State Space Methods and Examples for Computational Models of Human Movement

H. Hemami[1] & M. Hemami[2]

[1] Dept. of Electrical and Computer Engineering, The Ohio State University, Columbus, Ohio 43210, USA

[2] Dept. of Mechanical Engineering, Esfahan Inst. of technology, Esfahan, Iran

Correspondence: H. Hemami, Dept. of Electrical and Computer Engineering, The Ohio State University, Columbus, Ohio 43210, USA. Tel: 1-614-888-7739.E-mail: hemami.1@osu.edu

Abstract

In the past, multi-rigid-body systems have been formulated by Lagrangian and Hamiltonian dynamics, the Newton-Euler method and Kane's dynamic equations. Availability of large computers and versatile software systems enables us to formulate larger systems and analyze them computationally. In such circumstances, the probability of human error grows with the size of the system. The purpose of this work is to provide state space formulations that allow verification of computational results and be able to transport Lyapunov stability results across the these dynamics disciplines. The formulations are presented with matrices for all transformations and projections.

This work also investigates the constraint forces and their computation or elimination by different methods. A one-link constrained rigid body is considered first. The results are summarily extended to a three-link system. Six three-link rigid body sub modules are interconnected to describe, control and simulate many different maneuvers and activities of humans.

The actuators have alpha and gamma inputs, and pull but cannot push. The control strategy is based on Evarts' "attention set,", and is applied to the movement of one arm in a computational experiment.

Keywords: dynamics, state space, Newton-Euler, Lagrange, Hamilton, Kane

1. Introduction

Understanding the complexity of natural human and animal movements can only be unraveled by construction of large and anatomically accurate models that imitate the movement (Andriacchi, & Alexander, 2000; Gruber, Ruder, Denoth, & Schneider, 1998; Humphrey & Hemami, 2010; Ihlen, Skjaeret, & Vereijken, 2012; Lebiedowska, Wente, & Dufour, 2009). An important part of such a development is the construction of computational models that include all skeletal bones as single rigid bodies (Charles & Hogan, 2010; Onyshko & Winter, 1980; Rankin & Neptune, 2012). Computational and animation tools (Onyshko & Winter, 1980; Pandy & Berme, 1988; Popovich, Goswami, & Herr, 2005; Zheng & Hemami, 1986) are playing a more prominent role in modeling and understanding systems where direct experimental measurements are not possible or feasible. An important example is the control machinery of the central nervous system (CNS) involved in movement (Huston & passerello, 1971; Iqbal & Pai, 2000; Iqbal & Roy, 2009; Thelen, Anderson, & Delp, 2003; Wittenburg, 1989).

This paper is a preliminary step to develop a larger and more comprehensive skeletal model for studying human movement (Smith, 1973). The model must be able to imitate human movement (Blajer, & Czaplicki, 2001; Khosravi-Sichani, 1985; Khosravi-Sichani & Hemami, 1992; Khosravi-Sichani & Hemami, 1987), make and break contact (Blajer, 1997) with the environment and objects in the environment, and allow for inclusion of a large number actuators that imitate the geometry, physiology and structure of human musculature.

State space formulations are perhaps the only way to construct large skeletal models that yield to computer programming of rotational and translational motions (Popovich, Goswami, & Herr, 2005; Winter & Stark, 2012). Variable structure and sliding mode systems (Utkin, Guldner, & Shi, 1999; Lee & Utkin, 2007; Feng & Utkin, 1996) have traditionally used Lagrangian (LA) dynamics. Kane's method (K) is useful in the derivation of the

equations of motion and computations of the constraint forces (Fluit, Andesen, Kolk, Verdonschot, & Koop-man, 2014; O Reilly, 2008, 2010). A three-link rigid-body system, in Newton-Euler (NE) state space formulation, is used in sliding mode control and Lyapunov studies (Wu, 1996). In this regard, the application of state space (Bay, 1999) formulations of the rotational motion of one rigid body, anchored at its center of gravity (COG) (Minetti, Cisotti, & Mian, 2011), is a convenient way to compare the NE, LA, Hamilton (H) and K methods (Humphrey, Hemami, Barin, & Krishnamurthy, 2010; Jalics, Hemami, Clymer, & Groff, 1997; Schiehlen, 2006; Schwertassek & Roberson, 1986; Tomovic, Popovic, & Stein, 1991) for multi-body applications (Yoshioka, Nagano, Hay, & Fukashiro, 1986). The transformations between the different state spaces are one way to check the validity of the equations and results.

Based on the state space formulations of NE, we introduce a six-sub module, 18-link three-dimensional skeletal model. Each sub module is made of a three-link system. The six sub modules represent the two lower extremities (Pope, Crowninshield, Miller, & Johnson, 1976; Ren, Jones, & Howard, 2006), the torso (Minetti, Cisotti, & Mian, 2011; Winter & Stark, 1988), the two arm-lower arm-hand limbs (Charles & Hogan, 2010; Rankin & Neptune, 2012; Reich & Daunicht, 2000; Williams, Schmidt, Disselhorse-Klug, 2005). and the head and neck three segment. This 18-link module can also be used to model each hand and each foot. The hand module has a three-link palm to which five three-link fingers are attached. Therefore, a 90-link human skeletal model becomes available.

For ease of presentation, we consider point connections and holonomic constraints at all joints. All three-link sub modules have three translation and nine rotation degrees of freedom. Each sub module has its own internal muscle-like actuators that span joints and may span a segment or a sub module (Ajay & Pandy, 2007; Buchner, Hines, & Hemami, 1988; Winter & Stark, 2012).

The different sub modules are coupled by cross-module forces of connection and muscle-like actuators (Rankin & Neptune, 2012). The system has 72 degrees of freedom and five internal cross-joint forces. Two ground contact forces support the system. The degrees of freedom can be reduced, by projection, to 54 degrees of rotational freedom plus three degrees of translational freedom.

The control strategy here is based on Evarts' "attention set," (Evarts, Shinoda, & Wise, 1984; Evarts, 1979) i.e., the system is separated into two disjoint subsystems: the subsystem involved in the maneuver and the rest of the system staying motionless and in support of the active subsystem (Kuo & Zajac, 1993; Winter, Prince, Powell, & Zabjik, 1996). Both are controlled by co-activation (Williams, Schmidt, Disselhorst-Klug, & Rau, 2006; Blajer, 1997; Blajer & Schiehlen, 1992).

In Section Two, a single body is formulated in four different state spaces. Constraint forces are addressed in section Three. Multi rigid body systems and three-body sub modules are presented in Section Four. The six-sub module system and a particular actuation system are presented in Section Five. Computational experiments are briefly presented in Section Six. Discussions and conclusions follow in Section Seven.

2. Single Rigid Body

The state space formulation of the single rigid body is an important step in developing multi-rigid-body dynamics that do not use tensors (Pellionisz & Llinas, 1979, 1980) and are amenable to dimensional augmentation and dimensionality reduction by projection. All required matrices are synthesized from 3×3 matrices. The states are the three Euler or Bryant angles Θ and the angular velocity Ω (as expressed in BCS) for the NE method, They are Θ and $\dot{\Theta}$ in K and LA formulations, Θ and the angular momenta P in H method.

For systems with a natural skeleton, the NE formulation of the dynamics is easier to implement. For combinations of natural and man-made systems, such as an athlete with skates or bicycles in the air, Kane's method seems more appropriate. For deform-able and flexible bodies, the L and H methods may be more suitable. For understanding the energy conversion machinery of a single muscle, all four methodologies may be necessary.

Transformations between these state space formulations are desirable for the development of computational software development and use. All are compact and computer-adapted formulations that facilitate human interaction with computers (Marshall, Jensen, & Wood, 1985; Mansouri & Reinbolt, 2012). The LA and H methods provide energy functions that come handy in defining Lyapunov functions (Antsaklis & Michel, 1997; Winter, Prince, Powell, & Zabjik, 1996) and stability (Honarvar & Nakashima, 2012).

2.1 Newton-Euler Formulation

The NE method (Hemami, 2002) makes use of three matrices: A, B and C. Matrix A describes the transformation of vectors from the body coordinate system (BCS) to the inertial coordinate system (ICS). Matrix B relates $\dot{\Theta}$ and

Ω. Matrix C describes the constraints of interaction with, connection to and contact with other objects and with the environment. For examples of NE formulations, see (Amirouche, 1992; Hemami & Wyman, 2012) and Appendix A.

2.2 Kane's Formulation

The objective is to derive Kane's equations (Kane & Levinson, 1985; O Reilly, 2008) for a rigid body, connected to the origin of the inertial coordinate system (ICS) at its center of gravity. Let the three-vector T be the generalized torques acting on the system as defined by Kane. The key to the transformation is to relate the generalized holonomic torque vector T appearing in Kane's equations to the torque vector N applied to the BCS axes of the rigid body. The power p delivered by T is the inner product

$$p = T^{'} \dot{\Theta} \tag{1}$$

The same power in the NE method is given by

$$p = N^{'} \Omega \tag{2}$$

From the above two equations and the equality:

$$\dot{\Theta} = B(\Theta)\Omega, \tag{3}$$

it follows that

$$N^{'} B^{-1} = T^{'} \tag{4}$$

Let us define the transpose of B as B_t, the inverse B as B_i and the inverse of the transpose of B as B_{it}. Therefore,

$$T = B_{it}N$$

Thus, by multiplying the NE equations by B_{it}, one obtains Kane's equations. Conversely, by multiplying Kane's equations by B_t, one obtains NE equations.

There is one computational difficulty with matrix B. It is singular at

$$\Theta_2 = \pi/2$$

this problem can be computationally addressed by using a finite large number for the inverse of the cosine function at the critical value or change the coordinate system in that vicinity.

2.3 Lagrange's Formulation

The derivation of the Lagrange'e equations of motion (McCuskey, 1959; O Reilly, 2008) is carried out in (Hemami, 2002) starting with the Lagrangian (Lam, 1998; Wittenburg, 1989). We provide an alternative derivation here which is more general and may also be easily applicable to multi body systems. The equation for $\dot{\Theta}$ is differentiated with respect to time.

$$\ddot{\Theta} = -B(\dot{B}_i)\dot{\Theta} + B(\dot{\Omega}) \tag{5}$$

By multiplying both side of Eq. 5 by $B_{it}J(B_i)$, and recalling

$$f(\Omega) = -\check{\Omega}J\Omega,$$

one obtains the Lagrangian equations:

$$B_{it}JB_i\ddot{\Theta} = -B_{it}J\dot{B}_i\dot{\Theta} + B_{it}fB_i\dot{\Theta} + (B_{it})N \tag{6}$$

The above development points to an indirect method of deriving the Lagrangian angular acceleration variables: $((\ddot{\Theta})$ from available NE (or K) state space equations. One adds the angular accelerations as outputs to the state space formulation. These outputs are functions of the states and inputs, i.e., the structure is that of a Mealy machine as opposed to Moore machines where the output is a function of the state alone. The system has a feed through component (Bay, 1999).

2.4 Hamilton Formulation

In the Hamiltonian formulation, the state variables are defined as Θ and P, where

$$P = \partial L/\partial \dot{\Theta} \tag{7}$$

It follows that

$$H = 0.5 P_t J_L P.$$

and

$$\dot{P} = (\partial(P_t B)/\partial\Theta)(J^{-1})B_t P \tag{8}$$

The latter equation appears to be compact and simple, but it involves derivation of the tensor $\partial B/\partial\Theta$ or the simpler form $\partial P^T B/\partial\Theta$.

The relation between P and Ω is

$$P = B_{it}J\Omega \tag{9}$$

3. Forces of Constraint

Briefly, forces of constraint either appear in the equations of motion (Pope, Crowninshield, Miller, & Johnson, 1976; Hemami & Wyman, 1979; Popovich, Goswami, & Herr, 2005) or are not part of the equations of motion (Kane & Levinson, 1985). According to Kane's method, additional degrees of freedom are added such that the forces are computable.

When the forces appear in the dynamics, two methods exist to derive the forces. The physical constraints may be differentiated and combined with the system equations. The latter system can be simultaneously solved for the accelerations and the forces (Smith, 1973) Alternatively, the accelerations can be eliminated and the forces derived as functions of the state of the system and the inputs (Hemami & Wyman, 1979).

For the single rigid body system above, the procedure is as follows: According to Kane's method, one adds three translation degrees of freedom X to the hinge. Let X and Θ be the six generalized coordinates. Let Γ be the vector force of constraint at the center of gravity (COG). Let T be the sum of all torques acting on the body - some being independent couples and some, due to external forces, represented here by their resultant force acting at COG: F_i Let

$$F_r^* = [F_1^*, F_2^*, F_3^*].$$

This means

$$F_r^* = (J\dot{\Omega} + \breve{\Omega}J\Omega)B_i.$$

Similarly let

$$F_r = [F_1, F_2, F_3].$$

Kane's equations of motion, for rotation, are

$$F_r^* + F_r = 0.$$

For the translation, we have

$$F_t^* = -m\ddot{X}.$$

and

$$F_t = R_i + \Gamma.$$

Now, the constraints are imposed, i.e.,

$$\ddot{X} = 0.$$

From which it follows that Γ is equal to the negative of all applied forces.

4. Multi-segment Rigid Body Systems

With the above development, the Lagrangian equations of motion for a multi rigid body system can be derived. However, use of tensors for derivation and partial derivations may be necessary as the dimensions increase.

An alternative is to use K or NE dynamics in state space form, and add $\ddot{\Theta}$ as Mealy outputs of the latter as stated earlier. There is an inertial coordinate system (ICS) and every rigid body has its own body coordinate system (BCS). All six sub modules have identical dynamics. The three inertial axes are front, left and vertically upwards (a right hand coordinate system with roll, pitch and yaw.)

The state space of every rigid body consists of Θ and Ω. The 12- dimensional equations of the sub module are given in Appendix A (Hemami & Wyman, 2012; Hemami & Zheng, 2012). This system is supplied with muscle-like actuators that are centrally commanded (Ajay & Pandy, 2007; Buchner, Hines, & Hemami, 1988; Winter & Stark, 2012; Hogan, 1984; Hemami, Barin, & Pai, 2006; Pai, 2010; Saraf, Ramesh, Lennon, & Roberts, 2006). Standard controller design methods (Hemami & Stokes, 1983; Jalics, Hemami, & Clymer, 1997) and the inverse system (Blajer, & Czaplicki, 2001) with the classical feedback structures (Smith, 1958; Zames, 1981) are applicable here. Co-activation amounts to position and velocity feedback (Hemami, Barin, & Pai, 2006). Lyapunov stability can be used to show that the system is stable in the regions of interest. We further assume the CNS produces the trajectories.

4.1 The Sub modules Dynamics

A skeletal model of a human or animal could be constructed from connecting a large number of rigid bodies (Amirouche, 1992; Wittenburg, 1989)

While the large system could be constructed with individual rigid bodies, a better compromise would be to construct three rigid body sub modules first. The larger system would then be constructed from a number of such sub modules. The movements of the sub modules may be conceived, programmed and/or implemented by parts of the CNS. Therefore, the physical model of the skeleton may be in certain ways closer to the natural structures of the CNS.

There are three different sub module types in this system. The simplest one is that of the upper extremities and the neck and head. These three modules each have two internal forces of connection (constraint) and one external force. The two lower extremities have two internal and two external forces as shown in Figure 1.
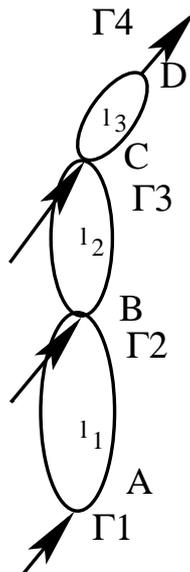
Figure 1. The three-link sub-module with two internal and two external forces. The lengths of the three segments are, respectively, l_1, l_2 and l_3.

4.2 The Torso Sub Module

The torso sub module, as shown in Figure 2, has five external forces and two internal forces. Initially, the BCS of the three links coincide with the ICS. The sequence of angles is roll, pitch, and yaw. The rotational space is considered first, and it is assumed that the torque vector N, expressed in the BCS operates on the system.
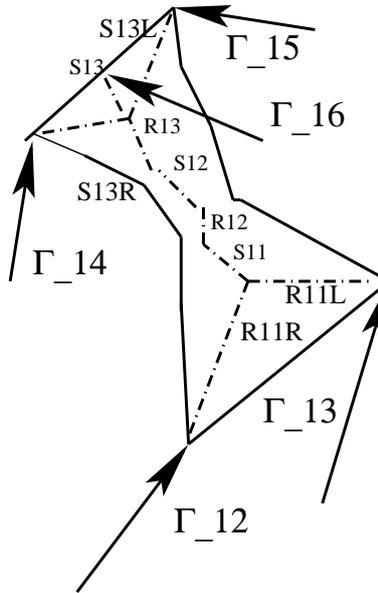
Figure 2. The three-link torso sub module with two internal and five external forces

The other two simpler sub modules (Hemami & Wyman, 2012; Hemami & Zheng, 2012) can be inferred from the torso sub module.

The 36-dimensional states of positions and angular and translational velocities are

$$Z = [X_1', \Theta_1', X_2', \Theta_2', \Theta_3', X_3']' \tag{10}$$

The velocity vector V is

$$V = [V_1', \Omega_1', V_2', \Omega_2', V_3', \Omega_3']' \tag{11}$$

A projection operator P eliminates the internal forces and maintains the five external forces.

4.3 Inter Sub Module Dynamics

The six sub module system is illustrated in Figure 3:
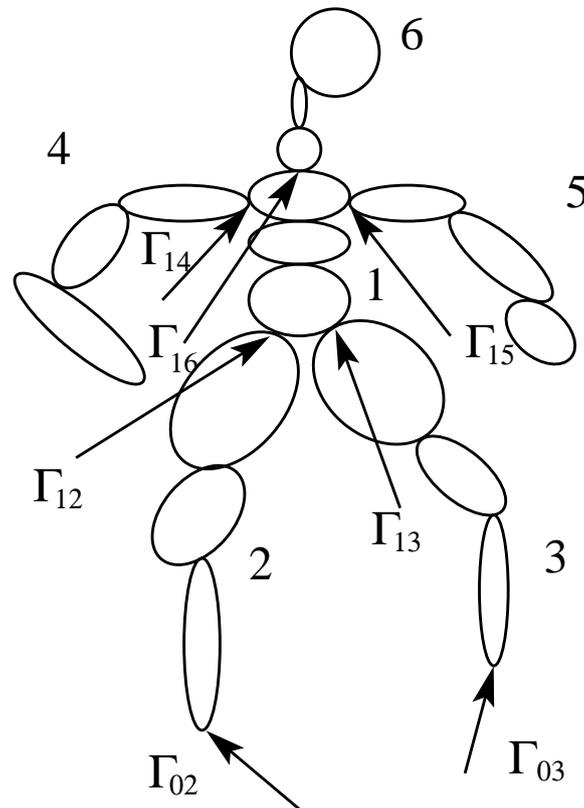
Figure 3. The six-sub module biped with five internal and two external support forces.

- Module 1 is the torso, composed of three segments.

- Module 2 is the right leg.

- Module 3 is the left leg

- Module 4 is the right hand.

- Module 5 is the left hand

- Module 6 is the three-segment neck and head.

The external forces are Γ_{02} and $\Gamma_0 3$ at the support points with the ground. The external forces on the torso are Γ_{12} and Γ_{13} for the left foot, Γ_{14} and Γ_{15} between the torso and the hands, and finally Γ_{16} between the torso and the neck Two alternative models of the dynamics are considered. In the first model, the system remains of dimension 72, and the formulation allows the joint forces and the forces of support to be available in the dynamics and continuously observable (Figure 4).
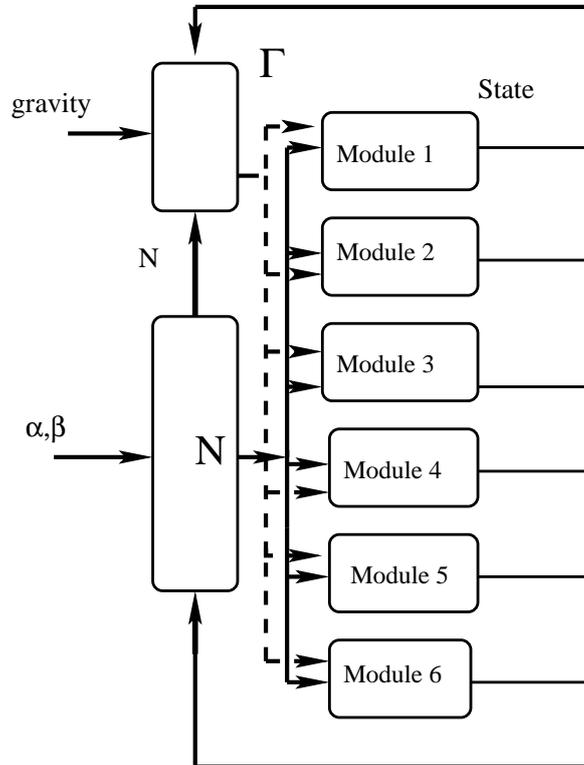
Figure 4. The block diagram of the six-sub module system

The alternative is to eliminate the five joint forces and reduce the dimension to 57. Only the two support forces remain in the equations. For the 57-dimensional case, the torso sub module is of dimension 12 and the remaining five sub modules each are of dimension nine. Both support forces go to zero when the system is in the air. The system can also stand on one leg. For simplicity, we only consider the 57-dimensional case in this paper. Because the neck, the hip and the shoulder joints all have three degrees of freedom, 15 holonomic constraints allow a projection, similar to the one defined in the Appendix A, to be developed as sketched below.

The equations for the six sub modules are symbolically written:

$$J_i \dot{V}_i = h_i$$

where

$$H = [h_1, h_2, h_3, h_4, h_5, h_6]' \tag{12}$$

Initially all segments are vertical and all are aligned with the inertial coordinate system (ICS). The three segments in every module are numbered, respectively, 1, 2 and 3, starting from the bottom. Every module has its own system of inputs: N_i where

$$i = 1....6.$$

There are inter module inputs N_{12}, N_{13}, N_{24}, N_{25} and N_{26} The translational motion of the torso module will be kept in the system equations and represents the translation component of the system:

$$X_a = X_{22}.$$

The translation of the system could be represented by the translation of one point of the six-sub module system. We consider the movement of the center of gravity of the mid-torso segment for this purpose.

Suppose the dynamics of the six-sub module system are programmed by the same subroutine that provides the 12 equations of motion from the 36-dimensional equations of motion of the three-link. The 24 position and velocity

variables in the equations of motion must be augmented to 36 equations as input to the subroutine. Therefore, a more accurate block diagram is the one presented in Figure 5 where the augmentation and reduction of dimension are marked and allow for inclusion of passive structures (Hemami & Hemami, 2014).
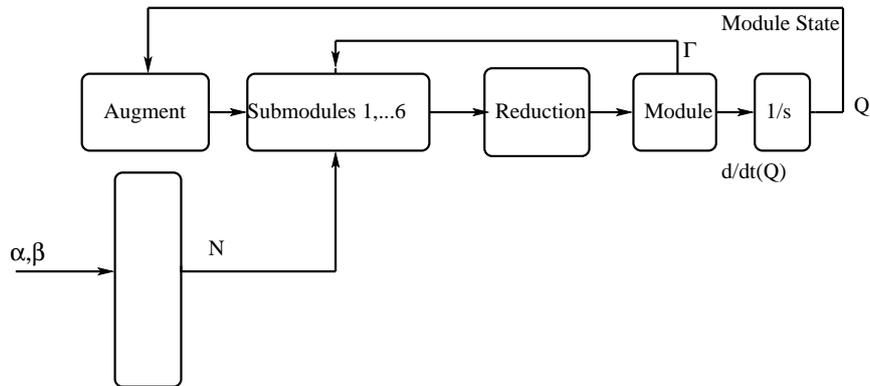
Figure 5. The six-sub module system where the expansion and reduction of the dimensinality are also marked to accommodate passive structures.

Let the individual segments of each sub module be numbered by 1, 2 and 3 upwards. The variables in the system have three subscripts: sub module number, segment number and attitude angle (1 for roll, 2 for pitch and 3 for yaw.) With the subscripts ij, i refers to the sub module and j refers to the segment. The torso segments are labeled 11, 12 and 13. The right foot, leg and thigh are labeled 21, 22 and 23. The left foot, leg and thigh are labeled 31, 32 and 33. The right upper arm, lower arm and hand are labeled 41, 42 and 43 - the arm is assumed to be stretched upward vertically. The left upper arm, lower arm and hand are labeled 51, 52 and 53.The neck and head segments are labeled 61, 62 and 63. For every one of the eighteen rigid bodies above, a 3×3 matrix A_{ij} defines the transformation from of the rigid body's BCS to ICS. We briefly introduce the contact constraints that relate the individual sub module's translation to that of the torso. The holonomic connection constraints that define the projection operator, as is done in appendix A for the torso module, are described here for the whole module. Let the translation of the torso module be X_m and let the remaining five translational vectors for the other five modules be $X22$, $X32$, $X42$, $X52$ and $X62$. It follows that the translations can be eliminated by the following projection:

$$
\begin{aligned}
X_m &= X_m \\
X22 &= X_m + A_{12}R12 - A_{11}S2l + A_{11}R1r - A_{23}S23 + A_{23}R23 - A_{22}S22 \\
X32 &= X_m + A_{12}R12 - A_{11}S21 + A_{11}R1l - A_{33}S33 + A_{33}R33 - A_{32}S32 \\
X42 &= X_m + A_{12}S12 - A_{23}R23 + A_{23}S1r - A_{41}R41 + A_{41}S41 - A_{42}R42 \\
X52 &= X_m + A_{22}S12 - A_{23}R23 + A_{23}S1l - A_{51}R51 + A_{51}S51 - A_{52}R52 \\
X62 &= X_m + A_{22}S12 - A_{23}R23 + A_{23}S13 - A_{61}R61 + A_{61}S61 - A_{62}R62
\end{aligned}
$$

(13)

In this projection, the translational degrees of freedom of the torso module are kept and the other 15 translation degrees of freedom as well as the forces between the segments are eliminated. Only the contact forces of the feet remain in the 57-dimensional dynamics. The first derivative of Eq.13 defines the projection operator for reducing the dimension of the system from 72 to 57. The second derivative helps carry out the reduction and derivation of the reduced order system equations. Let Ha be the latter projection operator. This operator is parsed into six horizontal strips: the first 12 rows pertain to module 1, and the next successive nine rows sequentially are related to modules 2, 3, 4, 5 and 6. These parsed strips are named, respectively,

$$Ha1, Ha2, Ha3, Ha4, Ha5, Ha6.$$

The inner product of each of these strips with H, as defined in Eq. 12, defines the dynamics of the individual sub modules and their couplings.

To complete the model, an additional module is necessary to compute the forces of support while the contact with the ground is in effect for either individual foot or for both feet. This step is well understood (Hemami & Wyman,

1979, 1980). It is also to be noted that, up to this point (namely, the development of the dynamic sub-modules) all operations are of the inner product type and summation. Only integration of the dynamics and computation of the support forces requires inversion of appropriate matrices.

5. Control and Support

A set of N moments of force act on the module. These inputs will be specified later. Each moment of force is brought about by a pair of muscle-like actuators (Ajay & Pandy, 2007; Brooks, 1986; Kandel, Schwartz, & Jessell, 1991). Three pairs of muscles (Ajay & Pandy, 2007) control each of the three degrees of freedom, respectively, at every joint. We assume the system is on tiptoe and the toe extensors and flexors that control the sagittal motion of the toes can be replicated in the frontal and medial planes such that there is one pair of actuators for every degree of freedom. Similarly we assume three pairs at the ankle and three pairs at the knee joint. The structures of the nine pairs of muscles may not be anatomically correct for humans, but they simplify the modeling and simulations here. The model here can be made more anatomically correct by adding a three-segment foot and more representative musculature (Humphrey, 2009). For bi-articular (such as the gastrocnemius) and multi-articular (such as the rectus femoris) muscles, modeling difficulties arise partly because the function of many such intertwined structures are not well understood or articulated. Slings and caps can be added here.

The precise understanding of the function of the cap is necessary in order to formulate the action of arbitrary muscles in the dynamics of any musculo-skeletal system. Briefly, if one uses Lagrangian dynamics, the incremental work dW performed by an actuator on the system is given by $F.dl$ where F and dl are, respectively, the muscle force and the incremental change in the length of the muscle. Let Θ be the vector of the totality of the angles of the skeletal elements. It follows that the effect of the force F can be entered in the dynamics as:

$$(\partial l/\partial \Theta)F.$$

Therefore, it is important to be able to specify the dependence of the length of the muscle on θ. To clarify this point, we consider a sagittal plane approximation to a sling where an actuator spans the toes, the bottom of the foot and around the heel. It is connected to the tendon of another actuator as shown in Figure 6.
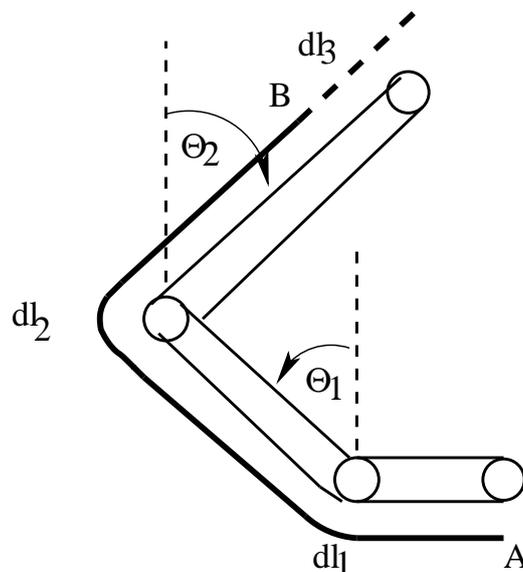
Figure 6. An actuator spanning two joints and, at one end, connected to a tendon

There are three links shown in Figure 6. Link one is at angle zero. As links 2 and 3 rotate, two caps accommodate the change of direction for the tendon of the muscle. The change of length of the muscle could possibly depend on a number of angles θ. Let the radius of cap 1 be r_1, the radius of cap 2 be r_2 and let the change of the tendon shown in the Figure be Δl_3.

The incremental change of the length of the muscle is

$$\Delta l_1 + \Delta l_2 + \Delta l_3$$

or

$$\Delta l = r_1 \theta_1 + r_2(\theta_2 - \theta_1) + \Delta l_3,$$

where Δl_3 could potentially depend on several more angles of the system.

The first term in the above expression can summarize one function of the five flexors: flexor digitorum brevis, quadratus plantae, flexor hallucis brevis, flexor digiti minimi brevis and lumbricales (Warfel, 1978). When the toes are firmly in touch with the ground and the limb is unloaded, the action of lifting the heel is facilitated and can be fast due to the collective action of these five muscles. This means there will be a moment of force acting on the foot to plantar-flex it, and the heel lifts easily due to the simultaneous effort of these five flexors.

This action amounts to having available feedback and/or feed-forward by these five muscles for the purpose of lifting the heel when the lower limb is not loaded. These feed-backs amount to state feedback in the system theoretic sense: it is shown below that linear state feedback stabilizes the attitude of the rigid body system in physical space.

For attitude control (Ren, Jones, & Howard, 2006; Honarvar & Nakashima, 2012), gravity compensation (Minetti, Cisotti, & Mian, 2011), inclusion of constraints and actuation (Pope, Crowninshield, Miller, & Johnson, 1976; Gruber, Ruder, Denoth, & Schneider, 1998), it is necessary to relate the rigid body attitude (in BCS) to the ICS system (see Eq. 3):

6. Computational Experiment

The objective is to carry the arm through a point to point movement, such as pointing (Soechting & Flanders, 1989) and directing music, and then carry the reverse movement to get the arm back to its original position.

The arm has nine degrees of freedom and the motion is observed imagined or intended in the ICS system. A neural network (Arbib, Erdi, & Szentagothai, 1998; Bullock, Grossberg, & Gyenther, 1996; Kandel, Schwartz, & Jessell, 1991) with two populations of neurons is designed (Houk, 1995; Ajemian & Bullock, 2000; Hemami & Dariush, 2012) to transform the ICS trajectory to the trajectories of four of the nine Bryant angles. All other Bryant angles and velocities are zero. The fingers position in ICS is described by

$$X_p = A_1 L_1 + A_2 L_2 + A_3 L_3 \tag{14}$$

Suppose this motion is, arbitrarily, carried out by pitch and yaw of the arm, yaw of the lower arm and yaw of the hand which is assumed to be the same as the yaw of the arm. This reduces the computational problem to three unknown angles that are the solution of the following three equations:

$$\begin{bmatrix} cos(\theta_{12})cos(\theta_{13}) \\ sin(\theta_{13}) \\ -sin(\theta_{12})cos(\theta_{13}) \end{bmatrix} l_1 + \begin{bmatrix} cos(\theta_{22}) \\ 0 \\ -sin(\theta_{22}) \end{bmatrix} (l_2 + l_3) = X_p \tag{15}$$

Given a desired $X_p(t)$, expressed in ICS, which is received for example from visual observation, the task of the central controller is to solve for the three Bryant angles in Eq. 15 above as functions of time. We make use of the three neural population circuits described in Hemami and Dariush (2012) to construct the Bryant angles from $X_p(t)$. Suppose the Bryant angles, in a simpler form and for two seconds, are parameterized as:

$$
\begin{aligned}
\theta_{12} &= -pi/4 + (pi/8) * sin(pi * t) \\
\theta_{13} &= pi/4 * cos(pi * t) \\
\theta_{22} &= pi/8 * cos(pi * t) \\
\theta_{32} &= pi/8 * cos(pi * t)
\end{aligned}
$$

The arm motion was simulated to demonstrate the feasibility of the three-link sub modules and the control strategy of co activation (Hogan, 1984). All the physical parameters are listed in Appendix B. The four Bryant angle inputs

and the actual four Bryant outputs of the system are shown in Figure 7. The desired trajectory of the tip of the fingers, and the actual trajectory of the fingers in the ICS system are shown in Figure 8.
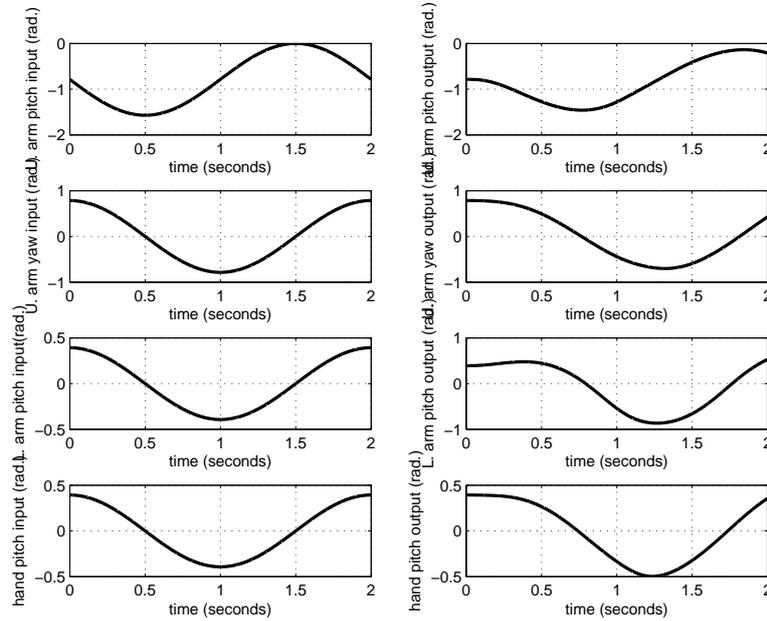
Figure 7. The four Bryant input angles (arm pitch and yaw and lower arm and hand yaw angles) are shown on the left side. The same angles as outputs of the three-link system are shown on the right side.
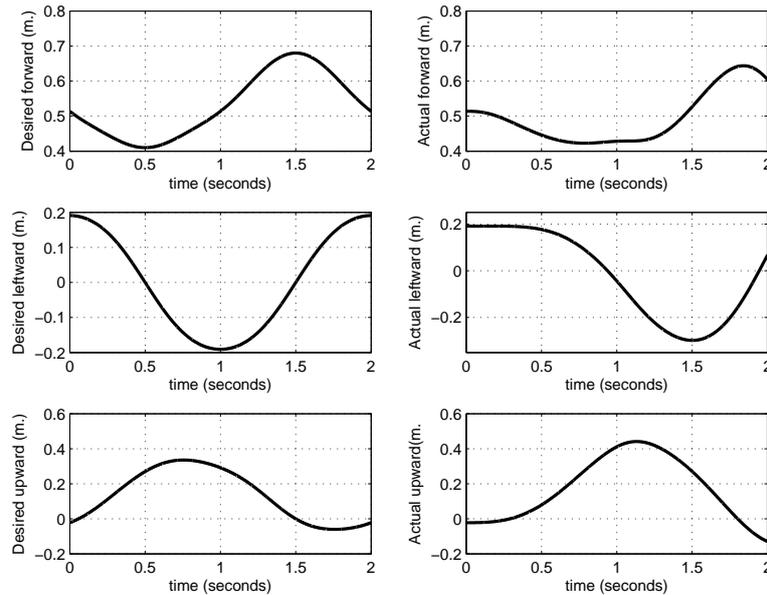
Figure 8. The desired trajectory of the hand and the actual trajectory in the ICS are shown.

7. Discussion and Conclusions

We have contrasted different state space formulations for a rigid body. These state space formulations can be extended to multiple rigid body systems where dimensionality reduction procedures are easy to implement by

projection operators. With the addition of Mealy outputs, the K or NE simulation programs can be adapted to compute Lagrangian outputs.

We have developed a six sub module computational tool for studying the machinery, control and coordination of human movement. For simplicity, all joints in the modules and between modules were assumed to have three degrees of freedom. Anatomically accurate systems of muscles can be added to this model. One may vary the number of muscles, disable them, and try different synergies. One can compute internal and support forces to compare with available experimental measurements. One may add more modules to have a more accurate spinal column, add modular tail structures for modeling animals as well as hand, foot, digit and toe modules. For testing the six-sub modules system, a pair of agonist-antagonist actuators were added for each of the 54 rotational degrees of freedom. The system can be supplied with a variety of sensory modalities: vision, vestibular, and tactile for more comprehensive and instructive models. Parallel computations (Hebb, 1949; Adeli, 1992; Singer, 2000) can be implemented to make the system more physiologically realistic.

The current model has two points of contact, with the ground providing support. This means all the movements are on tiptoe and mass-less toes provide the support. The human foot is very complex. One of the many challenges is to identify, define and precisely describe a three-link three-dimensional module for each foot (Humphrey & Hemami, 2010). Specifically, one of the many problems in formulating the foot is to articulate the dynamics of the joint between the toes and the metatarsals, sometimes refereed to as a sling. Briefly stated, one of the simpler functions of the sling is to change the direction of the force of the muscle or several muscles. This is similar to the function of the kneecap for changing the direction of action of the rectus femoris' heads at the knee (Rasch & Burke, 1978; Warfel, 1978). The mechanism is needed for almost all bi-articular muscles that span a joint or segment.

Philosophically speaking, one may cite some advantages for this computational method. The ideas presented here may be, pedagogically, superior in teaching rigid-body dynamics. They yield to more efficient and labor-saving methods of simulation and system integration. Finally, many internal states can be added and are candidates to be traced in the CNS of natural systems, and, therefore, help to unravel some of the magnificence of the brain mysteries in signal processing.

Acknowledgments

Appendix A, The Torso Model

Consider a three-body system with five external forces as shown in Figure 2. The first body is the lower torso. The second one is the middle torso. The third body is the upper torso. Let bodies one and two be connected together at one point. Let bodies two and three be connected at one point. There are seven forces involved in the torso system. Five are external and two are internal. Let external forces Γ_{12} and Γ_{13} be external forces applied, respectively, to the lower torso by the right and left thigh. Let Γ_{14} and Γ_{15} be the forces exerted on the upper torso by the arms. Let Γ_{16} be the force exerted on the body by the neck-head complex. There are also two forces $\Gamma2$ and Γ_3 at the connection points between lower and middle torso and between the middle and upper torso. We provide a first projection here to eliminate the two internal torso forces.

Two steps are carried out here:

Step 1: Specify the sub modules equations in a format that is common to all six sub modules. Step 2: Project the above equations onto the 12-degree-of-freedom space where the internal forces of constraint are eliminated.

Let the coordinates of the contact points of the thighs (right and left) in the lower torso be $R2_r$ and $R2_l$, respectively. Let the coordinates of the contact point of the lower torso with the middle torso, in the lower torso coordinate system, be S_11. Consider the mid-torso segment. The two contact points with the lower and upper torso points are, similarly R_12 and S_12. Let the coordinates of the contact points of the upper torso with the middle torso, neck and right and left arm be, respectively, $R13$, $S13$, $S1ri$, and $S1le$.

Let X_j, V_j, Θ_j and Ω_j be, respectively, the translational and rotational states of the segments of the torso

$$j = 1, 2, 3$$

segments. Let G_j be the vector of gravity for body j. Let N_j be the moment of input forces acting on body j. Let the internal constraint forces between the mid-torso and the lower and upper torso be, respectively, Γ_2 and Γ_3. The Newton-Euler equations of motion for the three segment torso, as free bodies, are:

$$\dot{X}_1 = V_1$$
$$M_1\dot{V}_1 = G_1 + \Gamma_{12} + \Gamma_{23} - \Gamma_2$$
$$\dot{\Theta}_1 = B_1(\Theta_1)\Omega_1$$
$$J_1\dot{\Omega}_1 = f(\Omega_1) + N_1 + \check{R}\check{1}rA_1^T\Gamma_{12} + \check{R}\check{1}lA_1^T\Gamma_{13} - \check{S}\check{1}_1A_1^T\Gamma_2$$

$$(16)$$

The second body represents middle torso

$$\dot{X}_2 = V_2$$
$$M_2\dot{V}_2 = G_2 + \Gamma_2 - \Gamma_3$$
$$\dot{\Theta}_2 = B_2(\Theta_2)\Omega_2$$
$$J_2\dot{\Omega}_2 = f(\Omega_2) + N_2 + \check{R}\check{1}2A_2^T\Gamma_2 - S\check{1}2A_2^T\Gamma_3$$

$$(17)$$

The equations of the upper torso with the three connections to the arms and the neck are

$$\dot{X}_3 = V_3$$
$$M_3\dot{V}_3 = G_3 + \Gamma_3 - \Gamma_{14} - \Gamma_{15} - \Gamma_{16}$$
$$\dot{\Theta}_3 = B_3(\Theta_3)\Omega_3$$
$$J_3\dot{\Omega}_3 = f(\Omega_3) + N_3 + \check{R}\check{1}3A_3^T\Gamma_3 - S\check{1}rA_3^T\Gamma_{14} - S\check{1}lA_3^T\Gamma_{15} - S\check{1}3A_3^T\Gamma_{16}$$

$$(18)$$

Let

$$\Gamma = [\Gamma_{12}, \Gamma_{13}, \Gamma_2, \Gamma_3, \Gamma_{14}, \Gamma_{15}, \Gamma_{16}]'.$$

With these definitions, the 18 Newton-Euler equations can be summarily written as

$$J1_{18\times18}\dot{V} = N_1 + f_1 + G_1 + H1_c\Gamma \tag{19}$$

where V is defined in Eq. 11, and where the coefficient of Γ is an 18×21 matrix $H1_c$ and where N_1, f_1 and G_1 are 18-vectors.

$$H1_c = \begin{bmatrix} I_{3\times3} & I_{3\times3} & -I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ \check{R}\check{1}rA_1^T & \check{R}\check{1}lA_1^T & -S\check{1}1_1A_1^T & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & -I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & \check{R}\check{1}2A_2^T & -S\check{1}2A_2^T & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} & -I_{3\times3} & -I_{3\times3} & -I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \check{R}\check{1}3A_3^T & -S\check{1}rA_3^T & -S\check{1}lA_3^T & -S\check{1}3A_3^T \end{bmatrix}. \tag{20}$$

Let Z be the 18-dimensional vector of translational and rotational positions states, defined by Eq. 10: Let the center of gravity of the mid-torso segment be $X_m = X_2$ and define vector W:

$$W = [\Theta_1^T, X_m{}^T, \Theta_2{}^T, \Theta_3^T]^T.$$

The projection from Z to W

$$Z = P2(W).$$

allows one to derive the equation of motion for the torso module. By this projection, six degrees of translational freedom as well as Γ_2 and Γ_3 are eliminated. The projection P is derived from the six holonomic connection constraint equations

$$X_1 = X_m + A_2 R12 - A_1 S11. \tag{21}$$
$$X_2 = X_m$$
$$X_3 = X_m + A_2 S12 - A_3 R13.$$
$$\Theta_1 = \Theta_1$$
$$\Theta_2 = \Theta_2$$
$$\Theta_3 = \Theta_3$$

We differentiate Eq. 22 with respect to time to derive the projection operator. The result of the differentiation is as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix} = (H1_a) \begin{bmatrix} \Omega_1 \\ V_m \\ \Omega_2 \\ \Omega_3 \end{bmatrix} \tag{22}$$

For convenience, let

$$V_n = [\Omega_1, V_m, \Omega_2, \Omega_3]'$$

The 18×12 matrix $H1_a$ is

$$H1_a = \begin{bmatrix} +A_1 S\check{1}1 & I_{3\times3} & -A_2 R\check{1}2 & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & -A_2 S\check{1}2 & +A_3 R\check{1}3 \\ I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix}.$$

Differentiating Eq. 22 once more, with respect to time, renders

$$\begin{bmatrix} \partial V_1/\partial t \\ \partial V_2/\partial t \\ \partial V_3/\partial t \\ \partial \Omega_1/\partial t \\ \partial \Omega_2/\partial t \\ \partial \Omega_3/\partial t \end{bmatrix} = (H1_a) \begin{bmatrix} \partial V_1/\partial t \\ \partial \Omega_1/\partial t \\ \partial \Omega_2/\partial t \\ \partial \Omega_3/\partial t \end{bmatrix} + H1_b$$

The 18-vector $H1_b$ is

$$H1_b = \begin{bmatrix} -A_1(\check{\Omega}_1)^2(S11) + A_2(\check{\Omega}_2)^2(R22) \\ 0_{3\times1} \\ (+A_2(\check{\Omega}_2)^2)(S12) - A_3(\check{\Omega}_3)^2 R13 \\ 0_{3\times1} \\ 0_{3\times1} \\ 0_{3\times1} \end{bmatrix}.$$

Table 1. Physical Parameters of the Arm

parameter	symbol	value	unit
mass of upper arm	m1	2.40	kg
mass of forearm	m2	2.00	kg
mass of hand	m3	1.20	kg
principal moment of inertia	j41x	0.0080	kg m^2
principal moment of inertia	j41y	0.0080	kg m^2
principal moment of inertia	j41z	0.0010	kg m^2
principal moment of inertia	j42x	0.0070	kg m^2
principal moment of inertia	j42y	0.0070	kg m^2
principal moment of inertia	j42z	0.0040	kg m^2
principal moment of inertia	j43x	0.0070	kg m^2
principal moment of inertia	j43y	0.0070	kg m^2
principal moment of inertia	j43z	0.0050	kg m^2
cog of upper arm	k1	0.14	m
cog of forearm	k2	0.13	m
cog of hand	k3	0.07	m
l1 of upper arm	l1	0.27	m
l2 of forearm	l2	0.25	m
l3 of hand	l3	0.16	m
gravity	g	10.0	m/s^2

Substitution of Eq. 23 in Eq. 19 and multiplying both side by $H1_a^T$ results in the 12-dimension torso dynamics As a consequence of the reduction, Γ_2, Γ_3 get eliminated. The 12-dimensional equations of motion are

$$H1_a{}^T J1 H1_a \dot{V}_n = H1_a{}^T (N_1 + f_1 + G_1 + H1_c \Gamma) - H1_a{}^T J1 H1_b. \tag{23}$$

The 12×21 matrix

$$H_d = H1_a^T H1_c$$

is the coefficient of the seven external forces

$$H1_d = \begin{bmatrix} (-S1\breve{l} + R1r)A_1^T & (-S1\breve{1} + R11)A_1^T & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ I_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} & -I_{3\times3} & -I_{3\times3} & -I_{3\times3} \\ -R\breve{1}2A_2^T & R\breve{1}2A_2^T & 0_{3\times3} & 0_{3\times3} & -S\breve{1}2A_2^T & -S\breve{1}2A_2^T & -S\breve{1}2A_2^T \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & (R13\breve{-}S1\breve{r})A_2^T & (R13\breve{-}S1\breve{l})A_2^T & (R13\breve{-}S13)A_2^T \end{bmatrix}$$

As can be seen from matrix $H1_d$, columns 3 and 4 are all zero. This means forces Γ_2 and Γ_3 are eliminated. For the leg modules, with one force on the bottom and one force on the top of the three link, the matrix $H1_d$ will have only two columns similar to the columns one and six. For the arm and head modules, there will be only one column in the corresponding $H1_d$ similar to column one of $H1_d$, above.

Appendix B, Numerical Parameters

The physical parameters of the arm are given in Table 1. The muscular gains are given in Table 2 24 The upper left 3×3 matrix represents the amplification factor of three pairs of muscles that move the arm about a stationary shoulder. The middle 6×3 right side of the matrix represents the action of the three pairs of muscles at the wrist that move the hand relative to the forearm.

The linear viscosity of the muscle, in the first phase, is assumed to be 0.2 of the gain of that muscle.

$$Gain \; = \; \begin{bmatrix} 10 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & -1.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.2 \end{bmatrix}$$

$$(24)$$

References

Adeli, H. (1992). *Parallel Processing in Computational Mechanics* (1st ed.). Mar- cel Dekker, New York.

Ajay, S., & Pandy, M. G. (2007). A neuromusculoskeletal tracking method for estimating individual muscle forces in human movement. *Journal of Biomechanics, 40*(13), 3013-3022.

Ajemian, R., Bullock, D., & S., G. (2000). Kinematic coordinates in which motor cortical cells encode movement direction. *J. Neuro-physiology*, 2191-2203.

Amirouche, F. (1992). *Computational Methods in Multi-body Dynamics* (2nd ed.). Prentice Hall, Englewoods, NJ.

Andriacchi, T. P., & Alexander, E. J. (2000). Studies of human locomo- tion: past, present and future. *Journal of Biomechanics, 33*(10), 1217-1224.

Antsaklis, P., & Michel, A. (1997). *Linear Systems*. The Mc-Graw Hill Companies, Inc., New York.

Arbib, M., Erdi, P., & Szentagothai, J. (1998). *Neural Organization, Structure, Function, and Dynamics*. A Brad- ford Book, The MIT Press, Cambridge, Mass.

Bay, J. S. (1999). *Fundamentals of Linear State Space Systems*. WCB, McGraw- Hill, Boston.

Blajer, W. (1997). A geometric unification of constrained system dynamics.*Multibody System Dynamics, 1*, 3-21.

Blajer, W., & Czaplicki, A. (2001). Modeling and inverse simulation of somersault on the trampoline. *Journal of Biomechanics, 34*, 1619-1629.

Blajer, W., & Schiehlen, W. (1992). Walking without impacts as a mo- tion/force control problem. *Journal of Dynamic Systems, Measurement and Control, 114*, 660-665.

Brooks, V. (1986). *The neural basis of motor control*. Oxford University Press, New York.

Buchner, H., Hines, M., & Hemami, H. (1988). A dynamic model for finger interphalangeal coordination. *Journal of Biomechanics, 6*, 459-468.

Bullock, D., Grossberg, S., & Gyenther, F. (1996). Neural net- work modeling of sensory-motor control. In H. N. Zelaznik (Ed.), *Advances in Motor Learning and Control* (pp. 261-292), Human Kinetics, Champaign, Ill.

Charles, S. K., & Hogan, N. (2010). Dynamics of wrist rotations. *Journal of Biomechanics, 44*(4), 614-621.

Evarts, E. V. (1979). Brain mechanisms of movement. *Scientific American, 3*, 164-179.

Evarts, E., Shinoda, Y., & Wise, S. (1984). *Neurophysiological Approaches to Higher Brain Functions* (1st ed.). John Wiley and Sons, New York.

Feng, Z., & Utkin, V. (1996). Adaptive simulation and control of variable- structure control systems in sliding regimes. *Automatica 32*, 1037-1042.

Fluit, R., Andesen, M. S., Kolk, S., Verdonschot, N., & Koop- man, H. F. J. M. (2014). Prediction of ground reaction forces and moments during various activities of daily living. *Journal of Biomechanics, 47*(10), 2321-2329.

Gruber, K., Ruder, H., Denoth, J., & K. Schneider, K. (1998). A comparative study of impact dynamics: wobbling mass model versus rigid body models. *Journal of Biomechanics, 31*(5), 439-444.

Hebb, D. (1949). *The Organization of Behavior* (1st ed.). John Wiley and Sons.

Hemami, H. (2002). Towards a compact and computer-adapted formulation of the dynamics and stability of multi rigid body systems. *J. Automatic Control, Univ. of Belgrade, 12*, 64-69.

Hemami, H., & Dariush, B. (2012). Central mechanisms for force and motion- towards computational synthesis of human movement. *Neural Networks, 36*, 167-178.

Hemami, H., & Hemami, M. (2014). Modeling and monitoring of passive con- trol structures in human movement. *International Journal of Control, 87*, 1861-1876.

Hemami, H., & Stokes, B. (1983). Four neural circuit models and their role in the organization of voluntary move- ment. *Biological Cybernetics, 2*, 69-77.

Hemami, H., & Wyman, B. (1979). Modeling and control of constrained dy- namic systems with application to biped locomotion in the frontal plane. *IEEE Trans. on Automatic Control, 4*, 526-535.

Hemami, H., & Wyman, B. (1980). Indirect control of the forces of constraint in dynamic systems. *Journal of Dynamic Systems, Measurement and Con- trol*, 355-360.

Hemami, H., & Wyman, B. F. (2012). A simple strategy for jumping straight up. *Math. Biosciences*, 28-37.

Hemami, H., & Zheng, Y. F. (2012). A three-link module for modular dynam- ics and control of high-dimensional humanoids. *Proc. IEEE 2012 Inter. Conf. on Robotics and Automation*, 3109-3115.

Hemami, H., Barin, K., & Pai, Y.-C. (2006). Quantitative analysis of a sagittal biped under platform disturbance. *IEEE Trans. Neural Systems and Rehabilitation Engineering, 14*(4), 470-480.

Hogan, N. (1984). Adaptive control of mechanical impedance by coactivation of anagonist muscles. *IEEE Trans. AC*, 681-691.

Honarvar, M. H., & Nakashima, M. (2012). A new measure of upright stability. *Journal of Biomechanics, 47*(2), 560 -567.

Houk, J. C. (1995). Information processing in modular circuits linking basal gan- glia and cerebral cortex. In *Models of Information Processing in the Basal Ganglia*. The M.I.T. Press, Cambridge, Mass.

Humphrey, L. R. (2009). *Modeling of the Foot, Toes and Vestibular System in Human Balance*. PhD thesis, The Ohio State University, Columbus, Ohio.

Humphrey, L. R., & Hemami, H. (2010). A computational human model for exploring the role of the feet in balance. *Journal of Biomechanics*, 3199-3206.

Humphrey, L. R., Hemami, H., Barin, K., & Krishnamurthy, A. (2010). Simulated responses to support surface disturbances in a humanoid biped model with a vestibular-like apparatus. *IEEE Trans SMC, part C*(1), 109-119.

Huston R. L., & Passerello, C. E. (1971). On the dynamics of a human body model. *Journal of Biomechanics, 4*(5), 369 -378.

Ihlen, E. A. F., Skjaeret, N., & Vereijken, B. (2012). The influence of center-of-mass-movement on the variation in the structure of human postural sway. *Journal of Biomechanics, 46*(3), 484-490.

Iqbal, K., & Pai, Y.-C. (2000). Predicted region of stability for balance recov- ery motion at the knee joint can improve termination of forward movement. *J. Biomechanics*, 1619-1627.

Iqbal, K., & Roy, A. (2009). A novel theoretical framework for the dynamic stability analysis, movement control, and trajectory generation in a multi- segment biomechanical model. *J. of Biomechnical Eng, Trans. ASME, 131*(1), 002-013.

Jalics, L., Hemami, H., & Clymer, B. (1997). A control strategy for terrain adaptive bipedal locomotion. *Autonomous Robots, 4*, 243-257.

Jalics, L., Hemami, H., Clymer, B., & Groff, A. (1997). Rocking, tapping, and stepping, a prelude to dance. *Autonomous Robots, 4*, 227-242.

Kandel, E., Schwartz, J., & Jessell, T. (1991). *Principles of Neural Science* (3rd ed.). Elsevir, New Tork.

Kane, T. R., & Levinson, D. A. (1985). *Dynamics, Theory and Applications*. McGraw-Hill, New York.

Khosravi-Sichani, B. (1985). *Control of Multi-Linkage Planar Systems in the Air and on the Ground*. PhD thesis,

The Ohio State University.

Khosravi-Sichani, B., Hemami, H., & S., Y. (1992). Energy transformations in human movement by impact. *Journal of Biomechanics, 25*(8), 881-889.

Khosravi-Sichani, B., S., Y., & Hemami, H. (1987). Control of a four-link biped in a back somersault maneuver. *IEEE Trans. on Systems, Man, and Cybernetics, 17*(2), 303-311.

Kuo, A., & Zajac, F. (1993). A biomechanical analysis of muscle strength as a limiting factor in standing posture. *J. Biomechanics, 26*, 137-150.

Lam, S. H. (1998). On lagrangian dynamics and its control formulations. *Applied Mathematics and Computation, 91*, 259-284.

Lebiedowska, M. K., Wente, T. M., & Dufour, M. (2009). The influence of foot position in body dynamics. *Journal of Biomechanics, 42*, 762-766.

Lee, H., & Utkin, V. I. (2007). Chattering suppression methods in sliding mode control systems. *IFAC, Annual reviews in Control*, 180-189.

Mansouri, M., & Reinbolt, J. A. (2012). A platform for dynamic simula- tion and control of movement based on opensim and matlab. *Journal of Biomechanics, 45*(8), 1517-1521.

Marshall, R. N., Jensen, R. K., & Wood, G. A. (1985). A general new- tonian simulation of an n-segment open chain model. *Journal of Biome- chanics, 18*(5), 359-367.

McCuskey, W. (1959). *Introduction to Advanced Dynamics*. Addison Wesley Pub., Reading, Mass.

Minetti, A. E., Cisotti, C., & Mian, O. S. (2011). The mathematical description of the center of mass 3d path in human and animal locomotion. *Journal of Biomechanics, 44*(8), 1471-1477.

O Reilly, O. M. (2008). *Intermediate Dynamics for Engineers* (2nd ed.). Cam- bridge University Press, Cambridge, England.

O Reilly, O. M. (2010). *Engineering Dynamics - A preview* (2nd ed.). Springer, New York, NY.

Onyshko, S., & Winter, D. A. (1980). A mathematical model for the dynam- ics of human locomotion. *Journal of Biomechanics, 13*(4), 361-368.

Pai, D. K. (2010). Muscle mass in musculoskeletal models. *Journal of Biome- chanics, 43*(11), 2093- 2098.

Pandy, M. G., & Berme, M. (1988). A numerical method for simulating the dynamics of human walking. *Journal of Biomechanics, 21*(12), 1043-1051.

Pellionisz, A., & Llinas, R. (1979). Brain modeling by tensor network theory and computer simulation. the cere- bellum: distibuted processor for predictive coordination. *Neuroscience, 4*, 323-350.

Pellionisz, A., & Llinas, R. (1980). Tensorial approach to the geometry of brain function: cerebellar coordination via a metric tensor. *Neuroscience, 5*, 1125-1150.

Pope, M., Crowninshield, R., Miller, R., & Johnson, R. (1976). The static and dynamic behavior of the human knee in vivo. *Journal of Biome- chanics, 9*(7), 449- 452.

Popovich, M., Goswami, A., & Herr, H. (2005). ground reference points in legged locomotion: Definitions, biolog- ical trajectories and control im- plications. *The International Journal of Robotics Research, 12*, 1013-1032.

Rankin, J., & Neptune, R. (2012). Musculotendon lengths and moment arms for a three-dimensional upper- extremity model. *Journal of Biomechanics, 9*, 1739-1744.

Rasch, P. J., & Burke, R. K. (1978). *Kinesiology and Applied Anatomy* (6th ed.). Lea and Febiger, Philadelphia.

Reich, J., & Daunicht, W. J. (2000). A rigid body model of the forearm. *Journal of Biomechanics, 47*(5), 1020-1026.

Ren, L., Jones, R. K., & Howard, D. (2006). Predictive modeling of human walking over a complete gait cycle. *Journal of Biomechanics, 40*(7), 1567- 1574.

Saraf, H., Ramesh, K. T., Lennon, A. M., C., M. A., & Roberts, J. (2006). Mechanical properties of soft human tissues under dynamic loading. *Journal of Biomechanics, 40*(9), 1960-1967.

Schiehlen, W. (2006). Computational dynamics: theory and applications of multibody systems. *European Journal 0f Mechanics A/Solids, 25*, 566-594.

Schwertassek, R., & Roberson, R. (1986). A perspective on computer- oriented multibody dynamic formalisms and their implementations. In G. B. W. Schiehlen (Ed.), *Dy- namics of Multibody Systems Symposium, IUTAM*. IUTAM/IFTOMM, Udine.

Singer, W. (2000). Response synchronization, a neural code for relatedness. In J. J. Bolhuis (Ed.), *Brain, Perception, Memory* (pp. 35-48). Oxford University Press.

Smith, D. A. (1973). the automatic generation of a mathematical model for machinery systems. *Trans. ASME, J. Eng. Industry*.

Smith, O. (1958). Feedback Control Systems. McGraw-Hill, New York.

Soechting, J., & Flanders, M. (1989). Errors in pointing are due to approx- imations in sensorimotor transformations. *Journal of Neurophysiology, 2*, 595-607.

Thelen, D. G., Anderson, F. C., & Delp, S. L. (2003). Generating dy- namic simulations of movement using computed muscle control. *Journal of Biomechanics, 36*(3), 321-328.

Tomovic, R., Popovic, D., & Stein, R. (1991). *Nonanalytical Methods for Motor Control*. World Scientific, Singapore.

Utkin, V., Guldner, J., & Shi, J. (1999). *Sliding Mode Control in Elec- tromechanical Systems*. Taylor and Francis Limited, London.

Warfel, J. H. (1978). *The Extremities* (4th ed.). Lea and Febiger, Philadelphia.

Williams, S., Schmidt, R., Disselhorse-Klug, C. (2005). An upper body model for the kinematic analysis of the joint chain of the human arm. *Journal of Biomechanics, 39*(13), 2419-2429.

Williams, S., Schmidt, R., Disselhorst-Klug, C., & Rau, G. (2006). An upper body model for the kinematical analysis of the joint chain of the human arm. *Journal of Biomechanics, 13*, 2419-2429.

Winter, D., Prince, D., F. Powell, C., & Zabjik, K. (1996). A unified theory regarding A/P and M/I balance during quiet stance. *J. of Neuro- physiology, 75*, 2334-2343.

Winter, J. M., & Stark, L. (1988). Estimated mechanicalproperties of syner- gisticmuscles involved in movements of a variety of human joints. *Journal of Biomechanics, 21*(12), 1027-1041.

Winter, J. M., & Stark, L. (2012). Muscle dynamics during human move- ment: individual contributions. *Journal of Biomechanics*, 19(6), 471.

Wittenberg, J. (1977). *Dynamics of Systems of Rigid Bodies*. B.G. Teubner, Stuttgart.

Wittenburg, J. H. (1989). The dynamics of multibody systems-a brief review. *Acta Astronautica, 20*, 89-92.

Wu, Q. (1996). *Lyapunov stability of a class of base-excited inverted pendulums with application to bipedal loco- motion systems*. PhD thesis, University of Manitoba.

Wu, Q., Sepehri, N., Thornton-Trump, A., & Onyshko, S. (1995). An extended integral method to derive lyapunov functions for nonlinear sys- tems. *International Journal of Control, 3*, 717-736.

Yoshioka, S., Nagano, A., Hay, D. C., & Fukashiro, S. (1986). Abstract, the the minimum required muscle force for a sit-to-stand task. *Journal of Biomechanics, 19*(6), 471.

Zames, G. (1981). Feedback and optimal sensitivity: model reference transforma- tions, multiplicative seminorms, and approximate inverses. *IEEE Trans. Automatic Control, 26*, 301-320.

Zheng, Y.-F., & Hemami, H. (1986). Computation of multibody system dy- namics by a multiprocessor scheme. *IEEE Transactions On Systems, Man and Cybernetics, 1*, 102-110.

Copyrights