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Vibration Isolation of Lumped Masses Supported on Beam by Imposing Nodes Using Multiple Vibration Absorbers

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Abstract

In this paper, variable stiffness damped absorbers are used to isolate the substructures of Euler-Bernoulli beam, modelled as lumped masses, from vibrations. The novel algorithm is developed that can be used to determine the required absorber masses and resonance frequencies to impose nodes at selected locations on beam with the constraint of vibration amplitude of absorber mass. Numerical simulations are performed to show the effectiveness of the proposed algorithm. Experimental test is conducted on a cantilever beam with two absorbers to verify the numerical results.

Keywords: vibration absorber, node, cantilever beam, harmonic force, vibration isolation

1. Introduction

The tuned vibration absorber (TVA) was invented by (Frahm, 1911), since then, has been is an important engineering tool for vibration suppression. The first classical paper on dynamic vibration absorber was by (Ormondroyd & Den, 1928). Their vibration absorber consisted of a tuned spring-mass used to suppress the response of a harmonic oscillator. An excellent survey of passive, semi-active and active dynamic vibration absorbers was prepared by (Sun et al., 1995). Spring-mass systems which are used as vibration absorbers to minimize excesses vibrations in continuous structure have received considerable interest in recent years. (Young, 1928) was the first to consider the application of absorber to control the vibrations of a continuous structure i.e. cantilever beam, at the absorber attachment point with the absorber tuned to the first natural frequency of the beam. (Manikanahally & Crocker, 1993) employed vibration absorbers to suppress any number of significant modes. The method was successfully applied to a space structure modeled as a mass-loaded free–free beam subjected to localized harmonic excitation. (Esmailzadeh & Jalili, 1998) presented a procedure in designing DVA for a structurally damped beam system subjected to distributed force excitation. The absorber is modeled as spring-mass-damper system and the optimum tuning and damping ratios are determine to minimize the beam dynamic response at the resonance frequency at which they operate. Research on suppressing vibration in a region or the particular span of an elastic structure by using the spring mass vibration absorber has been reported recently. (Cha, 2004, 2005) employed spring-mass vibration absorber to reduce vibrations at desired locations by imposing node technique (Hao et al., 2011). Suppressed the hand-arm vibration in electric grass trimmer by installing tunable vibration absorber at optimum location. The focus of (Cha & Rinker, 2012) was on enforcing nodes at desired location of damped Euler-Bernoulli beam during forced harmonic excitations using damped vibration absorbers. An efficient method is developed which determines the restoring force exerted by the damped absorbers using Gaussian elimination and then the forces are used to determine the parameters of oscillators. (Hao & Ripin, 2013) applied imposing node technique to achieve very low vibration at handle location using two tunable vibration absorbers. The grass trimmer system is simplified as an arbitrary, supported beam constrained by 4 lumped masses. The Matlab routine *global search* is utilized to obtain the tuning frequencies of the absorbers. (Patil, & Awasare, 2016) developed an iterative procedure to find the required resonance frequencies of absorbers to impose node at selected locations on beam.

In the method proposed by Cha to induce multiple nodes, a set of nonlinear algebraic equations need to be solved simultaneously. Numerically, the solution of these equations is very computationally intensive because the convergence is often very slow. The limitations of the procedure developed by Patil & Awasare are that, the maximum allowable absorber amplitudes were not considered while finding the resonance frequencies of the absorbers. The purpose of this paper is 1) To develop mathematical model of the beam carrying multiple damped absorbers and to formulate the equation for imposing nodes at desired locations on the beam 2) To develop an algorithm to find the required absorber parameters to impose nodes at selected locations on the beam with constraint of the tolerable vibration amplitudes of the absorber mass.3) To perform numerical simulation on beam to show the utility of the proposed algorithm 4) To carry the experimental test to validate the numerical results.

2. Theory

Figure 1 shows an arbitrarily supported, Euler-Bernoulli beam with n tunable vibration absorbers attached at x_i. The absorber modeled as single degree of freedom spring-mass-damper system having mass m_i, stiffness k_i and damping coefficient c_i of the i th absorber. The lumped masses are supported at locations x_{mi} on the beam. The external harmonic force $f(t) = Fe^{j\omega_e t}$ is applied to the structure at x_f, where F represents the forcing amplitude, ω_e denotes the excitation frequency, and $j = \sqrt{-1}$

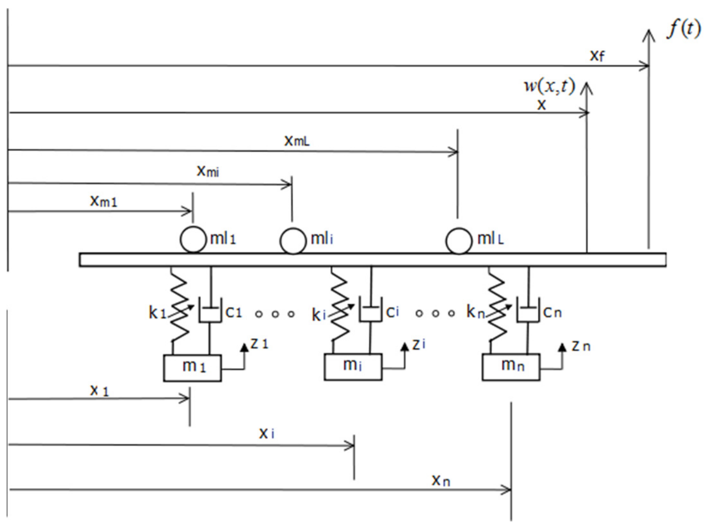

Figure 1. Beam with variable stiffness damped multiple absorbers subjected to localized harmonic excitations

Using the assumed-modes method the deflection of the beam at any point x along the structure is given by Leonard Meirovitch (2007).

$$w(x,t) = \sum_{i=1}^{N} \phi_i(x)\eta_i(t), \tag{1}$$

where N is the number of modes used in the assumed-modes expansion, $\phi_i(x)$ are the eigenfunctions of the undamped beam and $\eta_i(t)$ are corresponding generalized co-ordinates. Applying Lagrange's equations and assuming simple harmonic motion with same response frequency as the excitation frequency, the following equations of motion are obtained

$$\left(-\omega_e^{\,2}\begin{bmatrix}[M] & 0\\ 0^T & [m]\end{bmatrix} + j\omega_e\begin{bmatrix}[C] & [R_c]\\ [R_c]^T & [c]\end{bmatrix} + \begin{bmatrix}[K] & [R_k]\\ [R_k]^T & [k]\end{bmatrix}\right)\begin{bmatrix}\overline{\eta}\\ \overline{z}\end{bmatrix} = \begin{bmatrix}F\phi(x_f)\\ 0\end{bmatrix}, \tag{2}$$

where $\overline{\eta} = [\overline{\eta}_1 \quad \overline{\eta}_2 \quad \ldots \quad \overline{\eta}_N]^T$, $\overline{z} = [\overline{z}_1 \quad \overline{z}_2 \quad \ldots \quad \overline{z}_n]^T$, and the matrices $[m]$ $[c]$ and $[k]$ of size $n \times n$ are diagonal, whose i th elements are given by m_i, c_i and k_i respectively. The $N \times N$ $[M]$ $[C]$ and $[K]$ matrices of equation (2) are

$$[M] = [M^d] + \sum_{i=1}^{L} ml_i\phi(xm_i)\phi^T(xm_i), \quad [C] = [C^d] + \sum_{i=1}^{n} c_i\phi(x_i)\phi^T(x_i), \quad [K] = [K^d] + \sum_{i=1}^{n} k_i\phi(x_i)\phi^T(x_i) \tag{3}$$

where $[M^d]$, $[C^d]$ and $[K^d]$ are diagonal matrices whose i th elements are M_i, C_i, and K_i are the generalized masses, damping and stifnesses of beam. Vector of the eigenfunctions of the beam and the matrices $[R_c]$ and $[R_k]$

of size $N \times n$ are given by

$$\phi(x_i) = [\phi_1(x_i) \quad \phi_2(x_i) \ldots \phi_N(x_i)]^T, \qquad \phi(x_f) = [\phi_1(x_f) \quad \phi_2(x_f) \ldots \phi_N(x_f)]^T,$$

$$\phi(x_{mi}) = [\phi_1(x_{mi}) \quad \phi_2(x_{mi}) \ldots \phi_N(x_{mi})]^T \tag{4}$$

$$[R_c] = [-c_1\phi(x_1) \quad \ldots \quad -c_i\phi(x_i) \quad \ldots \quad -c_n\phi(x_n)] \quad [R_k] = [-k_1\phi(x_1) \quad \ldots \quad -k_i\phi(x_i) \quad \ldots \quad -k_n\phi(x_n)] \tag{5}$$

Using second equation of equation (2), the $\bar{z}_i$ are found to be

$$\bar{z}_i = \frac{\omega_i^2 + j\omega_e c_i/m_i}{\omega_i^2 - \omega_e^2 + j\omega_e c_i/m_i} \phi^T(x_i)\bar{\eta}, \qquad i = 1, \ldots n \tag{6}$$

In equation (6) resonance frequency ω_i and damping coefficient c_i of i th absorber are given by

$$\omega_i = \sqrt{k_i/m_i} \qquad c_i = 2\zeta_i\omega_i, \tag{7}$$

where ζ_i is the damping ratio of the i th absorber

Equation (6) is substituted into the first equation of equation (2) and then solving for $\bar{\eta}$ to obtain

$$\bar{\eta} = \left\{ -\omega_e^2[M] + j\omega_e[C^d] + [K^d] + \sum_{i=1}^{S} \sigma_i \phi(x_i)\phi^T(x_i) \right\}^{-1} F\phi(x_f) \tag{8}$$

In equation (8)

$$\sigma_i = \frac{(m_i\omega_i^2 + jc_i\omega_e)\omega_e^2}{\omega_e^2 - \omega_i^2 - j\,c_i/m_i\,\omega_e} \tag{9}$$

Substituting equation (8) in to equation (1), following equation is formulated, the solution of which gives the absorber parameters required to impose node at desired locations x_{nr} along the beam

$$W(x_{nr}) = \phi^T(x_{nr}) \left\{ -\omega_e^2[M] + j\omega_e[C^d] + [K^d] + \sum_{i=1}^{n} \frac{(m_i\omega_i^2 + jc_i\omega_e)\omega_e^2}{\omega_e^2 - \omega_i^2 - j\,c_i/m_i\,\omega_e} \phi(x_i)\phi^T(x_i) \right\}^{-1} F\phi(x_f) = 0 \qquad i = 1, \ldots n \tag{10}$$

Once the beam with its boundary conditions are specified, absorbers attachment locations x_i, excitation frequency ω_e and the excitation location x_f are known, equation (10) can be used to find the absorber parameters, mass of the absorber m_i and resonance frequencies ω_i, for given absorber damping ratio ζ_i at which displacements of beam $W(x_{nr})$ becomes zero to impose nodes at x_{nr}.

An algorithm is developed, which is based on finding the resonance frequency of absorber ω at which $|W(x_n)|$ is less than $|W(x_{nP})|$ and $|W(x_{nN})|$ i.e the absolute value of the displacement of the beam at node location is less than the absolute value of the displacement at previous and next to node location as shown in Figure 2. The condition $|W(x_n)| < |W(x_{nP})|$ and $|W(x_n)| < |W(x_{nN})|$, gives the absorber frequency ω for given mass m necessary to impose node.

The procedure to find the absorber masses m_1 and m_2 and frequencies of the absorbers ω_1 and ω_2 to impose two nodes is as follows

Algorithm to find the masses and corresponding resonance frequencies of the absorbers to impose two nodes

(1) Assume the lower value for the absorber masses m_1 and m_2.

(2) Set initial frequencies of the absorbers $\omega_1 < \omega_e$ and $\omega_2 < \omega_e$.

(3) Determine σ_1 and σ_2 from Equation (9).

(4) Compute $|W(x_{n1})|$ $|W(x_{n1P})|$ and $|W(x_{n1N})|$ using Equation (10).

(5) If $|W(x_{n1})| < |W(x_{n1P})|$ and $|W(x_{n1})| < |W(x_{n1N})|$, is the absorber mass and ω_1

(6) is the absorber frequency required to impose node. Else increase the frequency of absorber ω_1 in steps, compute $|W(x_{n1})|, |W(x_{n1P})|, |W(x_{n1N})|$, till above condition is achieved.

(7) Increase the mass m_1 of first absorber and repeat steps (3) to (5) till, $|\bar{z}_1| \leq |\bar{z}_{1\max}|$. Record the corresponding resonance frequency ω_1.

(8) Replace frequency and mass of first absorber in σ_1 with new frequency ω_1 and mass m_1 obtain from step number (6).

(9) Compute $|W(x_{n2})|$ $|W(x_{n2P})|$ and $|W(x_{n2N})|$ using Equation (10).

(10) If $|W(x_{n2})| < |W(x_{n2P})|$ and $|W(x_{n2})| < |W(x_{n2N})|$, m_2 is the absorber mass and ω_2 is the absorber frequency required to impose node.

(11) Else increase the frequency of absorber ω_2 in steps, compute $|W(x_{n2})|, |W(x_{n2P})|, |W(x_{n2N})|$, till above condition is achieved.

(12) Increase the mass m_2 of second absorber and repeat steps (8) to (10) till, $|\bar{z}_2| \leq |\bar{z}_{2\max}|$. Record the corresponding resonance frequency ω_2.

(13) Replace frequency and mass of second absorber in σ_2 with new frequency ω_2 and mass m_2 obtain from step number (11).

(14) Repeat procedure from step number (3) to (12) with revised σ_2.

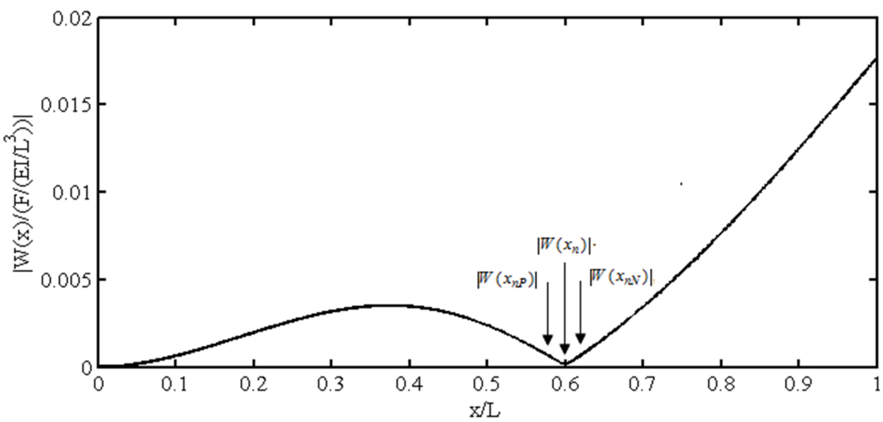

Figure 2. Illustration of condition $|W(x_n)| < |W(x_{nP})|$ and $|W(x_n)| < |W(x_{nN})|$, used to find frequency of absorber

3. Numerical Results

Because the assumed-mode method was used to formulate the equations of motions, the proposed procedure can be easily implemented to impose node along any arbitrary supported beam subjected to harmonic excitations. For cantilever beam, its normalized (with respect to mass per unit length, ρ, of the beam) eigenfunctions $\phi_i(x)$, generalised masses M_i and generalised stiffnesses K_i are given by

$$\phi_i(x) = \frac{1}{\sqrt{\rho L}}\left(\cos\beta_i x - \cosh\beta_i x + \frac{\sin\beta_i L - \sinh\beta_i L}{\cos\beta_i L + \cosh\beta_i L}(\sin\beta_i x - \sinh\beta_i x)\right) \quad (11)$$

$$M_i = 1 \qquad \text{and} \qquad K_i = (\beta_i L)^4\, EI/(\rho L^4) \quad (12)$$

where $\beta_i L$ satisfies the following transcendental equation

$$\cos\beta_i L \cosh\beta_i L = -1 \quad (13)$$

where E is Young's modulus, I is the moment of inertia of the cross-section of the beam.

In the following example the frequencies and and vibration amplitudes are non-dimensionalised by dividing by $\sqrt{EI/(\rho L^4)}$ and $F/(EI/L^3)$ respectively. Number of modes, $N = 15$ is used in the assumed-modes expansion. The value of the absorber frequency ω_i and mass m_i are incremented by $0.001\sqrt{EI/(\rho L^4)}$ and $0.0001\rho L$, in each iteration respectively. The absorber has a low damping to obtain the greatest vibration attenuation at the intended frequency.

Now consider the example of a uniform cantilever beam. It is desired that two nodes to be imposed, at $x_{n1} = 0.4L$ and $x_{n2} = 0.6L$, for $\omega_e = 64\sqrt{EI/(\rho L^4)}$ at $x_f = 1L$, for vibration isolation of lumped masses $ml_1 = 0.02\rho L$, $ml_2 = 0.04\rho L$ and $ml_3 = 0.02\rho L$ supported at $x_{m1} = 0.4L$, $x_{m2} = 0.5L$ and $x_{m3} = 0.6L$ respectively. The two absorbers are attached at location $x_1 = 0.3L$ and $x_2 = 0.65L$ on the beam. The absorber parameters, resonance frequencies and masses for given damping ratio, required to impose nodes obtained by using the algorithm developed are listed in Table-1.

Table 1. Summary of absorber parameters for the example of vibration isolation of lumped masses supported on uniform cantilever beam

Absorber	Damping ratio ξ_i	Attachment Locations x_i m	Mass m_i kg	Required Frequency ω_i rad/sec	Tolerable Mass amplitude $z_{i\max}$ m
First	0.001	0.4L	$0.0519\,\rho L$	$68.52\sqrt{EI/(\rho L^4)}$	$0.0015\,F/(EI/L^3)$
Second	0.001	0.6L	$0.0851\,\rho L$	$61.41\sqrt{EI/(\rho L^4)}$	$0.0035\,F/(EI/L^3)$

Figure 3, shows the steady state deformed shapes of cantilever beam with node at $0.4L$ and $0.6L$. Note the region on beam up to $0.6L$ experiences less vibration comparing to the beam without absorbers.

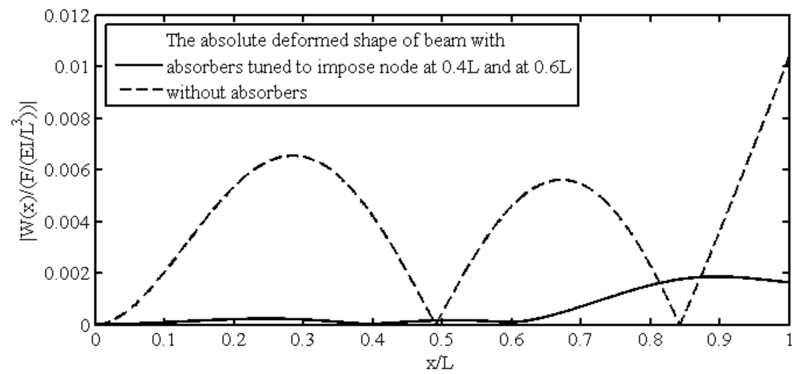

Figure 3. The absolute steady-state response of beam with and without absorbers to impose nodes at $0.4L$ and $0.6L$, for the absorber parameters listed in Table 1

4. Experimental Test

For imposing two nodes the dual cantilevered mass absorbers were designed and constructed as shown in Figure 5. The resonance frequency of device is adjusted by moving the masses towards or away from the base support, which alters the effective stiffness in the system and alters its resonance frequency. Note that the structural damping of the absorbers used is considered as equivalent viscous damping. Next in order to verify the numerical result, experimental test was conducted for the case of cantilever beam with two absorbers as shown in Figure 5. The two absorbers are attached at $0.3L$ and $0.65L$ and the harmonic input force is applied at the tip of the beam with frequency 152 Hz i.e $\omega_e = 64\sqrt{EI/(\rho L^4)}$. The force amplitude is kept constant at 5 N through the experiment and used to non-dimensionalize the displacements of the beam by dividing by $F/(EI/L^3)$. The system parameters and material properties used in experimental test are listed in Table 2. The toltal weights of absorber end masses attached at $0.3L$ and $0.65L$ are 0.25 Kg and 0.42 Kg respectively. The damping ratios of the both absorber are $\xi_1 = \xi_2 = 0.001$. The absorbers were tuned by moving the end masses in or out such that the displacement at $0.4L$ and $0.6L$ was minimized. After tuning of the absorbers the vibration amplitudes were measured at twenty points on the beam's surface by the accelerometer and recorded by vibration analyzer to plot experimental steady state response as shown in Figure 6. It is observed that the vibrations at the node locations of the beam are reduced to a minimum level. Comparing Figure 3, with Figure 6, it is observed that there is good agreement between the numerical results and experimental results.

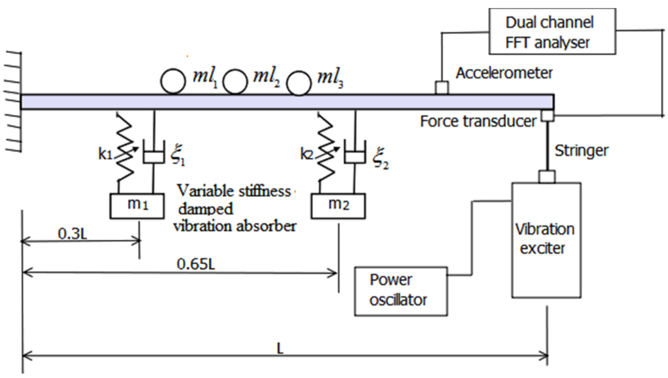

Figure 4. Illustration of experimental set up

Figure 5. Experimental test on cantilever beam supporting lumped masses with two absorbers tuned to impose nodes

Table 2. The system parameters and material properties used in the experimental test

Length of the beam L		1 m
Thickness of the beam t		0.01 m
Width of the beam b		0.065 m
Density of the beam		7830 Kg/m^3
Young's modulus of the beam E		2.1 x 10^{11} N/m^2
Mass per unit length of beam ρ		5 Kg/m
Parameter EI/L^3 used to nondimensionlised stiffness of beam		1137.5 N/m
Parameter $\sqrt{EI/(\rho L^4)}$, used to nondimensionlised frequencies		14.95 N/m/Kg
Parameter $F/EI/L^3$ used to nondimensionlised vibration amplitudes of beam and absorber masses		0.004 m
Weight of lumped masses	ml_1	0.1 Kg = 0.02 ρL
	ml_2	0.2Kg = 0.04 ρL
	ml_3	0.1 Kg = 0.02 ρL
Total weight of end masses of first absorber	m_1	0.25 Kg = 0.052 ρL
Total weight of end masses of second absorber	m_2	0.42 Kg = 0.085 ρL
Damping ratio of absorbers	$\xi_1 = \xi_2$	0.001

The frequency response plots for first and second tuned absorbers are depicted in Figure 7 (a) and Figure 7 (b) respectively. The frequency response plot shows the resonance frequency, of first absorber $\omega_1 = 164Hz \approx 68.9\sqrt{EI/(\rho L^4)}$ and of second absorber $\omega_2 = 145Hz \approx 61\sqrt{EI/(\rho L^4)}$, are in close match with numerical solution i.e $\omega_1 = 68.52\sqrt{EI/(\rho L^4)}$ and $\omega_2 = 61.42\sqrt{EI/(\rho L^4)}$. The frequency response plot for beam is as shown in Figure 8(a) and peaks give the first three natural frequencies of the cantilever beam 8 Hz, 53 Hz and 151 Hz respectively. The frequency response plot for beam with absorbers is shown in Figure 8(b), and observed that the response drops to a minimum at excitation frequency $152Hz \approx 64\sqrt{EI/(\rho L^4)}$ and peaks at $108Hz \approx 47\sqrt{EI/(\rho L^4)}$ and $162Hz \approx 68\sqrt{EI/(\rho L^4)}$.

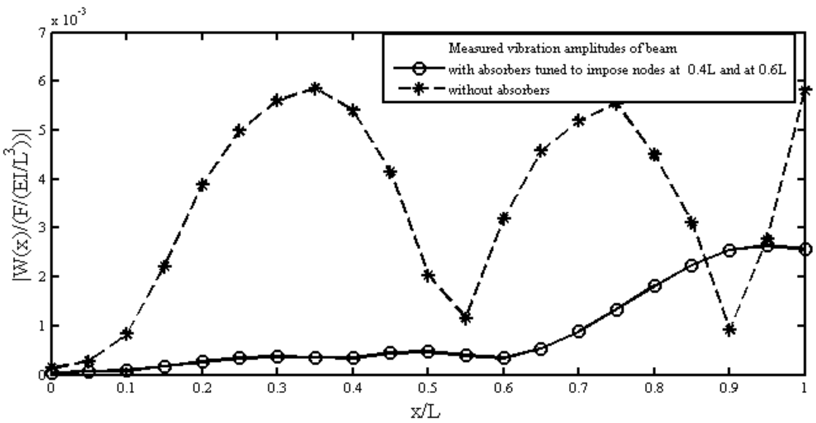

Figure 6. Measured vibration amplitude of beam with absorber tuned to impose nodes at $0.4L$ and $0.6L$ and without absorber

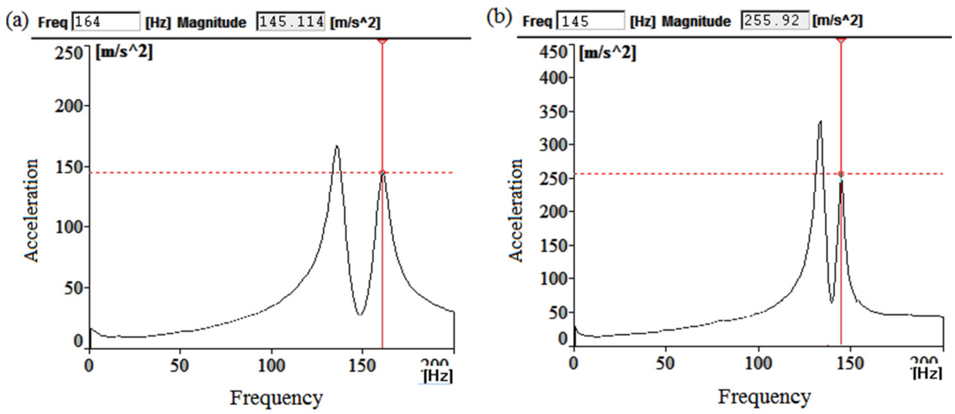

Figure 7. Frequency response of a) absorbers attached at $0.3L$ b) absorbers attached at $0.65L$, by experimental modal analysis when absorbers tuned to impose node at $0.4L$ and $0.6L$

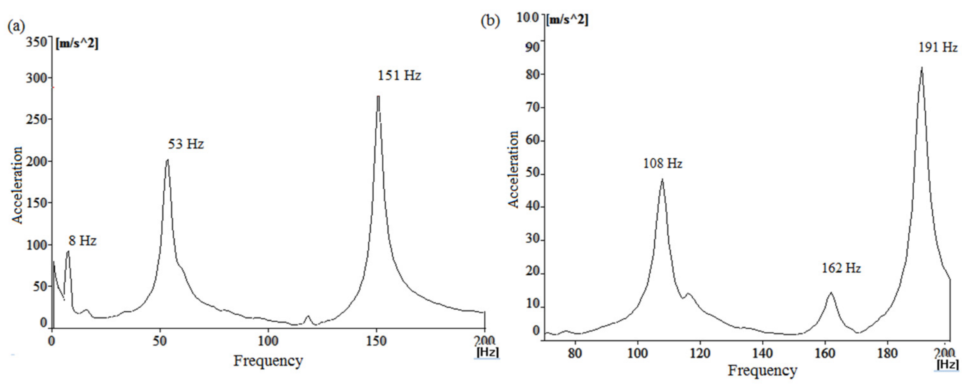

Figure 8. Frequency response of a) beam without absorber b) beam with absorbers when absorbers tuned to impose node at $0.4L$ and $0.6L$, by experimental modal analysis

5. Conclusions

This investigation presents novel algorithm to find the absorber parameters to impose nodes at chosen locations on beam. Once the generalized program is developed, it can be used to determine the feasible absorber parameters and can be easily modified to accommodate beam with different boundary conditions. For imposing multiple nodes, the procedure gives different combinations of two absorber masses and corresponding resonance frequencies required to impose nodes for given damping ratio. The design constraint on maximum allowable vibration amplitudes on absorber masses makes the proposed procedure more practical. The results show that by imposing nodes at appropriate locations, the vibrations are suppressed for the segment of beam thus isolating the lumped masses. The experimental results show good agreement with those obtained by numerical experiments.

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