32	79.47	61.9%	38.1%
11.0%	48.91	32.74	81.65	59.9%	40.1%
11.5%	48.67	35.35	84.02	57.9%	42.1%
12.0%	48.42	38.18	86.60	55.9%	44.1%
12.5%	48.18	41.26	89.44	53.9%	46.1%
13.0%	47.94	44.64	92.58	51.8%	48.2%
13.5%	47.70	48.37	96.08	49.7%	50.3%
14.0%	47.47	52.53	100.00	47.5%	52.5%
14.5%	47.23	57.22	104.45	45.2%	54.8%
15.0%	46.99	62.55	109.54	42.9%	57.1%
15.5%	46.76	68.71	115.47	40.5%	59.5%
16.0%	46.53	75.95	122.47	38.0%	62.0%
16.5%	46.29	84.64	130.93	35.4%	64.6%
17.0%	46.06	95.36	141.42	32.6%	67.4%

17.5%	45.83	109.09	154.92	29.6%	70.4%
18.0%	45.60	127.60	173.21	26.3%	73.7%

In Figure 2, we demonstrate the values of $\boldsymbol{m_1}$, $\boldsymbol{m_2}$ and m that are influenced by

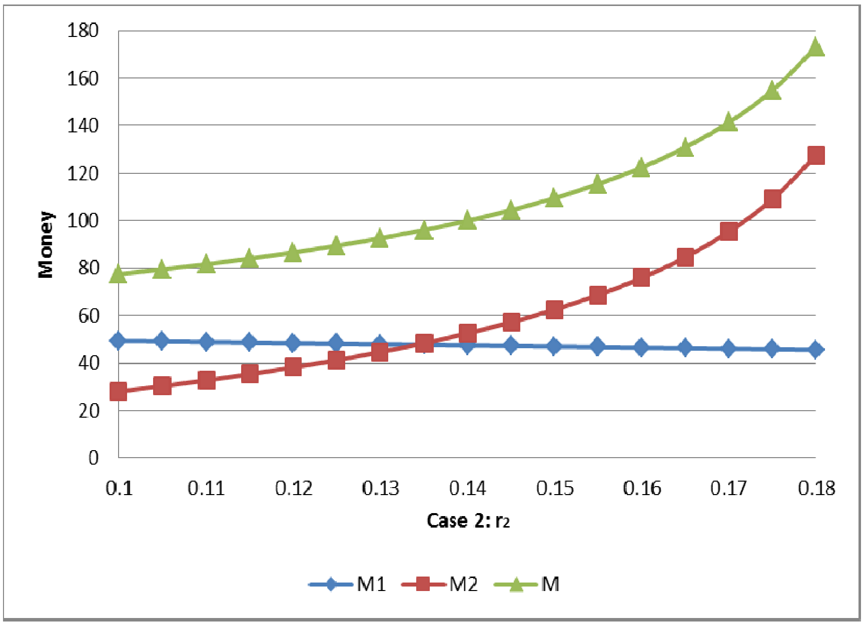

Figure 2. Case II

Using the equations (11), (13) and (14) of different $\boldsymbol{\gamma}$ functions as opposed to equations (4), (6) and (7) above in Case II and Case I, respectively, leads to several immediate propositions.

$\boldsymbol{m_I = m_{II} = \left(\frac{2bY}{r_m - r_2}\right)^{1/2}}$. The total liquid assets $\boldsymbol{m}$ withdrawn at each transaction is not affected by $\boldsymbol{\gamma}$ and is equal in both cases. We state in the proposition following Proposition 1, below: Regardless of the "deterioration rate" of the $\boldsymbol{\gamma}$ function the total withdrawal amount is the same. It is a positive function of the cost per transaction, b, as well as of the total purchase Y. Total money withdrawn, m, is a negative function of the gap between interest rates on savings deposits and demand deposits.

What is the ratio between $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ within each deterioration method of $\boldsymbol{\gamma}$?

Proposition 1: Since in Case 2 the deterioration starts in an accelerating shape, it leads to holding more liquid cash than in the case of linear deterioration. Therefore

$m_{1,II} > m_{1,I}$, thus, $m_{2,II} < m_{2,I}$. See the proof in the Appendix.

Comparative Static Analysis

In this section we intend to verify how the change in r_M affects the value of liquid withdrawal and the structure of the liquid assets. Equation (4), above, may be rewritten as:

$$(4) \quad m = \left(\frac{2bY}{\mathrm{r_M} - r_2}\right)^{1/2}$$

Thus $\frac{\partial m}{\partial r_2} = (2bY)^{\frac{1}{2}}\left(-\frac{1}{2}\right)(r_M - r_2)^{-\frac{3}{2}}(-1) = \frac{bY^{\frac{1}{2}}}{(r_M - r_2)^{\frac{3}{2}}} > 0$

For a higher interest rate for demand deposit it is worthwhile holding more liquid assets available in order to gain several benefits: A consumer saves transaction costs with fewer withdrawals, but he also holds more liquid assets that are not necessarily cash. Thus he can reduce losses arising from spontaneous purchasing by holding more m_2. This is shown from derivatives of (6) and (7) with respect to r_2 as follows:

(15) $\frac{\partial m_1}{\partial r_2} = -\frac{1}{4\varepsilon} < 0$

Thus,

(16) $\frac{\partial m_2}{\partial r_2} = \frac{\partial m}{\partial r_2} - \frac{\partial m_1}{\partial r_2} = \frac{bY^{\frac{1}{2}}}{(r_M - r_2)^{\frac{3}{2}}} + \frac{1}{4\varepsilon} > \frac{\partial m}{\partial r_2} > 0$

m_2 is increasing by more than the reduction inm_1. It is an increase beyond the increase in r_2. This indicates that the increase in r_2 leads to an increase in total m, that is held by consumers. The change in r_2 has two effects on the tendency to hold liquid assets. The increase in r_2 reduces the price of holding liquidity. Therefore the total demand for money, m, increases. In contrast, the price of holding cash m_1, increases so that the consumer holds more liquid assets for the sake of easier transactions but reduces his tendency to hold cash for spontaneous purchases.

4. Conclusions and Implications

From Case Iwe find the modified Baumol demand for liquid assets with different degrees of liquidity. Assuming that the devaluation of real value of any transaction is diminishing as a linear function of m_1 we determine the total liquid assets the consumer withdraws at each transaction, m, and the division between a very liquid asset (cash) that has no return, m_1, and a less liquid but still available mean of payment (such as a checking account balance with a low yet positive yield), m_2. We find that coefficient γ has an effect on the division between m_1 and m_2 but no influence on *m*.

We may conclude that a higher value of γ very significantly encourages the tendency of customers to avoid, as much as possible, holding the more liquid cash assets.

Sometime δ positively affects U. For example, a retailer may get good offers from a producer for quick and profitable purchase for cash transactions in the market. Retailers are thereby encouraged to hold large amounts of m_1 and to profit from spontaneous purchasing. Yet, more often the opposite holds true. The modified Baumol function allows for an intermediate liquid asset, m_2, with some positive yield. The yield is less than what would be acceptable for a non-liquid asset, r_M, but is not necessarily zero, as when cash is held.

The modified version redefines the original Baumol equation as:

$$m^* = \left(\frac{2bY}{r_m - r_2}\right)^{1/2}.$$

In Case II we developed another modified demand function for money with different influences of γ on m_1 and m_2. Case II differs from Case I in the assumption that γ, although diminishing (as in case I), is not a linear function of m_1. From Case II we can conclude that regardless of the "deterioration rate" of the γ function, the total money withdrawn is the same. It is a positive function of the cost per transaction, b, as well as of the total purchase *Y*. *m* is a negative function of the gap between interest rates on savings deposits and demand deposits, regardless of the characteristics of the γ function.

Regarding the internal division between m_1 and m_2 within each shape of the γfunction, we can conclude that m_1 mildly increases while m_2 decreases more sharply in Case II than in Case I. See Figure 2 in comparison to Figure 1.

In Case II at low levels of money coefficients differences (γ-δ), the money demand function is rigid. At high levels of money coefficients differences (γ-δ) the money demand function is flexible. The trend is reversed when considering the interest rate differentials. At low levels of interest rate differentials the money demand function is flexible, while at high levels of interest rate differentials the money demand function is rigid. See Figure 2. This conclusion may be implemented in government policy aimed at impacting the demand for money, either by changing interest rates or by investing in financial management education. Such policies affect the money coefficients and can be of greater influence when γ is a concave function in comparison to a linear function.

In Case I the interest rate differentials and the money coefficients behave similarly.

Further expansion of the present study could be directed to the formulation of a general model in which liquid assets can be categorized according to degree of liquidity. Assets with decreasing liquidity levels guarantee higher yield and reduce the impulsive purchasing effect, but also restrict the ability to conduct immediate, valuable and efficient transactions.

Worthy of further discussion is the investigation of the accurate relationship between yields and degrees of liquidity that may not only minimize the cost of holding liquid assets for efficient and valuable transactions, but also eliminate or at least reduce spontaneous transactions.

References

Amir, F. Z., Syed, F., Qureshi, A. N., Akbar, S. J., & Amin, M. F. (2014). Impulsive Buying Behavior of Women in Apparel Industry in Pakistan. *International Journal of Management Sciences*, *2*(7), 296-314. http://search.proquest.com/docview/1561350548?accountid=40023

Amromin, G., & Chakravorti, S. (2009). Whither Loose Change? The Diminishing Demand for Small-Denomination Currency. *Journal of Money*, *Credit and Banking*, *41*(2-3), 315-335. http://dx.doi.org/10.1111/j.1538-4616.2009.00207.x

Badgaiyan, A. J., & Verma, A. (2014). Intrinsic Factors Affecting Impulsive Buying Behavior—Evidence from India. *Journal of Retailing and Consumer Services*, *21*, 537-549. http://dx.doi.org/10.1016/j.jretconser.2014.04.003

Bates, T. W., Kahle, K. M., & Stulz, R. M. (2009). Why do U.S. Firms Hold So Much More Cash than They Used to? *The Journal of Finance*, *64*, 1985-2021. http://dx.doi.org/10.1111/j.1540-6261.2009.01492.x

Baumol, W. J. (1952). The Transactions Demand for Cash: An Inventory Theoretic Approach. *Quarterly Journal of Economics*, *66*, 545-556. http://dx.doi.org/10.2307/1882104

Carroll, C. D. (2001). A Theory of the Consumption Function, With and Without Liquidity Constraints. *Journal of Economic Perspectives*, *15*, 23-45. http://dx.doi.org/10.1257/jep.15.3.23

Chang, J. J., Chang, W. Y., Chang, L., Ching, C., & Wang, P. W. (2007). Equilibrium Dynamics in an Endogenous Growth Model of Money and Banking. *Journal of Money*, *Credit and Banking*, *39*(7), 1683-1710. http://dx.doi.org/10.1111/j.1538-4616.2007.00083.x

Da Costa, M., Marcelo, B., & Nagano, M. S. (2012). Cash Balance Management: A Comparison Between Genetic Algorithms and Particle Swarm Optimization. *Acta Scientiarum Techonology*, *34*, 373-379. http://dx.doi.org/10.4025/actascitechnol.v34i4.12194

Deaton, A. (1991). Saving and Liquidity Constraints. *Econometrica*, *59*, 1221-1248. http://dx.doi.org/10.1007/s00181-012-0677-y

Frenkel, J. A., & Jovanovic, B. (1980). On Transactions and Precautionary Demand for Money. *The Quarterly Journal of Economics*, *95*, 25-43. http://dx.doi.org/10.2307/1885347

Friedman, M. (1957). The Relation between the Permanent Income and Relative Income Hypotheses. In *A Theory of the Consumption Function* (pp. 157-182). National Bureau of Economic Research, Inc. Retrieved from http://www.jstor.org/10.1016/j.jedc.2011.03.012

Friedman, M. (1959). Testimony in Hearings before the Joint Economic Committee, part 4—The influence on prices of changes in the effective supply of money. *U.S. Government Printing Office*, 605-637.

Gonen, L. D., Weber, M., Tavor, T., & Spiegel, U. (2015). *A Modified Baumol Approach: Optimal Holding of Money* (Unpublished).

Gross, D., & Souleles, N. (2000). Consumer Response to Changes in Credit Supply: Evidence from Credit Card Data. *The Wharton Financial Institutions Center, Working Paper Series*, *1-10*, 1-23.

Harwani, S. V., & Singh, S. (2014). An Insight on Consumer Impulse Buying Behavior. *International Journal for Administration in Management, Commerce and Economics*, *2*, 124-130. http://dx.doi.org/10.1057/bm.2010.32

Kalckreuth, U. V., Schmidt, T., & Stix, H. (2014). Using Cash to Monitor Liquidity: Implications for Payments, Currency Demand, and Withdrawal Behavior. *Journal of Money, Credit and Banking*, *46*(8), 1753-1785. http://dx.doi.org/10.1111/jmcb.12165

Karbasivar, A. Y. H. (2011). Evaluating Effective Factors on Consumer Impulse Buying Behavior. *Asian Journal of Business Management Studies*, *2*, 174-181. http://dx.doi.org/10.1108/EJM-03-2011-0110

Khurana, I. K., Martin, X., & Pereira, R. (2006). Financial Development and the Cash Flow Sensitivity of Cash. *Journal of Financial and Quantitative Analysis*, *41*, 787-807. http://dx.doi.org/10.1017/S0022109000002647

Krueger, M. (2012). Money: A Market Microstructure Approach. *Journal of Money, Credit and Banking*, *44*(6), 1245-1258. http://dx.doi.org/10.1111/j.1538-4616.2012.00530.x

Ludvigson, S. (1999). Consumption and Credit: A Model of Time-Varying Liquidity Constraints. *The Review of Economics and Statistics*, *81*, 434-447. http://dx.doi.org/10.1162/003465399558364

Miller, M. H., & Orr, D. (1966). A Model of the Demand for Money by Firms. *The Quarterly Journal of Economics*, *80*, 413-435. http://dx.doi.org/10.2307/1880728

Moayery, M., Zamani, S., & Vazlfehoost, H. (2014). Effect of Visual Merchandising on Apparel Impulse Buying Behaviors among Iranian Young Adult Females. *Indian Journal of Science and Technology*, *7*(3), 360-366. http://search.proquest.com/docview/1525959276?accountid=40023

Orr, D. (1974). A Note on the Uselessness of Transaction Demand Models. *The Journal of Finance*, *29*, 1565-1572. http://dx.doi.org/10.1111/j.1540-6261.1974.tb03137.x

Patterson, K. D. (1991). A Non-Parametric Analysis of Personal Sector Decisions on Consumption, Liquid Assets and Leisure. *The Economic Journal*, *101*, 1103-1116. http://dx.doi.org/10.2307/2234429

Pissarides, C. A. (1978). Liquidity Consideration in the Theory of Consumption. *The Quarterly Journal of Economics*, *92*, 279-296. http://dx.doi.org/10.2307/1884163

Premachandra, I. M. (2004). A Diffusion Approximation Model for Managing Cash in Firms: An Alternative Approach to the Miller-Orr Model. *European Journal of Operational Research*, *157*, 218-226. http://dx.doi.org/10.1016/S0377-2217(03)00109-7

Rizwan, R. A., Vishnu, P., & Muhammad, A. A. (2014). Impact of Product Packaging on Consumer's Buying Behavior. *European Journal of Scientific Research*, *120*(2), 145-157.

Rook, D. W., & Hoch, S. J. (1985). Consuming Impulses. *Advances in Consumer Research*, *12*, 23-27.

Saleh, M. A. H. (2012). An Investigation of the Relationship Between Unplanned Buying and Post-Purchase Regret. *International Journal of Marketing Studies*, *4*, 106-120. http://dx.doi.org/10.5539/ijms.v4n4p106

Sprenkle, C. M. (1969). The Uselessness of Transaction Demand Models. *Journal of Finance*, 835-847. http://dx.doi.org/10.1111/j.1540-6261.1969.tb01696.x

Tapiero, C. S., & Zuckerman, D. (1980). A Note on the Optimal Control of a Cash Balance Problem. *Journal of Banking and Finance*, *4*, 345-352. http://dx.doi.org/10.1016/0378-4266(80)90013-8

Tobin, J. (1956) The Interest-Elasticity of Transactions Demand for Cash. *Review of Economics and Statistics*, *38*(3), 241-247. http://dx.doi.org/10.2307/1925776

Whalen, E. L. (1966). A Rationalization of the Precautionary Demand for Cash. *The Quarterly Journal of Economics*, *80*(2), 314-324. http://dx.doi.org/10.2307/1880695

Xiao, Z., Xie, Y. Q., & Ming, L. (2011). A Comparative Analysis of Consumer Behavior with Payments of Shopping Cards and Cash. *Beijing Information Science & Technology*, *School of Economics and Management*, 539-544.

Zellner, A., Huang, D. S., & Chau, L. C. (1965). Further Analysis of the Short-Run Consumption Function with Emphasis on the Role of Liquid Assets. *Econometrica*, *33*, 571-581. http://dx.doi.org/10.2307/1911752

Note

Note 1. Gender, age, marital status, number of children, education may also differently affect impulsive and compulsive purchasing.

Appendix

Let us denote:

$$m_{1,\mathrm{I}} \equiv X$$

$$m_{1,\mathrm{II}} \equiv e^{(2X-1)}$$

Thus, $X \underset{<}{\overset{>}{-}} e^{(2X-1)}$

For each X: $\ln X < 2X - 1$

That is, $m_{1,\mathrm{II}} > m_{1,\mathrm{I}}$ for each X

Therefore, $m_{2,\mathrm{II}} < m_{2,\mathrm{I}}$ for each X