

Damper with Porous Anisotropic Ring

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Abstract

The solution of a problem of coefficient of damper drive with a double-layer porous ring and squeeze film of lubrication is presented at its combined feed, and also the account of permeability of porous layers. Novelty of the solution is simultaneous introduction in analytical model of variety of variable factors which were considered separately earlier.

Keywords: Hydrodynamics, finite-dimensional damper, forced series lubricant feed, porous ring, permeability anisotropy in the radial and circumferential directions

1. Introduction

Damping vibrations of the various physical nature play an important role in increasing the general resource of bearing assemblies by decreasing the level of vibrations, transmitted to the body.

Damping effect is defined by the transmission coefficient value of the damper, which depends on the structure and the material of the last.

The considered damper represents the analogue of the journal bearing with the outer ring, made from porous sintered material, and the oil layer between this ring and the inner element of the damper. The liquid lubricant, the layer of which participates in damping, is fed into the running clearance of the damper under pressure while the consequent change of its feed direction from the radial into axial one. Besides, when solving this problem the permeability anisotropy of the damper porous bushing is taken into consideration.

In similar tasks, dedicated to the hydrodynamic designs of the radial journal bearings of the finite length with porous bushings (Cusano & Conri, 1974; Akhverdiev & Molenko, 2002; Akhverdiev, Mukutadze, Novgorodova, & Cherkasova, 2013; Cusano & Funk, 1977; Mukutadze, Aleksandrova, Konstantinov, & Shevchenko, 2012; Zadorozhnaya, 2015; Rozhdestvensky & Zadorozhnaya, 2014; Tolpinskaya, 1986; Rahmatajadi, 2010; Akhverdiev, Kochetova, & Mukutadze, 2009; Akhverdiev, Kopotun, & Mukutadze, 2007; Akhverdiev & Kopotun, 2005), their permeability is considered to be constant, and the feed lubricant direction is not taken into consideration. Such set of variable factors cannot provide the stable liquid mode.

The work is devoted to the design model development of the heterogeneous porous bearing of the finite length when forced lubricant feed is available (Akhverdiev, Mukutadze, Fleck, Zadorozhnaya, & Polyakov, 2013). Generalization of this task for cases, when the permeability is changing both in the radial and circumferential directions will allow solving the requested task.

So uniting into the common design complex of the factors, the indicated above increases the design models accuracy and approximates their results to the practice requirements.

2. Problem Setting and Solution

We use then solving the analogue in the damper and radial journal bearing with porous bushing operating conditions. Motion equations of the rotor for the shaft center nonstationary motion in the directions ξ and η (Figure 1) can be written as:

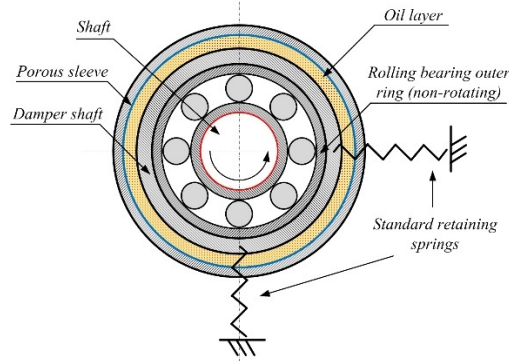


Figure 1. Damper design with the compressed oily film and porous cage

$$m \left[\frac{d^2 e}{dt^2} - e \left(\frac{d\phi}{dt} \right)^2 \right] = F_\xi - [W - K_y (Y + \delta_y)] \sin \phi - K_x (X + \delta_x) \cos \phi + u \omega^2 \cos (\phi - \omega t), \tag{1}$$

$$m \left[e \frac{d^2 \phi}{dt^2} + 2 \left(\frac{de}{dt} \right) \left(\frac{d\phi}{dt} \right) \right] = F_\eta - [W - K_y (Y + \delta_y)] \cos \phi + K_x (X + \delta_x) \sin \phi - u \omega^2 \sin (\phi - \omega t). \tag{2}$$

We assume the operating load W is static and directed in accordance with the design on the picture 2. The damper load causes initial displacements, defined by the formulas $\delta_x = 0$ and $\delta_y = W/K_y$. Having accepted that $K_x = K_y = K$, $\varepsilon = \dot{e}/C$, $T = \omega_r t$ and $X = e \cos \phi$, $Y = -e \sin \phi$, it is possible to present the equations (1) and (2), as follows:

$$\ddot{\varepsilon} - \varepsilon \dot{\phi}^2 = \frac{F_\xi}{mC\omega_r^2} - \frac{K \cdot \varepsilon}{m\omega_r^2} + \frac{u}{mC} \left(\frac{\omega}{\omega_r} \right)^2 \cos \left[\phi - \left(\frac{\omega}{\omega_r} \right) T \right], \tag{3}$$

$$\varepsilon \dot{\phi} + 2\dot{\varepsilon} \phi = \frac{F_\eta}{mC\omega_r^2} - \frac{u}{mC} \left(\frac{\omega}{\omega_r} \right)^2 \sin \left[\phi - \left(\frac{\omega}{\omega_r} \right) T \right]. \tag{4}$$

Force values F_ξ and F_η we get by pressure integration in the lubricant layer by the parameters ξ and η . With this aim it is necessary preliminary to solve the equation for pressures in the porous ring and the liquid oil layer, as well as agree these solutions by the boundary line.

It is necessary to note, that the considered damper (see Figure 1) represents itself the analogue of the radial bearing with the porous bushing. That's why at first it is considered the non-stationary laminar flow of the liquid lubricant while forced lubricant feedin the radial and axial directions.

The bearing with the heterogeneous porous layer is considered to be statical, and the shaft motion – preset (see Figure 2). The porous layer permeability is preset by the following dependence

$$k' = A_0 e^{k_1 \left(\frac{z}{L} \right) \frac{y}{H}}. \tag{5}$$

Here A_0 is preset constant value, $k_1(z/L)$ is known non-dimensional function, L is bearing length, H is porous layer thickness.

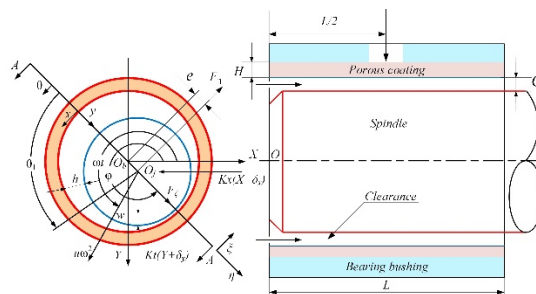


Figure 2. Finite radial bearing with porous ring

Further we will consider that the coordinate of the surface $y=-H$, the porous layer permeability in the direction of the axis z is changed by the normal law, and the lubricant feed pressure is subordinated to the parabolic relation.

Equation, defining the lubricant flow in the porous matrix, is represented by

$$\frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} + k_1 \left(\frac{z}{L} \right) \frac{1}{H} \frac{\partial p^*}{\partial y} + \frac{y}{H} \frac{\partial p^*}{\partial z} \frac{\partial k_1}{\partial z} = 0, \quad (6)$$

where, z are rectangular coordinates, p^* is hydrodynamic pressure in the porous layer.

The pressure in the lubricant layer (between the shaft and the bushing) is defined on the basis of Reynolds modified equation (Gear, 1972) within the limits of the short bearing model

$$\frac{\partial}{\partial z} \left(h^3 \frac{\partial p}{\partial z} \right) = 6\mu \left[\left(\omega_b + \omega_j - 2\omega_L - 2 \frac{d\Phi}{dt} \right) \frac{dh}{d\theta} + 2 \frac{de}{dt} \cos \theta \right] - 12\mu v_0 \Big|_{y=0}, \quad (7)$$

where $h = c(1+\varepsilon \cos \theta)$ is lubricant layer thickness;

c is radial clearance; x, y, z are rectangular coordinates;

e is shaft eccentricity; ω_b is bushing angular speed;

ω_j is shaft angular speed; ω_L is load angular speed;

v_0 is speed component indirection to y on the boundary between the porous bearing and the lubricant layer.

Further the load angular speeds and bearing bushing are accepted as equal to zero.

The value v_0 is subordinated to Darcy's law

$$v_0 = - \frac{k^*}{\mu} \left(\frac{\partial p^*}{\partial y} \right) \Big|_{y=0}, \quad (8)$$

Where k^* is porous layer material permeability.

We will proceed to the non-dimensional parameters by formulas

$$p^* = \frac{p^* C^2}{\mu R_0^2 \omega_j}, \quad P = \frac{p C^2}{\mu R_0^2 \omega_j}, \quad Z = \frac{2z}{L}, \quad Y = \frac{y}{H},$$

$$k^* = A_0 k, \quad k = e^{\frac{\beta Z^2}{4}}, \quad \Phi = \frac{A_0 H}{C^3}, \quad \tilde{p}_g = \frac{p_g C^2}{\mu R_0^2 \omega_j}, \quad \tilde{p}_a = \frac{p_a C^2}{\mu R_0^2 \omega_j}. \quad (9)$$

We will set the law of lubricant feed on the surface $Y = -1$, as well as porous layer permeability on this surface as

$$\tilde{p}_g = \tilde{p}_a + \tilde{p}_g (Z^2 - 1), \quad \tilde{p}_g = \text{const}, \quad k = e^{-\beta \frac{Z^2}{4}}. \quad (10)$$

Substituting (8)–(10) into the equations (6) and (7), we will get:

$$\frac{\partial^2 P^*}{\partial Y^2} + 4 \left(\frac{H}{L} \right)^2 \frac{\partial^2 P^*}{\partial Z^2} + \beta \frac{Z^2}{4} \frac{\partial P^*}{\partial Y} + \left(\frac{H}{L} \right)^2 \frac{\beta Z}{2} Y \frac{\partial P^*}{\partial Z} = 0, \quad (11)$$

$$\frac{\partial^2 P}{\partial Z^2} = \frac{12 \left(\frac{L}{D} \right)^2}{(1 + \varepsilon \cos \theta)^3} \left[\varepsilon \left(\phi - \frac{1}{2} \right) \sin \theta + \dot{\varepsilon} \cos \theta \right] + \frac{3\Phi}{(1 + \varepsilon \cos \theta)^3 \left(\frac{H}{L} \right)^2} \left(\frac{\partial P^*}{\partial Y} \right) \Big|_{y=0}. \quad (12)$$

Boundary conditions for equation (11) and (12) are accepted accordingly as:

$$P^* = P \text{ when } Y=0; \quad p^* = \tilde{p}_g \text{ when } Y=-1;$$

$$P^* = P = \tilde{p}_a \text{ when } Z=-1; \quad P^* = P = \tilde{p}_a \text{ when } Z=1; \quad (13)$$

$$P^* = P \text{ when } Y=0; \quad \frac{\partial P^*}{\partial Y} = 0 \text{ when } Y=-1;$$

$$P^* = P = \tilde{p}_H \text{ when } Z=0; \quad P^* = P = \tilde{p}_K \text{ when } Z=1, \quad (14)$$

where

$$\tilde{p}_H = \frac{p_H C^2}{\mu R_0^2 \omega_j}; \quad \tilde{p}_K = \frac{p_K C^2}{\mu R_0^2 \omega_j}; \quad a = \frac{\tilde{p}_K - \tilde{p}_H}{2}; \quad b = \frac{\tilde{p}_K + \tilde{p}_H}{2};$$

\tilde{p}_g is lubricant feed pressure; \tilde{p}_a is atmospheric pressure; \tilde{p}_H is pressure in the initial cross section; \tilde{p}_k is pressure in the final cross section.

Assuming the porous layer thickness to be small, the equation (11) we will average by the lubricant layer thickness. Then the equation (11) will be written as

$$\int_0^1 \left(\frac{\partial^2 P^*}{\partial Y^2} + 4 \left(\frac{H}{L} \right)^2 \frac{\partial^2 P^*}{\partial Z^2} + \beta \frac{Z^2}{4} \frac{\partial P^*}{\partial Y} + \frac{1}{2} \left(\frac{H}{L} \right)^2 \beta Z Y \frac{\partial P^*}{\partial Z} \right) dY = 0. \quad (15)$$

Equation (15) solution, satisfying the main boundary conditions (13), we will search as

$$P^* = A_1 Y^3 + A_2 Y^2 + A_3 Y + \tilde{P}_a + P_1(Z, 0). \quad (16)$$

Substituting (16) in (15), taking into consideration the boundary conditions (13), we will lead to the following system of equations:

$$\begin{aligned} -A_1 + A_2 - A_3 + P_1 + \tilde{P}_a &= \tilde{P}_g; \\ A_1 + 2P_1 - 2A_3 - 2\tilde{P}_g(Z^3 - 1) + \beta \frac{Z^2}{4} (-A_1 + A_2 - A_3) + \left(\frac{H}{L} \right)^2 \beta Z \left(-\frac{1}{5} \frac{\partial A_1}{\partial Z} + \frac{1}{4} \frac{\partial A_2}{\partial Z} - \frac{1}{3} \frac{\partial A_3}{\partial Z} + \frac{1}{2} \frac{\partial P_1}{\partial Z} \right) + 4 \left(\frac{H}{L} \right)^2 \left(\frac{1}{4} \frac{\partial^2 A_1}{\partial Z^2} - \frac{1}{3} \frac{\partial^2 A_2}{\partial Z^2} + \frac{1}{2} \frac{\partial^2 A_3}{\partial Z^2} - \frac{\partial^2 P_1}{\partial Z^2} \right) &= 0. \end{aligned}$$

Assuming

$$\frac{2}{3} \left(\frac{H}{L} \right)^2 \frac{\partial^2 A_3}{\partial Z^2} - \frac{\beta \tilde{P}_g}{4} (Z^4 - Z^2) - 2\tilde{P}_g (Z^2 - 1) = 0 \quad (17)$$

and solving the equation (12) taking into consideration (17), we will get the following formula:

$$\begin{aligned} P_1 &= \frac{12(L/D)^2}{(1 + \varepsilon \cos \theta)^3} \left[\varepsilon \left(\phi - \frac{1}{2} \right) \sin \theta + \varepsilon \cos \theta \right] (Z^2 - 1) + \\ &+ \frac{9\Phi}{2(1 + \varepsilon \cos \theta)^3} \left(\frac{L}{H} \right)^4 \left[\frac{\beta}{480} \tilde{P}_g \left(\frac{Z^8}{14} - \frac{Z^6}{3} + \frac{11}{42} \right) + \frac{1}{12} \tilde{P}_g \left(\frac{Z^6}{15} - Z^4 + \frac{14}{15} \right) + \frac{1}{4} \tilde{P}_g \left(\frac{\beta}{40} + \frac{5}{3} \right) (Z^2 - 1) \right]. \end{aligned} \quad (18)$$

Then

$$\begin{aligned} P &= aZ + b + P_1(Z, \theta) = \frac{\tilde{p}_k - \tilde{p}_H}{2} Z + \frac{\tilde{p}_k + \tilde{p}_H}{2} + \frac{12(L/D)^2}{(1 + \varepsilon \cos \theta)^3} \left[\varepsilon \left(\phi - \frac{1}{2} \right) \sin \theta + \varepsilon \cos \theta \right] (Z^2 - 1) + \\ &+ \frac{9\Phi}{2(1 + \varepsilon \cos \theta)^3} \left(\frac{L}{H} \right)^4 \left[\frac{\beta}{480} \tilde{P}_g \left(\frac{Z^8}{14} - \frac{Z^6}{3} + \frac{11}{42} \right) + \frac{1}{12} \tilde{P}_g \left(\frac{Z^6}{15} - Z^4 + \frac{14}{15} \right) + \frac{1}{4} \tilde{P}_g \left(\frac{\beta}{40} + \frac{5}{3} \right) (Z^2 - 1) \right]. \end{aligned} \quad (19)$$

We will proceed to the case of axial lubricant feed through the clearance.

The equation (11) we will average by the clearance:

$$\int_0^1 \left[\frac{\partial^2 P^*}{\partial Y^2} + \left(\frac{H}{L} \right)^2 \frac{\partial^2 P^*}{\partial Z^2} + \beta Z^2 \frac{\partial P^*}{\partial Y} + 2\beta Z Y \left(\frac{H}{L} \right)^2 \frac{\partial P^*}{\partial Z} \right] dY = 0.$$

The equations (11) and (12) solution taking into consideration boundary conditions (14) we will search as

$$P = aZ + b + P_1(Z, \theta), \quad P^* = A_1 Y^3 + A_2 Y^2 + A_3 Y + aZ + b + P_1. \quad (20)$$

Substituting (20) in (11), taking into consideration boundary conditions (14) we will have

$$3A_1 - 2A_2 + A_3 = 0,$$

$$2A_1 - A_2 - A_3 + \beta Z^2 (-A_1 + A_2 - A_3) + 2 \left(\frac{H}{L} \right)^2 \beta Z \left(-\frac{1}{5} \frac{\partial A_1}{\partial Z} + \frac{1}{4} \frac{\partial A_2}{\partial Z} - \frac{1}{3} \frac{\partial A_3}{\partial Z} + \frac{a}{2} + \frac{1}{2} \frac{\partial P_1}{\partial Z} \right) + \left(\frac{H}{L} \right)^2 \left(\frac{1}{4} \frac{\partial^2 A_1}{\partial Z^2} - \frac{1}{3} \frac{\partial^2 A_2}{\partial Z^2} - \frac{\partial^2 P_1}{\partial Z^2} + \frac{1}{2} \frac{\partial^2 A_3}{\partial Z^2} \right) = 0.$$

Assuming

$$\left(\frac{H}{L} \right)^2 \beta Z a + \frac{4}{3} \left(\frac{H}{L} \right)^2 \frac{\partial^2 A_3}{\partial Z^2} = 0 \quad (21)$$

And solving the equation (21) with boundary conditions (14), we will get

$$A_3 = \frac{1}{2} a \beta (Z^3 - Z). \quad (22)$$

With consideration of (22) the equation (12) solution will be written as

$$P_1 = \frac{24\left(\frac{L}{D}\right)^2}{(1+\varepsilon \cos \theta)^3} \left[\varepsilon \left(\phi - \frac{1}{2} \right) \sin \theta + \dot{\varepsilon} \cos \theta \right] (Z^2 - Z) + \frac{6\Phi a \beta}{(1+\varepsilon \cos \theta)^3 \left(\frac{H}{L}\right)^2} \left(\frac{Z^5}{20} - \frac{Z^3}{6} + \frac{1}{6} Z^2 - \frac{Z}{20} \right). \quad (23)$$

Then P for axial lubricant feed we will get as

$$P = aZ + b + P_1(Z, \theta) = \frac{\bar{P}_K - \bar{P}_H}{2} z + \frac{\bar{P}_K + \bar{P}_H}{2} + \frac{24\left(\frac{L}{D}\right)^2}{(1+\varepsilon \cos \theta)^3} \left[\varepsilon \left(\phi - \frac{1}{2} \right) \sin \theta + \dot{\varepsilon} \cos \theta \right] (Z^2 - Z) + \frac{6\Phi a \beta}{(1+\varepsilon \cos \theta)^3 \left(\frac{H}{L}\right)^2} \left(\frac{Z^5}{20} - \frac{Z^3}{6} + \frac{1}{6} Z^2 - \frac{Z}{20} \right). \quad (24)$$

We will proceed to define the intensification in the oily layer.

In the considered case this intensification is calculated by integrating by the positive area of the pressure distribution.

In case of the incomplete filling with the lubricant we have:

in case of the lubricant feed in the radial direction of the bearing:

$$F_{\xi} = -\frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_{\theta_1}^{\theta_1+\pi} P \cos \theta d\theta dZ = \frac{\mu R^3 \omega_j L}{2C^2} \left[2b \sin \theta_1 + 4 \left(\frac{L}{D} \right)^2 \varepsilon^2 (\cos 3\theta_1 + 3 \cos \theta_1) - \frac{\Phi \bar{P}_g}{70 \left(\frac{H}{L}\right)^4} \left(\frac{107\beta}{48} + 136 \right) \left(2 \sin \theta_1 + \frac{3\varepsilon\pi}{2} \right) \right]; \quad (25)$$

$$F_{\eta} = \frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_{\theta_1}^{\theta_1+\pi} P \sin \theta d\theta dZ = \frac{\mu R_0^3 \omega_j L}{2C^2} \left[2b \cos \theta_1 + 4\varepsilon \left(\frac{L}{D} \right)^2 (\pi + 3\varepsilon \sin \theta_1 - \varepsilon \sin 3\theta_1) + \frac{\Phi \bar{P}_g \cos \theta_1}{35 \left(\frac{H}{L}\right)^4} \left(\frac{107\beta}{48} + 136 \right) \right]. \quad (26)$$

Here the formula for P is defined by the formula (19);

in case of the lubricant feed in the axial direction:

$$F_{\xi} = -\frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_{\theta_1}^{\theta_1+\pi} P \cos \theta d\theta dZ = \frac{\mu R^3 \omega_j L}{C^2} \left[b \sin \theta_1 + 4\varepsilon^2 \left(\frac{L}{D} \right)^2 (\cos 3\theta_1 + 3 \cos \theta_1) + \frac{\Phi a \beta}{7 \left(\frac{H}{L}\right)^2} \left(2 \sin \theta_1 + \frac{3\varepsilon\pi}{2} \right) \right]; \quad (27)$$

$$F_{\eta} = \frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_{\theta_1}^{\theta_1+\pi} P \sin \theta d\theta dZ = \frac{\mu R_0^3 \omega_j L}{C^2} \left[b \cos \theta_1 + 4\varepsilon \left(\frac{L}{D} \right)^2 (\pi + 3\varepsilon \sin \theta_1 - \varepsilon \sin 3\theta_1) + \frac{2\Phi a \beta \cos \theta_1}{7 \left(\frac{H}{L}\right)^2} \right], \quad (28)$$

Where P is defined by the formula (24).

In case of the complete clearance filling with the lubricant we will have:

a) in case of the lubricant feed in the radial direction

$$F_{\xi} = -\frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_0^{2\pi} P \cos \theta d\theta dZ = \frac{\pi \mu R^3 \omega_j L^3}{2C^2} \left[\frac{16\varepsilon}{D^2} - \frac{3\Phi \bar{P}_g \varepsilon L^2}{70H^4} \left(\frac{107\beta}{48} + 136 \right) \right]; \quad (29)$$

$$F_{\eta} = \frac{\mu R^3 \omega_j L}{2C^2} \int_{-1}^1 \int_0^{2\pi} P \sin \theta d\theta dZ = -\frac{8\pi \mu R^3 L^3 \omega_j \varepsilon \left(\phi - \frac{1}{2} \right)}{D^2 C^2}, \quad (30)$$

Here P is defined by the formula (19);

b) in case of the lubricant feed in the axial direction

$$F_{\xi} = -\frac{\mu R^3 \omega_j L}{C^2} \int_{-1}^1 \int_0^{2\pi} P \cos \theta d\theta dZ = \frac{2\pi \mu R^3 L^3 \omega_j}{C^2} \left(\frac{32\varepsilon}{D^2} - \frac{6\Phi a \beta \varepsilon}{7H^2} \right); \quad (31)$$

$$F_{\eta} = \frac{\mu R^3 \omega_j L}{C^2} \int_{-1}^1 \int_0^{2\pi} P \sin \theta d\theta dZ = -\frac{16\mu R^3 L^3 \omega_j \pi \left(\phi - \frac{1}{2} \right)}{D^2 C^2}, \quad (32)$$

Where P is defined by the formula (24).

Substituting the obtained analytic expressions F_ξ and F_η in the equations (3) and (4), we will get:

– in case of the incomplete clearance filling with the lubricant:

a) in case of the lubricant feed in the radial direction

$$\frac{\mu R^3 L}{2mC^3 \omega_r} \left[2b \sin \theta_1 + 4 \left(\frac{L}{D} \right)^2 \varepsilon^2 (\cos 3\theta_1 + 3 \cos \theta_1) - \frac{\Phi \bar{P}_g}{70 \left(\frac{H}{L} \right)^4} \left(2 \sin \theta_1 + \frac{3\varepsilon\pi}{2} \right) \left(\frac{107\beta}{48} + 136 \right) \right] + \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \cos \left[\varphi + \left(\frac{\omega}{\omega_r} \right) T \right] - \frac{K \cdot \varepsilon}{m\omega_r^2} = 0; \quad (33)$$

$$\frac{\mu R_0^3 L}{2mC^3 \omega_r} \left[2b \cos \theta_1 + 4\varepsilon \left(\frac{L}{D} \right)^2 (\pi + 3\varepsilon \sin \theta_1 - \varepsilon \sin 3\theta_1) + \frac{\Phi \bar{P}_g \cos \theta_1}{35 \left(\frac{H}{L} \right)^4} \left(\frac{107\beta}{48} + 136 \right) \right] - \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \sin \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right] = 0; \quad (34)$$

b) in case of the lubricant feed in the axial direction

$$\frac{\mu R^3 L}{mC^3 \omega_r} \left[b \sin \theta_1 + 4\varepsilon^2 \left(\frac{L}{D} \right)^2 (\cos 3\theta_1 + 3 \cos \theta_1) + \frac{\Phi a \beta}{7 \left(\frac{H}{L} \right)^2} \left(2 \sin \theta_1 + \frac{3\varepsilon\pi}{2} \right) \right] + \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \cos \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right] - \frac{K \cdot \varepsilon}{m\omega_r^2} = ; \quad (35)$$

$$\frac{\mu R_0^3 L}{mC^3 \omega_r} \left[b \cos \theta_1 + 4\varepsilon \left(\frac{L}{D} \right)^2 (\pi + 3\varepsilon \sin \theta_1 - \varepsilon \sin 3\theta_1) + \frac{2\Phi a \beta \cos \theta_1}{7 \left(\frac{H}{L} \right)^2} \right] - \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \sin \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right] = 0; \quad (36)$$

– in case of the complete bearing clearance filling with the lubricant we will have

a) in case of the lubricant feed in the radial direction

$$\ddot{\varepsilon} - \varepsilon \dot{\varphi}^2 = \frac{\mu R^3 L \pi}{mC^3 \omega_r} \left[\frac{16\varepsilon}{D^2} + \frac{3\Phi \varepsilon L^2 \bar{P}_g}{70 H^4} \left(\frac{107\beta}{48} + 136 \right) \right] + \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \cos \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right] - \frac{K \cdot \varepsilon}{m\omega_r^2}; \quad (37)$$

$$\varepsilon \ddot{\varphi} + 2\dot{\varepsilon} \dot{\varphi} = - \frac{8\pi \mu R^3 L^3 \varepsilon \left(\dot{\varphi} - \frac{1}{2} \right)}{mD^2 C^3 \omega_r} - \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \sin \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right]; \quad (38)$$

b) in case of the lubricant feed in the axial direction

$$\ddot{\varepsilon} - \varepsilon \dot{\varphi}^2 = \frac{2\pi \mu R^3 L^3}{mC^3 \omega_r} \left(\frac{32\varepsilon}{D^2} - \frac{6\Phi a \beta \varepsilon}{7 H^2} \right) + \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \cos \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right] - \frac{K \cdot \varepsilon}{m\omega_r^2}; \quad (39)$$

$$\varepsilon \ddot{\varphi} + 2\dot{\varepsilon} \dot{\varphi} = - \frac{16\pi \mu R^3 L^3 \left(\dot{\varphi} - \frac{1}{2} \right)}{mD^2 C^3 \omega_r} - \frac{u}{mc} \left(\frac{\omega}{\omega_r} \right)^2 \sin \left[\varphi - \left(\frac{\omega}{\omega_r} \right) T \right]. \quad (40)$$

We will insert the notation

$$B = \frac{\mu R^3 L}{mC^3 \omega_r}, U = \frac{u}{mc}, \omega_s = \sqrt{K/m}, \Omega_s = \frac{\omega_s}{\omega_r}, \Omega = \frac{\omega}{\omega_r}, \beta = \varphi - \Omega T,$$

Where B is damper parameter; U is dimensionless imbalance; ω_s is rotor self-frequency; Ω_s and Ω is shaft self and angular frequency.

Equation (33)–(36), describing shaft centre stationary motion, and (37) – (40), describing its non-stationary motion, gets as follows:

a) given the shaft stationary motion:

$$B \left[2b \sin \theta_1 + 4 \left(\frac{L}{D} \right)^2 \varepsilon^2 (\cos 3\theta_1 + 3 \cos \theta_1) - \frac{\Phi \bar{P}_g}{70 \left(\frac{H}{L} \right)^4} \left(2 \sin \theta_1 + \frac{3\varepsilon\pi}{2} \right) \left(\frac{107\beta}{48} + 136 \right) \right] + U \Omega^2 \cos \beta - \Omega_s^2 \varepsilon = 0; \quad (41)$$

$$B \left[2b \cos \theta_1 + 4\varepsilon \left(\frac{L}{D} \right)^2 (\pi + 3\varepsilon \sin \theta_1 - \varepsilon \sin 3\theta_1) + \frac{\Phi \bar{P}_g \cos \theta_1}{35 \left(\frac{H}{L} \right)^4} \left(\frac{107\beta}{48} + 136 \right) \right] - U \Omega^2 \sin \beta = 0. \quad (42)$$

$$B \left[b \sin \theta_1 + 4\epsilon^2 \left(\frac{L}{D} \right)^2 (\cos 3\theta_1 + 3 \cos \theta_1) + \frac{\Phi a \beta}{7 \left(\frac{H}{L} \right)^2} \left(2 \sin \theta_1 + \frac{3\epsilon \pi}{2} \right) \right] + U \Omega^2 \cos \beta - \Omega^2 \epsilon = 0; \quad (43)$$

$$B \left[b \cos \theta_1 + 4\epsilon \left(\frac{L}{D} \right)^2 (\pi + 3\epsilon \sin \theta_1 - \epsilon \sin 3\theta_1) + \frac{2\Phi a \beta \cos \theta_1}{7 \left(\frac{H}{L} \right)^2} \right] - U \Omega^2 \sin \beta = 0; \quad (44)$$

b) given the shaft non-stationary motion

$$\ddot{\epsilon} - \epsilon \dot{\phi}^2 = BL^2 \pi \left[\frac{16\dot{\epsilon}}{D^2} + \frac{\dot{P}_g}{70H^4} \frac{3\Phi \epsilon L^2}{70H^4} \left(\frac{107\beta}{48} + 136 \right) \right] + U \Omega^2 \cos \beta - \Omega^2 \epsilon; \quad (45)$$

$$\epsilon \ddot{\phi} + 2\dot{\epsilon} \dot{\phi} = \frac{8B\pi L^2 \epsilon \left(\dot{\beta} + \Omega - \frac{1}{2} \right)}{D^2} - U \Omega^2 \sin \beta; \quad (46)$$

$$\ddot{\epsilon} - \epsilon \dot{\phi}^2 = 2B\pi L^2 \left(\frac{32\dot{\epsilon}}{D^2} - \frac{6\Phi a \beta \epsilon}{7H^2} \right) + U \Omega^2 \cos \beta - \Omega^2 \epsilon; \quad (47)$$

$$\epsilon \ddot{\phi} + 2\dot{\epsilon} \dot{\phi} = - \frac{16B\pi L^2 \left(\dot{\beta} + \Omega - \frac{1}{2} \right)}{D^2} - U \Omega^2 \sin \beta. \quad (48)$$

Equations (41)–(48) have been calculated by the numerical method, developed by Gear (1972). The results of the numerical analysis are presented in the Figures 3–6. All they comply with the zero initial speeds and the final perturbation of the initial shaft position.

3. Findings and Their Discussion

Transmission coefficient is defined as the ratio of the scalar force, transmitted to the body, to the scalar centrifugal force of the disbalance, namely at the totally fixed support the transmission coefficient is equal to one. It is necessary to note, that under several conditions of tribosystem operation, the damper is even capable to reinforce the imbalance impact. Due to that it is very important to define the operating condition and the damper outer porous ring permeability, which would lead to the damping of the transmitted force.

Scalar transmitted force F_{TP} we will define as RSS (root of sum of squares) from the sum of squares of its constituents

$$|F_{TP}| = \left[(F_{\xi} - Ke)^2 + F_{\eta}^2 \right]^{\frac{1}{2}}. \quad (49)$$

For the case of the lubricant feed into the radial and axial directions when the shaft center stationary motion the module is defined with the help of the formulas (25)–(28), and when the nonstationary – with the help of the formulas (29)–(32).

As the disbalance module is equal to $u\omega^2$, then the transmission coefficient T_r can be represented by the formula:

$$T_r = \frac{|F_{TP}|}{u\omega^2} = \frac{\left[(F_{\xi} - Ke)^2 + F_{\eta}^2 \right]^{\frac{1}{2}}}{u\omega^2}. \quad (50)$$

For the shaft center stationary and nonstationary motion while lubricant feed in the radial and axial directions the transmission coefficient T_r is defined with the help of the formula (49).

The dampers of the described structure provide the significant reduction of the operational load variation effect and shaft disbalance on the journal rolling contact bearings.

So, on the basis of the fulfilled calculations it is established, that in the researched area the considered dampers with the porous ring while forced lubricant feed in the radial and axial directions in consideration of the porous layer permeability in the models reduce effectively the transmitted forces of the disbalance. By the results of the numerical calculations the graphs are constructed, given on the pictures 3–6. The analysis of the given computes models and graphs allows making the following conclusions.

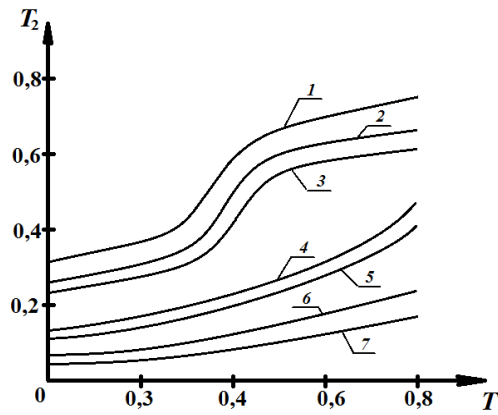


Figure 3. Dependence of the stationary transmission coefficient from T parameter:

$$B=0,1; \Omega=1,1; \frac{H}{L}=0,1; \Phi = \frac{A_0 H}{C^3}; \beta=0,01; \bar{p}_g=1,1; \bar{p}_a=1,1; \bar{p}_H=0,04 \text{ мПа}; \bar{p}_K=0,03 \text{ мПа};$$

$$\varepsilon(0)=0,8; \theta_1=0; \theta_2=\pi:$$

- 1) $\Phi = 0,001$; 2) $\Phi = 0,0015$; 3) $\Phi = 0,02$;
 4) $\Phi = 0,01$; 5) $\Phi = 0,015$; 6) $\Phi = 0,03$; 7) $\Phi = 0,04$

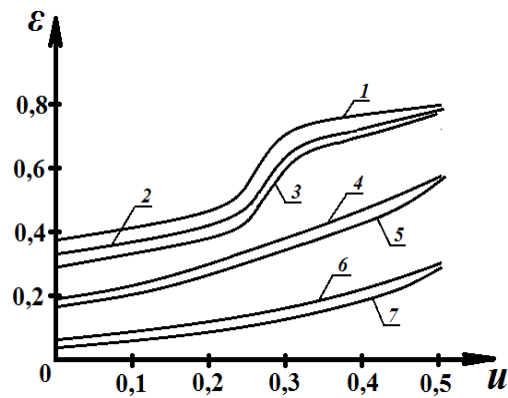


Figure 4. Dependence of the stationary eccentricity ε of damper component from the disbalance eccentricity U :

$$B=0,2; \Omega=1,1; \frac{H}{L}=0,1; \Phi = \frac{A_0 H}{C^3}; \beta=0,1; \bar{p}_g=1,5; \bar{p}_a=1,5; \bar{p}_H=0,04 \text{ мПа}; \bar{p}_K=0,03 \text{ мПа};$$

$$\varepsilon(0)=0,8; \theta_1=0; \theta_2=\pi:$$

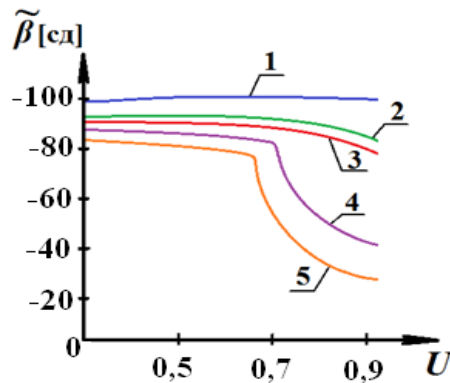


Figure 5. Dependence of the non-stationary eccentricity ratio of the damper from the disbalance eccentricity:

$$B = 0,4; U = 0,3; \Omega = 1,1; \Omega_s = 0,5; \dot{\varepsilon}(0) = 0,1;$$

$$\varepsilon(0) = 0,8; H/L = 0,1; \theta_1 = 0; \theta_2 = 2\pi;$$

1) $\Phi = 0,001$; 2) $\Phi = 0,005$; 3) $\Phi = 0,004$;
4) $\Phi = 0,01$; 5) $\Phi = 0,03$

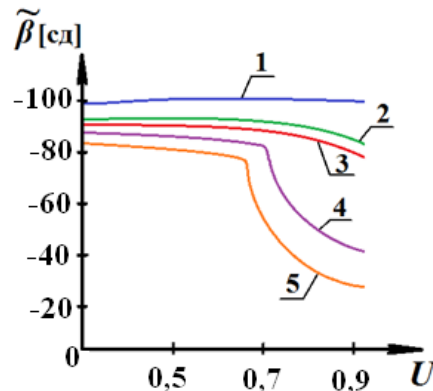


Figure 6. Dependence of the disbalance eccentricity from non-stationary eccentricity ratio of the damper:

$$B = 0,4; U = 0,2; \Omega = 1,1; \Omega_s = 0,6; \dot{\varepsilon}(0) = 0;$$

$$\varepsilon(0) = 0,9; \beta(0) = ?; \dot{\beta}(0) = 0; H/L = 0,1;$$

$$\theta_1 = 0; \theta_2 = 2\pi;$$

1) $\Phi = 0,001$; 2) $\Phi = 0,005$; 3) $\Phi = 0,0055$;
4) $\Phi = 0,015$; 5) $\Phi = 0,035$

4. Conclusion

The obtained findings show, that the dampers of the considered structure (with porous anisotropy outer ring and the compressed oily layer) provide due to the transmission coefficient the sufficient reducing of the vibration impact on the radial bearings.

Besides, it is noted, that the sequent forced lubricant feed in the axial and radial directions provides steadier operation both of dampers, and the radial bearings with porous bushing.

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