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# New Bi-modular Material Approach to Buckling Problem of Reinforced Concrete Columns

Ahmad Salah Edeen Nassef[1,2] & Mohammed A. Dahim[2]

[1] Faculty of Engineering, Matria – Helwan University, Cairo, Egypt

[2] Faculty of Engineering, Najran University, Najran, Kingdom of Saudi Arabia

Correspondence: Ahmad Salah Edeen Nassef, Faculty of Engineering, Matria – Helwan University, Cairo, Egypt. E-mail: ahmad_nassef@hotmail.com

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**Abstract**

This paper was investigating the buckling problem of reinforced concrete columns considering the reinforced concrete as bi – modular material. Governing differential equations was driven. The relation between the non-dimensional transverse deflection and non-dimensional distance between centroid axis and the neutral axis "eccentricity" was drawn to enable the solution of the governing differential equation. The new approach was verified with different experimental results and different codes of practice.

**Keywords:** buckling, columns, Bi – modular, modulus of elasticity, reinforced concrete

## 1. Introduction:

Bi – modular material term was recognized, where some materials have different elastic behavior when they are loaded in tension and compression. Both fiber reinforced and composite materials have different moduli in tension and compression as displayed in Table (1).

Table 1. Representative Tension and Compression Moduli Relationship for Fiber Reinforced and Composite Materials

| Material | Representative Moduli Relationship |
|---|---|
| **Glass/Epoxy** | $E_t = 1.2\ E_c$ |
| **Boron/Epoxy** | $E_c = 1.2\ E_t$ |
| **Graphite/Epoxy** | $E_t = 1.4\ E_c$ |
| **Carbon/Carbon** | $E_t = 2 - 5\ E_c$ |
| **ZTA Graphite** | $E_c = 1.2\ E_t$ |
| **ATJ-S Graphite** | $E_t = 1.2\ E_c$ |
| **Concrete** | $E_t = 1\text{-}1.3\ E_c$ |

From the above Table, one can note that no unique pattern of a larger tension than compression moduli or vice versa exists. Concrete is also considered as bi-modular material, where the tensile Young's moduli of concrete and mortar mixtures were measured using a direct tension test (Isamu Yoshitake et al., 2012). The results show that the tensile moduli are approximately 1.0–1.3-times larger than the compressive moduli. Two material models were widely used in dealing with bi-modular material within the engineering research .The first model is the criterion of positive-negative signs in the longitudinal strain proposed (Bert, 1977). This model can be applied to orthotropic materials. The other model is the criterion of positive-negative signs of principal stress proposed (Ambartsumyan, 1986). This model can be applied to isotropic materials, where The basic assumptions of this model are:

(1) The investigated body is continuous, homogeneous, and isotropic.

(2) Small elastic deformation is assumed, and the general law of continuum mechanics is applicable.

(3) When the principal stresses are uniformly positive or uniformly negative, the three basic equations are essentially the same as those of classical elastic theory; when the signs of the principal stresses are different, the equilibrium equation and the geometry equation are identical to those of classical elastic theory, with the exception of the physical equations (constitutive equations) where the stress-strain relationship is bilinear in the elasticity theory of different moduli.

The Ambartsumyan material model was compared the criteria for consistent material and found to violate the requirement of symmetry compliance, this improved model called the weighted compliance matrix (WCM) material, which can be extended by deduction to more complicated situations (R. M. Jones 1977). The basic assumptions of this model and its development, several innovative computational methods, and some important engineering applications were reviewed (Jun-Yi Sun et al., 2010).

The transfer-matrix approach was used to determine the small-deflection static behavior of bi-modulus beams, including transverse shear deformation. The effects of axial load and non-natural boundary conditions were considered. Exact closed-form solutions were also presented for special cases in which the neutral-surface location was constant along the beam axis (Tran & Bert, 1982).

The bifurcation buckling of a uniform, slender, cantilever column constructed of a bi-modular material was treated by three different approximate techniques: finite difference, segmentation (transfer matrix), and energy (Bert & Ko, 1985). The buckling and post-buckling analysis of bi-modulus circular and annular plates was treated and the problem was solved using annular finite elements. The constitutive matrix based on the "Jones model" was used for the analysis (Srinivasan & Ramachandra, 1989). Some composite materials and synthetic fibers with known bi-modular behavior were compared to wood and wood fibers in an attempt to find a new model for wood and wood fiber (Conners & Medvecz, 1992). An analytical model for Euler buckling was developed for composite structural members with open or closed thin-walled sections. Failure envelopes for some commercially available structural shapes were presented and the presented analytical model could be used to predict the behavior of any new material (Barbero & Raftoyiannis, 1993). The analytical solution was deduced for bending-compression column subject to combined loadings by the flowing coordinate system and phased integration method. The finite element program was compiled for calculation, and the comparison between the result of finite element and analytical solution were given too (Yao & Ye, 2004).

The flexural problem was chosen to investigate the response of bi-modular material, since the two regions, one of tension and one of compression, can be identified easily using simple intuition. The stress was assumed continuous across the boundary of the two regions and the discontinuities of stress across the boundary of the two regions problems were considered (Michel Destrade et al., 2009).

A semi-analytical method for the critical buckling loads of variable–cross section slender rods with different moduli was developed based on the variational principle. By developing a nonlinear iterative program and using the variational iteration method, the critical buckling loads were obtained. Then, buckling tests and numerical simulation were conducted for slender rods made from graphite with different moduli (Yao & Ma, 2013).

A numerical model to simulate the nonlinear behavior of slender RC columns considering the long-term deformations of concrete was presented (Kwak & Kim, 2006). The effect of a material uncertainty on the buckling of inhomogeneous reinforced concrete columns was investigated (Shahsavar et al., 2013), material properties affect the critical value of the buckling loads. Also sensitivity analysis of critical loads of various parameters such as $E$, $I$ and $L$ was investigated.

An investigation of reinforced concrete columns and beam-columns were carried out (Tim & Hansen, 2002). A linear elastic – perfectly plastic material behavior of the reinforcement and a parabolic material behavior of the concrete with no tensile strength were assumed, the behavior of columns and beam-columns are analyzed numerically and compared with experimental data

## 2. Neutral Axis Position:

Consider a uniform slender rod of length $\boldsymbol{L}$, with rectangular cross section of dimensions $\boldsymbol{b \times h}$. The column own weight is neglected and the column is subjected to the compression force $\boldsymbol{P}$. When the compression increases to the critical buckling load the bending deformation occurs in the slender column and the cross section is divided into a tension zone and compressive zone as shown in the Figure 1.

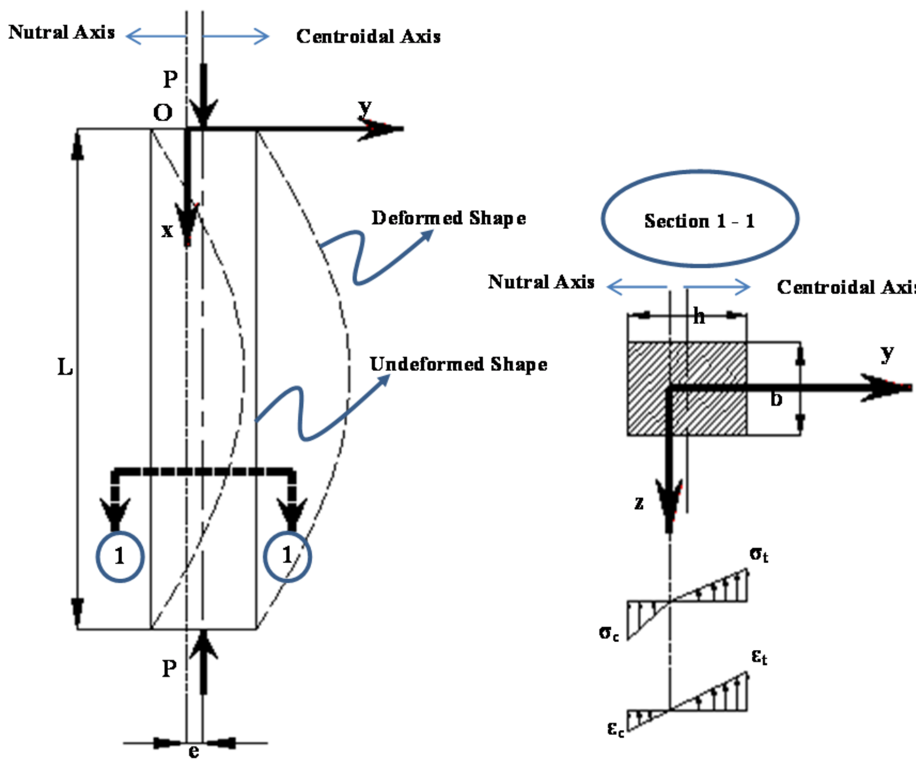


Figure 1. Geometrical properties of Considered Column and Cross Section 1 – 1 of Column With Stress and Strain Distribution

One should note that the neutral axis of the cross section will not coincide with the centroidal axis, where the column is constructed of bi-modular material, and it can be deduced for stress and strain distribution shown in the cross section.

As shown in Figure 1, the origin point was chosen to lie on the neutral axis and hence when the centroidal axis became at the right of the neutral axis the eccentricity distance ***e*** will be positive and when it became at the left of the neutral axis, it will be negative.

One should not that neutral should not be out of the section to ensure that the concept of bi-modular material is satisfied. From Figure 1, the position of the neutral axis can be determined as follows:

$$\frac{\varepsilon_t}{\varepsilon_c} = \frac{\frac{h}{2}+e}{\frac{h}{2}-e} \tag{1}$$

$$\frac{\varepsilon_t}{\varepsilon_c} = \frac{\frac{1}{2}+\frac{e}{h}}{\frac{1}{2}-\frac{e}{h}} \tag{2}$$

Put $\frac{e}{h} = \bar{e}$, and it will be called non-dimensional eccentricity.

$$\frac{\varepsilon_t}{\varepsilon_c} = \frac{\frac{1}{2}+\bar{e}}{\frac{1}{2}-\bar{e}} \tag{3}$$

Hence:

$$\frac{\sigma_t}{\sigma_c} = \frac{E_t\left(\frac{1}{2}+\bar{e}\right)}{E_c\left(\frac{1}{2}-\bar{e}\right)} \tag{4}$$

$$\frac{\sigma_t}{\sigma_c} = E^* \frac{\left(\frac{1}{2}+\bar{e}\right)}{\left(\frac{1}{2}-\bar{e}\right)} \tag{5}$$

Where:

$E^*$is the bi-modular ratio$\frac{E_t}{E_c}$.

The compression force ***P***at any cross section can be calculated as:

$$P = \frac{1}{2}\sigma_c b\left(\frac{h}{2} - e\right) - \frac{1}{2}\sigma_t b\left(\frac{h}{2} + e\right) \tag{6}$$

By substitution of equation (5) in equation (6), one can obtain that:

$$P = \frac{\sigma_c bh}{2\left(\frac{1}{2} - \bar{e}\right)}\left[\left(\frac{1}{2} - \bar{e}\right)^2 - E^*\left(\frac{1}{2} + \bar{e}\right)^2\right] \tag{7}$$

Also, The moment can be obtained as:

$$M = \frac{1}{6}\sigma_t b\left(\frac{h}{2} + e\right) * (h - e) * b + \frac{1}{6}\sigma_c b\left(\frac{h}{2} - e\right) * (h + e) * b \tag{8}$$

By substitution of equation (5) in equation (8), one can obtain that:

$$M = \frac{\sigma_c bh^2}{6\left(\frac{1}{2} - \bar{e}\right)}\left[\left(\frac{1}{2} - \bar{e}\right)^2(1 + \bar{e}) + E^*\left(\frac{1}{2} + \bar{e}\right)^2(1 - \bar{e})\right] \tag{9}$$

From the equilibrium at any section:

$$M = -P * W \tag{10}$$

Where, $W$ is the transverse deflection of the column.

By substitution from equations (7) and (9) in (10), one obtains:

$$\bar{W} = \frac{1}{3}\frac{E^*\left(\frac{1}{2} + \bar{e}\right)^2(1 - \bar{e}) + \left(\frac{1}{2} - \bar{e}\right)^2(1 + \bar{e})}{E^*\left(\frac{1}{2} + \bar{e}\right)^2 - \left(\frac{1}{2} - \bar{e}\right)^2} \tag{11}$$

Where:

$\bar{W} = \frac{W}{h}$is the non dimensional transverse deflection.

Equation (11) is the non-dimensional relation between deflection and the eccentricity (the position of the neutral axis), one should note that $|\bar{e}| < \frac{1}{2}$ to keep the concept of bi-modular material.

The following Figure 2 shows plots of the Equation 11:

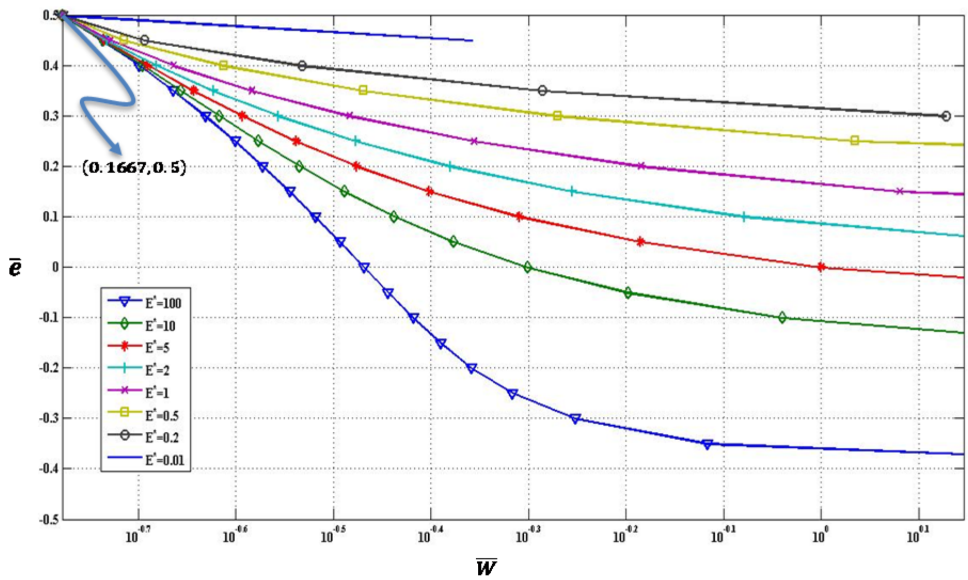


Figure 2. Relation Between Non-dimensional Transverse Deflection and Non-dimensional Eccentricity

It can be noted from the above Figure 2 that the bi-modular section of the column is considered from the non-dimensional eccentricity of $\frac{1}{2}$, which is called the boundary of the core of the considered section.

## 3. Eigenvalue Analysis

Herein, the axial deformation of the column is neglected, small transverse deflection and plane cross section assumptions are also taken. The bending normal strain of a random point in the x- direction can be expressed as:

$$\varepsilon_x = y\frac{d^2W}{dx^2} \tag{12}$$

Then, for section of bi-modular material:

$$\sigma_t = E_t * y\frac{d^2W}{dx^2}\ , \quad \sigma_c = E_c * y\frac{d^2W}{dx^2} \tag{13}$$

Where:

$\sigma_t$ and $\sigma_c$ are the tension and compression stresses.

The bending moment at any section can be determined as:

$$M = \int_0^{\frac{h}{2}+e} b\sigma_t y dy + \int_0^{\frac{h}{2}-e} b\sigma_c y dy \tag{14}$$

By substitution from the equation (14) into equation (15), one can obtain:

$$M = bE_t\frac{\left(\frac{h}{2}+e\right)^3}{3}\frac{d^2W}{dx^2} + bE_c\frac{\left(\frac{h}{2}-e\right)^3}{3}\frac{d^2W}{dx^2} \tag{15}$$

The above equation (16) can be put in the following form:

$$M = \frac{bh^3}{3}E_c\left[\left(\frac{1}{2}+\bar{e}\right)^3 E^* + \left(\frac{1}{2}-\bar{e}\right)^3\right]\frac{d^2W}{dx^2} \tag{16}$$

By substitution from equation (17) in equation (10), one obtains:

$$\frac{bh^3}{3}E_c\left[\left(\frac{1}{2}+\bar{e}\right)^3 E^* + \left(\frac{1}{2}-\bar{e}\right)^3\right]\frac{d^2W}{dx^2} = -P * W \tag{17}$$

Equation (18) can be rearranged and put in non-dimensional form as:

$$\frac{d^2\bar{W}}{d\bar{x}^2} = -\bar{P}\frac{\bar{W}}{\left[\left(\frac{1}{2}+\bar{e}\right)^3 E^* + \left(\frac{1}{2}-\bar{e}\right)^3\right]} \tag{18}$$

Where:

$\bar{x} = \frac{x}{L}$ is the nondimentional coordinate.

$\bar{P} = \frac{3PL^2}{bh^3E_c}$ is the nondimensiol force.

As previously mentioned that bi-modular section is considered from the non-dimensional eccentricity of $1/2$, hence the column of bi-modular material should not be governed by the single differential equation.

The Euler's formula should be applied to the parts that don't behave as a bi-modular section, then the governing equation for these parts is:

$$\frac{d^2\bar{W}}{d\bar{x}^2} = -\frac{\bar{P}}{4}\bar{W} \tag{19}$$

*3.1 Boundary Conditions*

$$\left.\begin{aligned} &\bar{W}(0) = 0, \quad \frac{d^2\bar{W}}{d\bar{x}^2}(0) = 0 \\ &\bar{W}(1) = 0, \quad \frac{d^2\bar{W}}{d\bar{x}^2}(1) = 0 \end{aligned}\right\} \tag{20}$$

*3.2 Continuity Conditions*

Again at the section where the column transferred from unimodular to bi-modular and vise versa the non-dimensional transverse deflection $\bar{W} = \frac{1}{6}$ (i. e $\bar{e} = \frac{1}{2}$).

**5. Solution of Governing Differential Equation:**

One can note that both the governing differential equations (18) and (19) are Eigen and like Eigenvalue form, and it can be noted that the equation (18) should be solved simultaneously with equation (11). The equation (11) has been plotted to get the corresponding non-dimensional eccentricity $\bar{e}$ for buckling nondimensional transverse deflection, consequently the critical buckling load can be determined.

It is well known that this is an instability ($\bar{W} \to \infty$ ), so determining the corresponding value of nondimensional eccentricity $\bar{e}$ is the main target through the plot of the nondimensional eccentricity versus nondimentional transverse deflection on semi log scale.

Assume the solution of the governing differential equation (19) is in the well known form:

$$\bar{W} = Asin\left(\sqrt{\lambda}\bar{x}\right) + Bcos\left(\sqrt{\lambda}\bar{x}\right) \tag{21}$$

Where:

$$\lambda = \frac{\bar{P}}{\left[\left(\frac{1}{2}+\bar{e}\right)^3 E^* + \left(\frac{1}{2}-\bar{e}\right)^3\right]} \tag{22}$$

From boundary condition, equation (20):

$\bar{W}(0) = 0 = 0 + B$

Hence $B = 0$

And, where $\bar{W}(1) = 0 = Asin\left(\sqrt{\lambda}\bar{x}\right)$

Hence $A = 0$, which leads to trivial solution.

$$\text{Or } \sqrt{\lambda} = n\pi \tag{23}$$

Where $n = 0, 1, 2, ....$

Then:

$$\pi^2 = \frac{\bar{P}}{\left[\left(\frac{1}{2}+\bar{e}\right)^3 E^* + \left(\frac{1}{2}-\bar{e}\right)^3\right]} \tag{24}$$

**6. Bi-modular Material Approach For Reinforced Concrete Columns:**

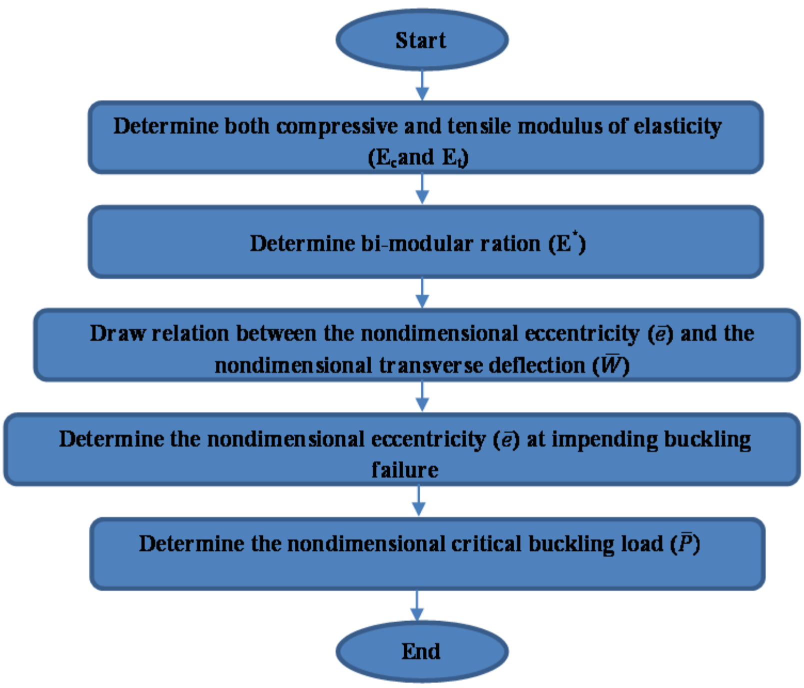


Figure 2. New Approach Procedure

Modern researches show that the concrete can be considered as bi-modular material where the tensile modulus of elasticity is about 1 - 1.3 times the compressive modulus of elasticity, this property leads to a new approach to dealing buckling problem of reinforced concrete columns, where at buckling state the stress distribution will be as shown in Figure 1 then the new solution of the buckling problem can be summarized in the following Figure 2.

**7. Results and Discussion**

O.Bauman (Tim Gudmand-Høyer and Lars Zenke Hansen 2002) investigated simply supported, concentrically loaded columns with cross section 200× 100 mm, W.Gehler and A.Hütter (Tim Gudmand-Høyer and Lars Zenke Hansen 2002) investigated columns with cross section 160× 140 mm and F. N. Pannell and J. L. Robinson (Tim Gudmand-Høyer and Lars Zenke Hansen 2002) investigated columns with cross section 95.3× 63.5 mm, different slenderness and reinforcement ratios were considered. The behavior of columns was analyzed numerically, according to the new approach and compared with the different experimental data. The following Tables (2), (3) and (4) show the results of the new approach and experimental data.

Table 2. Comparison of Bi-modular Approach and *W. Gehler and A. Hütter* Experimental Data

| **b(m)** | **h(m)** | **$f_c$(Mpa)** | **L/h** | **$E_c$(Mpa)** | **$E_t$(Mpa)** | **$E^*$** | **$P_{new\ App.}$** | **$P_{Exp.}$(KN)** | **$P_{new\ App.}/P_{Exp.}$** |
|---|---|---|---|---|---|---|---|---|---|
| 0.16 | 0.14 | 19.3 | 40 | 19300 | 2.05E+04 | 1.062176 | 229.3238 | 249.55 | 0.9189 |
| 0.16 | 0.14 | 19.4 | 30 | 19400 | 2.05E+04 | 1.056701 | 408.7111 | 391.9 | 1.0428 |
| 0.16 | 0.14 | 20.7 | 25 | 20700 | 2.35E+04 | 1.135266 | 651.971 | 515.6 | 1.2644 |
| 0.16 | 0.14 | 24 | 40 | 24000 | 3.00E+04 | 1.25 | 311.1429 | 305.7 | 1.0178 |
| 0.16 | 0.14 | 13.4 | 30 | 13400 | 1.34E+04 | 1 | 274.5228 | 257.7 | 1.0652 |

Table 3. Comparison of Bi-modular Approach and *F. N. Pannell and J. L. Robinson* Experimental Data

| **b(m)** | **h(m)** | **$f_c$(Mpa)** | **L/h** | **$E_c$(Mpa)** | **$E_t$(Mpa)** | **E*** | **$P_{new\ App.}$** | **$P_{Exp.}$(KN)** | **$P_{new\ App.}/P_{Exp.}$** |
|---|---|---|---|---|---|---|---|---|---|
| 0.0953 | 0.0635 | 19.1 | 41.6 | 19100 | 2.05E+04 | 1.0732 | 56.9919 | 60.9 | 0.9358 |
| 0.0953 | 0.0635 | 18.3 | 41.6 | 18300 | 1.83E+04 | 1 | 52.6743 | 74.7 | 0.7051 |
| 0.0953 | 0.0635 | 17 | 27.2 | 17000 | 1.70E+04 | 1 | 114.457 | 99.6 | 1.1491 |
| 0.0953 | 0.0635 | 21.3 | 32 | 21300 | 2.40E+04 | 1.1267 | 110.180 | 98.7 | 1.116 |

Table 4. Comparison of Bi-modular Approach and *O. Bauman* Experimental Data

| **b(m)** | **h(m)** | **$f_c$(Mpa)** | **L/h** | **$E_c$(Mpa)** | **$E_t$(Mpa)** | **$E^*$** | **$P_{new\ App.}$** | **$P_{Exp.}$(KN)** | **$P_{new\ App.}/P_{Exp.}$** |
|---|---|---|---|---|---|---|---|---|---|
| 0.178 | 0.14 | 27.7 | 32.1 | 27700 | 3.15E+04 | 1.1371 | 589.247 | 685.4 | 0.8597 |
| 0.198 | 0.098 | 26.2 | 32.8 | 26200 | 3.10E+04 | 1.1832 | 424.597 | 392.8 | 1.080 |
| 0.2 | 0.1 | 26.2 | 32.1 | 26200 | 3.10E+04 | 1.1832 | 456.934 | 402.6 | 1.134 |
| 0.25 | 0.16 | 35.3 | 40.7 | 35300 | 3.53E+04 | 1 | 701.6397 | 667.8 | 1.0506 |

From the above Tables (2), (3) and (4), good agreement has been found. Also for more verification of the bi-modular approach the results of calculations according to the some codes of practice like Danish Code of Practice [18] and ACI [17] have been compared with the new approach as shown in the following Table (5).

Table 5. Comparison of Bi-modular Approach and *Danish & ACI Codes of Practice*

| b(m) | h(m) | $f_c$(Mpa) | L/h | $E_c$(Mpa) | $E_t$(Mpa) | $P_{new\ App.}$ | $P_{ACI.}$(KN) | $P_{DAN.}$(KN) | $P_{new\ App.}/P_{DAN.}$ | $P_{new\ App.}/P_{ACI.}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.16 | 0.14 | 19.3 | 40 | 19300 | 2.05E+04 | 229.323 | 237.942 | 160.074 | 1.4326 | 0.9637 |
| 0.0953 | 0.0635 | 19.1 | 41.6 | 19100 | 2.05E+04 | 56.991 | 59.123 | 49.550 | 1.1501 | 0.9639 |
| 0.25 | 0.16 | 35.3 | 40.7 | 35300 | 3.53E+04 | 701.639 | 555.040 | 506.22 | 1.386 | 1.2641 |

A good, but a bit conservative, agreement has been found which may due to the conservative nature of the codes of practice.

To stand on the efficiency of the new bi-modular approach, the critical buckling load of bi-modular approach with respect to the critical buckling load of experimental and DAN & ACI codes results versus the bimodular ratio $E^*$ was plotted as shown in the Figure 3.

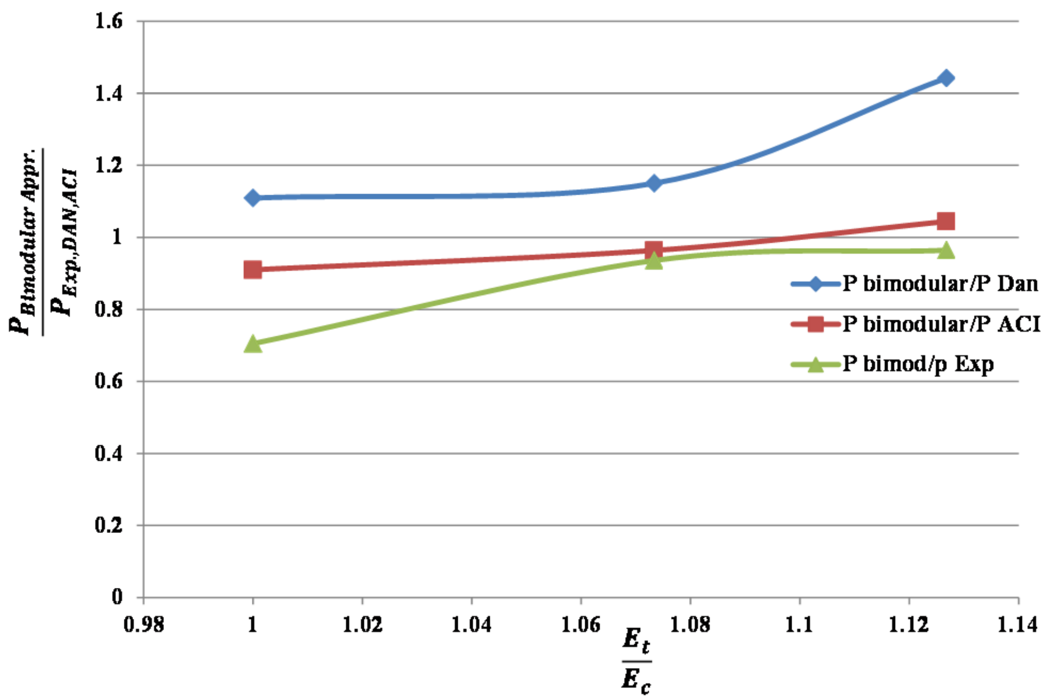


Figure 3. Bimodular Approach Results With Respect to Experiment and DAN & ACI Codes Results Versus Bi-modular ratio ($E^*$)

From the above figure, it can be noted that as the bi-modular ratio increase the results of bimodular approach became close to the experimental results, which mean that this approach made a good model for the buckling problem of the column. Also, it can be noted that as the bi-modular ratio increase above 1.1 the results of the critical buckling load according to the codes of practice began to diverge from the results of experiments.

## 8. Conclusion

The study of the buckling problem of reinforced concrete columns using bi-modular concept gives good agreement with experimental and codes of practice results and leads to:

- As the bimodular ratio $\left(E^* = \frac{E_t}{E_c}\right)$ of the concrete increases the new bi-modular approach is efficient in studying the reinforced concrete columns buckling problem.
- As the bimodular ratio $\left(E^* = \frac{E_t}{E_c}\right)$ reaches 1.1 the DAN and ACI codes of practice diverge away from experimental results.

As a future work, the reinforcement ratio effect should be studied to improve the bimodular approach

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**Notation**

$A$ is the constant of transverse deflection function.

$B$ is the constant of transverse deflection function.

$b$ is the width of the column cross section.

$h$ is the depth of the column cross section.

$L$ is the length of the column.

$P$ is the concentric compression force.

$\varepsilon_t$ is the tensile strain.

$\varepsilon_c$ is the compressive strain.

$e$ is the eccentricity between neutral and centroidal axes.

$E^*$ is the bi-modular ratio $\frac{E_t}{E_c}$.

$E_t$ is the tensile modulus of elasticity.

$E_c$ is the compressive modulus of elasticity.

$\bar{e}$ is the non-dimensional eccentricity.

$\sigma_t$ is the tensile stress.

$\sigma_c$ is the compressive stress.

$M$ is the moment.

$W$ is the transverse deflection of the column.

$x$ is the coordinate in the direction of the column length.

$\bar{x}$ is the nondimentional coordinate.

$\bar{P}$ is the nondimensiol concentric force.

$\varepsilon_x$ is the strain in the x- direction.

$\lambda$ is the eigen value.

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