The Influence of Design Parameters on Straight Bevel Gear Dynamics

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Abstract

This study analyzes the influence of design parameters on the dynamics of straight bevel gears by constructing a model that allows variation in the shaft angle, pressure angle, and backlash. According to the statistical analysis, the order of influence of these parameters on weight is shaft angle > pressure angle > backlash. When the shaft angle is 90°, the statistical results show the drive is stable and the stress fluctuation level is low. The pressure angle, on the other hand, can affect the gear's dynamic property by influencing the driving component force on the gear and the component force on the shaft. The results for the shaft and pressure angles are used to determine the appropriate backlash. Overall, the analysis not only provides designers with an important reference but explains the dominance in the market of gear designs with a 90° shaft angle and a 20° pressure angle.

Keywords: bevel gears, shaft angle, pressure angle, backlash, dynamics, stress

1. Introduction

Bevel gears, primarily of spiral, hypoid, and straight types, are widely used on the power transmission of concurrent axes and alternating axes. In general, the shaft angle of a bevel gear is designed to be 90° with a maximum reduction ratio of 10, leading to high efficiency and high precision. In particular, the design must meet the spatial surface meshing condition, and the gear blank and tooth form designs must be able to vary with different cutting methods (Litvin et al., 1987). The calculation of bevel gears is thus more complicated than for other gear types (Fong & Tsay, 1991, 1992) and requires the development of special design software. The most common type of bevel gear is the straight bevel gear, which is applied to power transmissions between concurrent axes. This gear is characterized by a straight tooth whose depth decreases gradually from large to tip end. The theoretical tooth form is involute (Al-Daccak et al., 1994), but the tooth form designs vary with different manufacturing methods. Compared to other bevel gears, however (e.g. the spiral bevel gear), the configuration is simple and the gear easily made, forged, and shaped. As a result, the bevel differential mechanism is used extensively in the market.

As regards research on the straight bevel gear, most studies have focused on the theoretical mathematical model of gear (Tsai & Chin, 1987; Chang & Tsay, 2000; Figliolini & Angeles, 2004), teeth contact analysis (Jiao & Cao, 2011; Kolivand et al., 2015), finite element modeling and analysis (Nalluveettil & Muthuveerappan, 1993), teeth deflection analysis (Gagnon et al., 1997), manufacturing techniques and cutting methods (Özel et al., 2005; Shih & Fong, 2008; Shih, 2012; Fuentes et al., 2011; Tsuji et al., 2013; Kawasaki et al., 2015), measurement technique (Kawasaki & Shinma, 2009), lubrication regime (Ghahnavieh et al., 2014) and dynamic behavior of two-stage straight bevel gear (Yassine et al., 2014). To the best of our knowledge, however, no work exists on the influence of dynamic contact stress and gear design parameters on straight bevel gear dynamics. Hence, this study assesses the influence of a straight bevel gear's main design parameters (shaft angle, pressure angle, and backlash) on its dynamics by constructing a dynamics analysis model that weights the influence of the various parameters. The results provide a useful reference for straight bevel gear design.

2. Dynamics Analysis Model for a Straight Bevel Gear

2.1 Straight Bevel Gear Modeling

First, using Solidworks, a straight bevel gear CAD model is built in which the tooth form is created based on a mathematical model. Here, the modulus is 2, the number of teeth is 15, the pressure angle is 20°, the shaft angle is 90°, and the backlash is 0.2. The building of this model is described below and linked to the illustrations in Figure 1:

Step1: A blank is created with a shaft angle of 90° (a);

Step2: The involute profile is drawn on the gear plane (b);

Step3: A straight bevel gear tooth form is cut along the conical point of involute profile (c);

Step4: The required teeth are cut in a ring array to complete a straight bevel gear (d); and then

Step5: Another straight bevel gear (45 teeth) is built following the above steps, and the two straight bevel gears are combined (e).
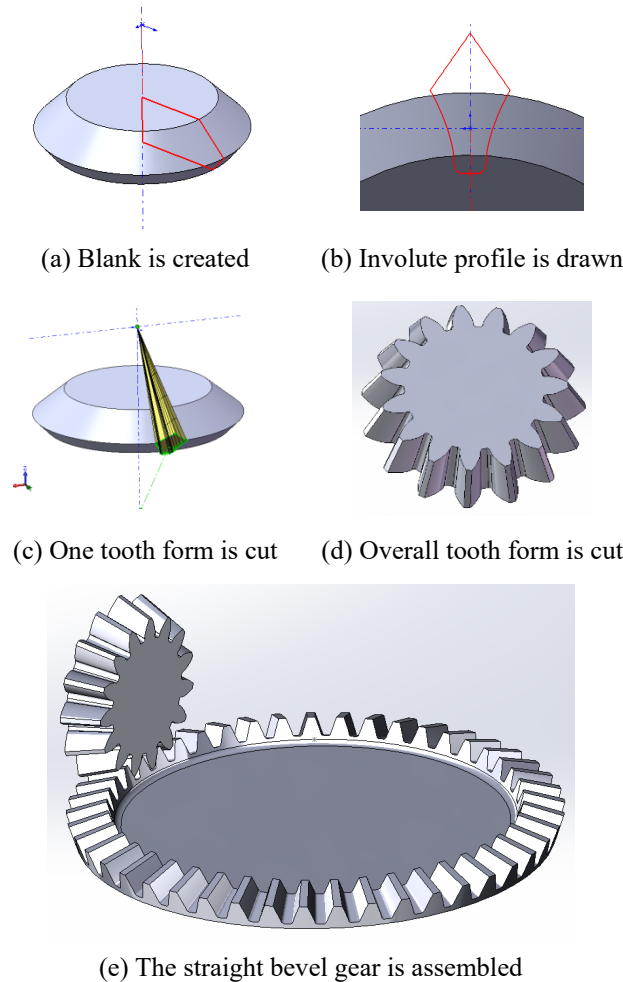

(a) Blank is created   (b) Involute profile is drawn

(c) One tooth form is cut   (d) Overall tooth form is cut

(e) The straight bevel gear is assembled

Figure 1. Construction of the straight bevel gear

2.2 Dynamics Analysis Model

To construct the dynamic analysis model, it employs a systematic dynamics calculation model for an experimentally validated cycloidal speed reducer (Hsieh, 2014), whose applications include testing traditional, improved, and novel designs and identifying their shortcomings and advantages (Hsieh, 2014). These modeling and calculation techniques (Hsieh, 2014) allow constructing a dynamic analysis model of the straight bevel gear. In this paper, the gear modulus is fixed at 2, the driving gear is set at 15 teeth, and the driven gear is at 45 teeth. The simulated conditions include total run duration of 1.5 sec and the driving gear rotation speed set at a constant 120 rpm (720 deg/s). The contact pattern is a 3D collision mode, and gravity acceleration is 9.81 m/sec^2. The material is alloy steel whose mechanical properties are given in Table 1.

In the stress analysis, stress can be calculated by the finite element method using SolidWorks (Hsieh, 2014). During bevel gear motion, the variations in gear contact stress can be measured over time. First, the triangular mesh is calculated. If the values obtained when calculating gear engagement are to be accurate, the grid density

in the place without gear contact can be low, but the grid density in the place with gear contact must be high. Grid computation is shown in Figure 2(a) and stress calculation in Figure 2(b).

The material used in all models and the friction factor are the same. Assuming the loading at the driven gear is fixed as 5 N-m, the effect on dynamics was carried out by giving different driving torque, which includes 15 N-m, 30 N-m, 45 N-m, 60 N-m, 75 N-m and 90 N-m, respectively. Thus, different parameter design values are assigned to the straight bevel gear model that result in different motion and stress variations. The tooth profile design parameters are pressure angle, shaft angle, and backlash, whose parameter value settings are listed in Table 2. To assess the influence of parameter design on the straight bevel gear's dynamics, these parameter values are cross related, producing 27 groups as simulation subjects in each driving torque case.

Table 1. Mechanical properties of the material

Material	Mechanical properties N/mm^2			
	Elastic modulus	Shear modulus	Tensile strength	Yield strength
Alloy steel	210,000	79,000	723.1025	620.422

Table 2. Gear design parameter values in each driving torque case

Numerical	Parameter		
	Pressure angle	Backlash	Shaft angle
1	14.5°	0.1	45°
2	20°	0.3	60°
3	25°	0.5	90°
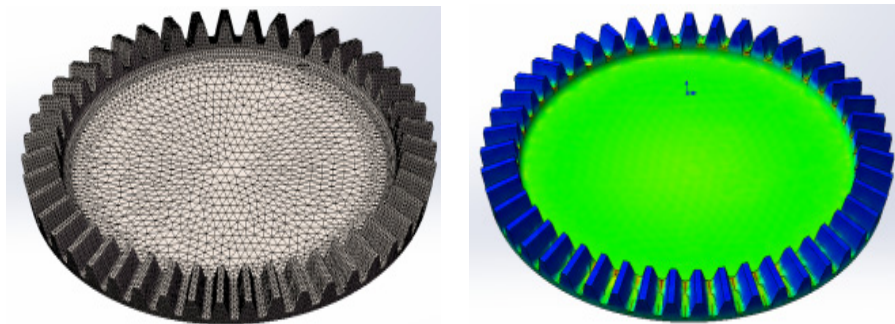
(a) Grid chart   (b) Schematic diagram of stress

Figure 2. Stress analysis model

2.3 Dynamics Analysis

In the dynamics analysis, for example, the driving torque is 15 N-m, the fixed pressure angle is 20°, the backlash is 0.1 mm, and the variable shaft angle is 45°, 60°, and 90° respectively. The following results are obtained based on the data from one revolution of the driving gear. The time interval of calculation is 0.0013889 s. Figures 3 and 4 show the analytic results of the angular velocity and angular acceleration of the driven gear. The velocity fluctuation is regular in all three shaft angle designs, but the 60° shaft angle has the maximum fluctuation. The angular acceleration analysis further reveals that the 60° shaft angle has the maximum peak values. Although one explanation could be larger vibrations, when the shaft angle is 90°, the minimum angular velocity fluctuation and acceleration reach peak values, so the motion is relatively stable and the vibration slight.
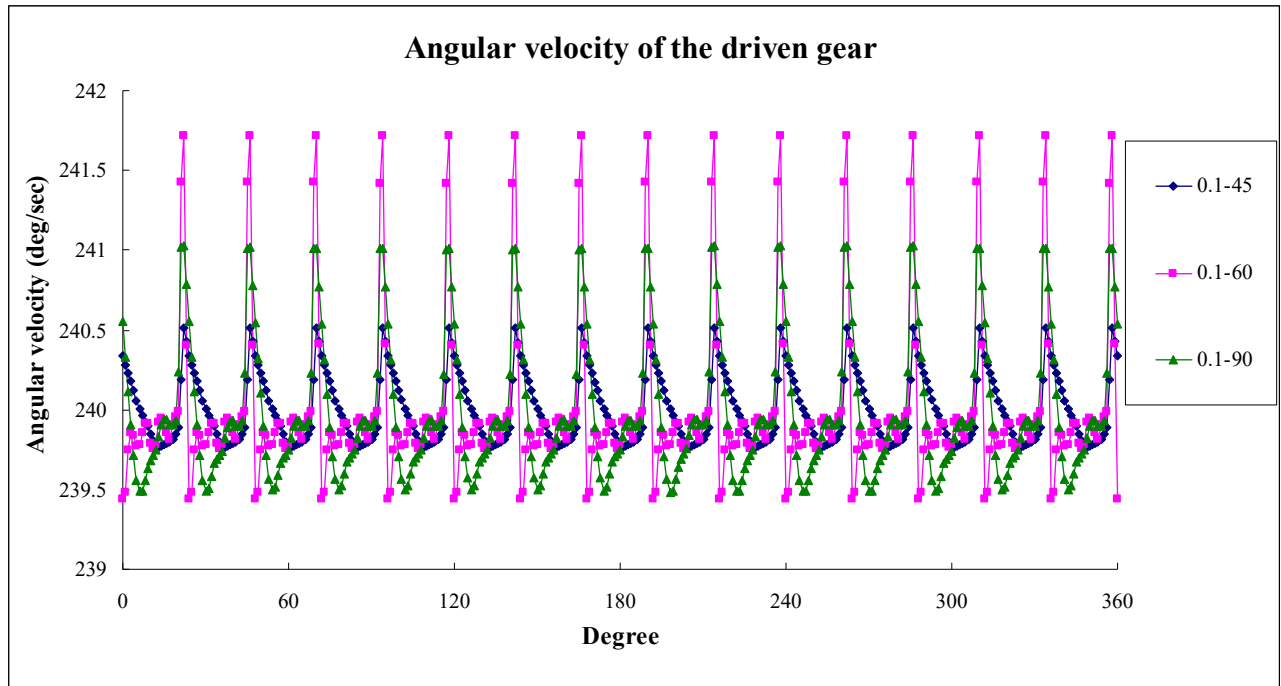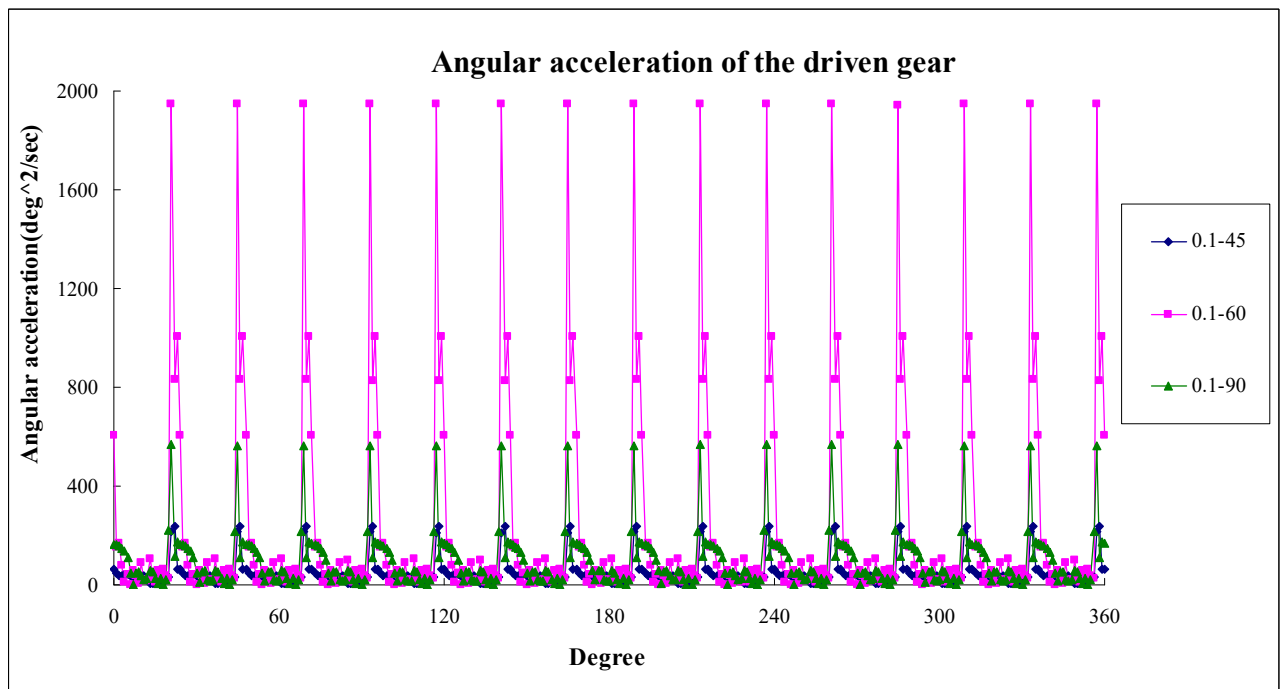
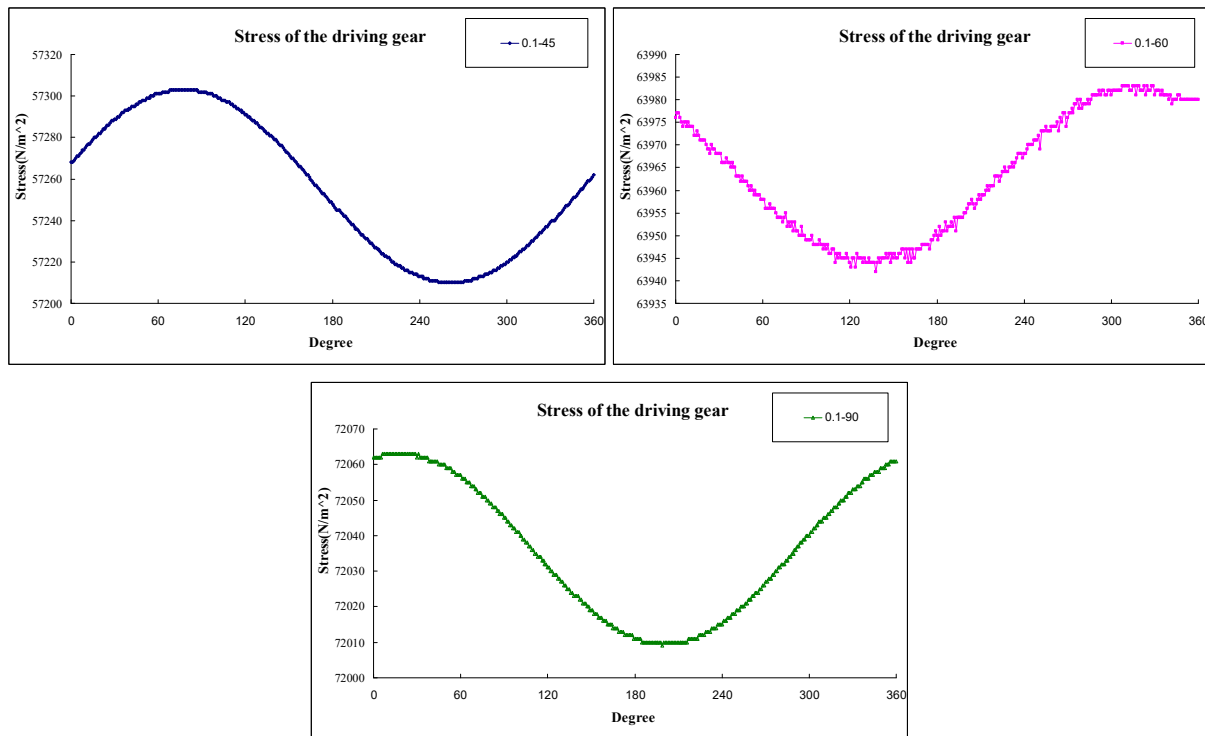Figure 3. Angular velocity of the driven gear

Figure 4. Angular acceleration of the driven gear

(a)

(b)

Figure 5. Stress variation of driving gear

According to the stress analysis, the axial component force increases with the shaft angle, while the stress of the driving gear increases gradually (Figure 5(a)). As Figure 5(b) shows, the three stress curves of the driving gear fluctuate regularly but slightly. When the shaft angle is 90°, the average stress is maximum about 72000 N/m^2; when the shaft angle is 60°, the average stress is about 64000 N/m^2; and when the shaft angle is 45°, the average stress is minimum about 57000 N/m^2.
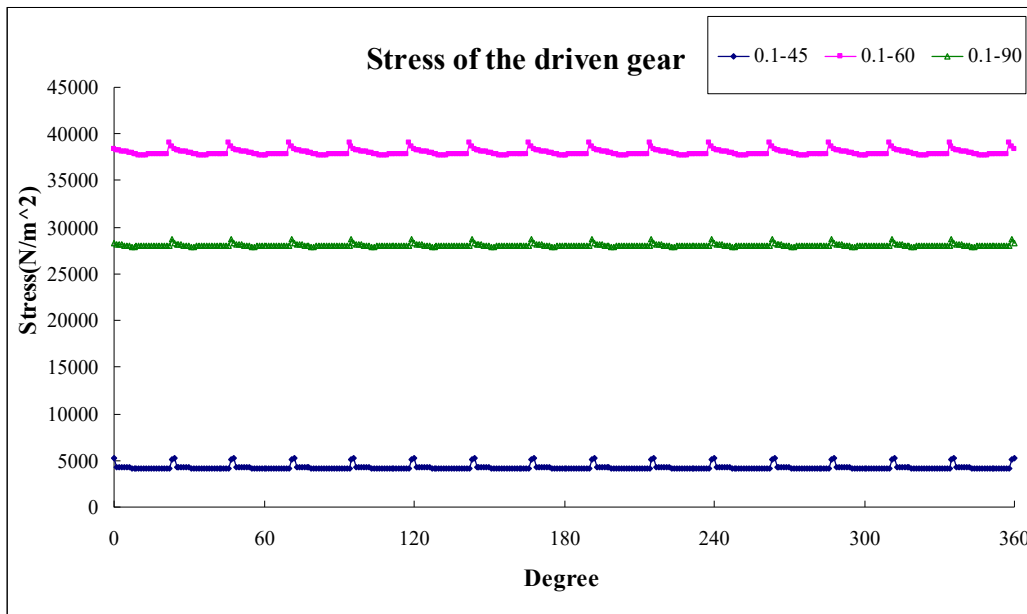
(a)
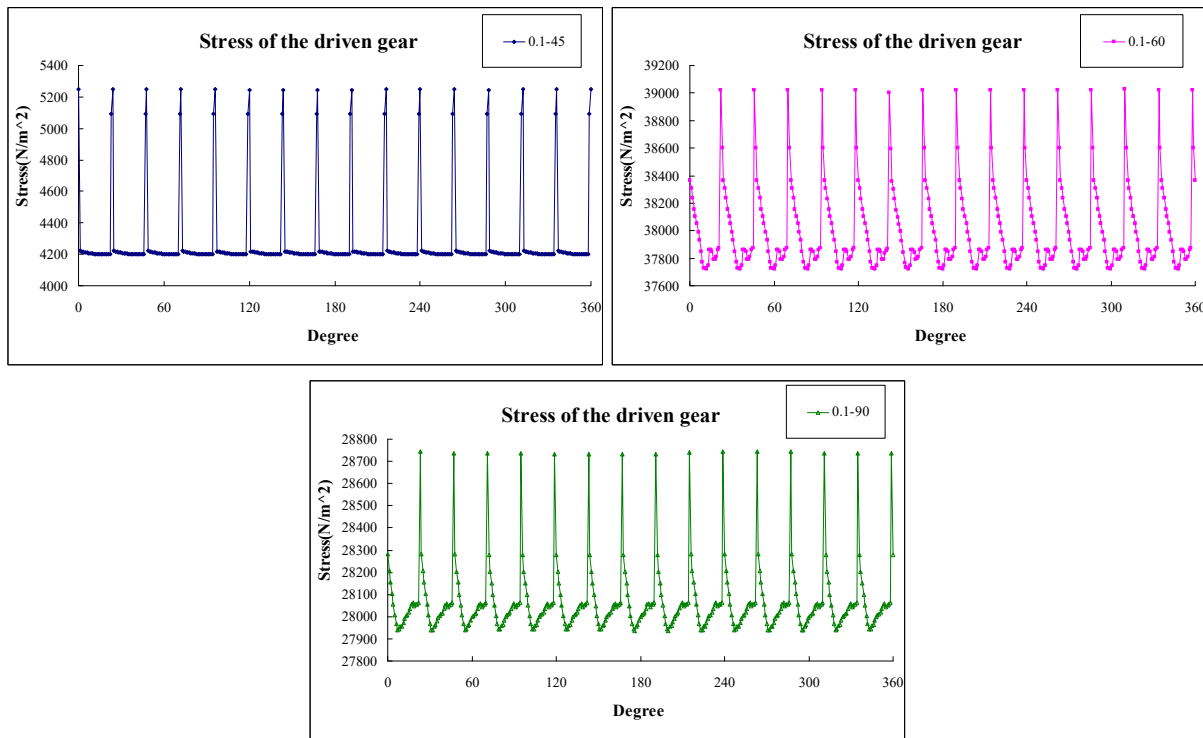
(b)

Figure 6. Stress variation of driven gear

The stress of the driven gear is as shown in Figure 6. The stress peaks are all regular (Figure 6(b)). In this case, the maximum average stress is the shaft angle 60° after that is the shaft angle 90°, and the minimum average stress is the shaft angle 45°. Since the stress ripple coefficient (Hsieh, 2014) can indicate the stress fluctuation level, the calculation result shows the hierarchy of the stress ripple level is as shaft angle 45°>shaft angle 60° >shaft angle 90°. This analysis thus suggests a shaft angle of 90° as an ideal design.

The stress analysis can indicate the contact stress condition, thus would affect the life of the components. For comparing, each driving torque case includes 27 sets of stress data.

Since the rotation speed setting is on the driving gear, only the stress of the driven gear is analyzed and discussed. The equation of standard deviation can be expressed as

$$S_d^{(i)} = \frac{1}{N}\sqrt{N\Sigma\sigma^2 - (\Sigma\sigma)^2} \,,\ \ i = 1 \sim 27 \tag{1}$$

where N is the number of stress data for one revolution of the driving gear. σ is the stress data of the driven gear.

In order to compare the stress variation level, the coefficient of variation can be defined as

$$C_v^{(i)} = \frac{S_d}{\bar{\sigma}} \times 100 \,,\ \ i = 1 \sim 27 \tag{2}$$

In which, $\bar{\sigma}$ means the average stress of the driven gear.

Here, the influences of the backlash, pressure angle and the shaft angle parameters, the equations are defined respectively as

$$B_m = \sum_{j=1}^{9} C_{v,m}^{(j)} \,,\ \ m = 0.1, 0.3, 0.5 \tag{3}$$

$$P_n = \sum_{j=1}^{9} C_{v,n}^{(j)} \,,\ \ n = 14.5, 20, 25 \tag{4}$$

$$S_o = \sum_{j=1}^{9} C_{v,o}^{(j)} \,,\ \ o = 45, 60, 90 \tag{5}$$

In this paper, the tendency of a whole influence is calculated by sum of variation coefficient of each driving torque case as follows:

$$SB_m = B_m^{(15)} + B_m^{(30)} + B_m^{(45)} + B_m^{(60)} + B_m^{(75)} + B_m^{(90)} \,,\ \ m = 0.1, 0.3, 0.5 \tag{6}$$

$$SP_n = P_n^{(15)} + P_n^{(30)} + P_n^{(45)} + P_n^{(60)} + P_n^{(75)} + P_n^{(90)} \,,\ \ n = 14.5, 20, 25 \tag{7}$$

$$SS_o = S_o^{(15)} + S_o^{(30)} + S_o^{(45)} + S_o^{(60)} + S_o^{(75)} + S_o^{(90)} \,,\ \ o = 45, 60, 90 \tag{8}$$

3. Statistical Results and Discussion

### 3.1 Influence on Dynamics of the Backlash Parameter

As shown in Figure 7 (a), when the backlash is small (0.1 mm), the gear collision frequency and gear vibration will increase and the gear contact travel will extend to increase the stress. Thus, the variation coefficients in most driving torque cases are the largest. The whole tendency shows that the stress ripple level would relieve with the increasing of backlash as shown in Figure 8(a). When the backlash is large, in contrast, the collision between gears can be reduced, although there will be energy loss in the power transmission, reducing transmission efficiency. The maximum variation coefficient in backlash parameter is the smallest (see Figure 8) among the three parameters, thus it has the minimum influence comparing to the other parameters. Accordingly, the main influence on stress and fluctuation occurs at the shaft angle and pressure angle, an appropriate backlash value should be determined based on their design and working condition.

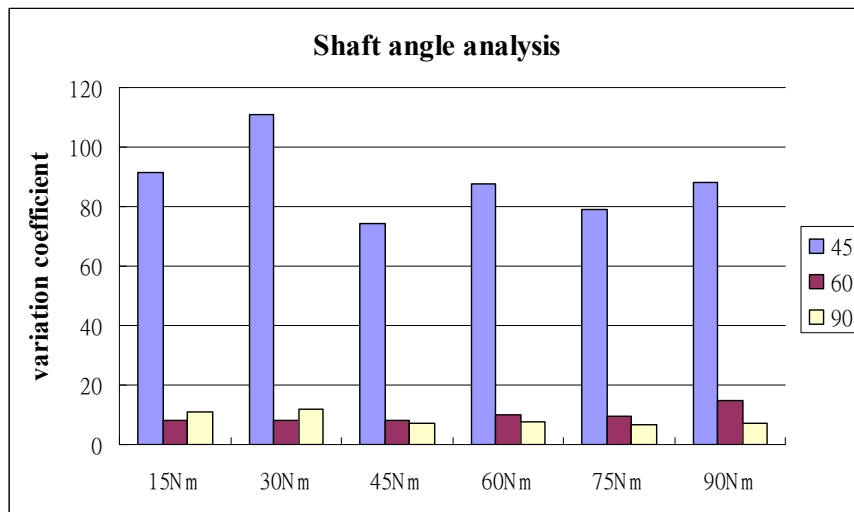### 3.2 Influence on Dynamics of the Pressure Angle Parameter

The variation coefficient results show the influence degree of the pressure angle parameter is between the backlash and shaft angle parameters in Figures 7 and 8. In the driven gear, the stress ripple level would be increased with the decreasing of the pressure angle especially at 14.5°, which load and fluctuation are the largest. Apparently, the gear driving component force and stress fluctuation increase as the pressure angle decreases. On the other hand, the larger the pressure angle (25°), the lower the gear driving component force, and the stress fluctuation decreases while the load on the shaft increases. Hence, according to the stress statistical result, a pressure angle of 20° is a better choice.
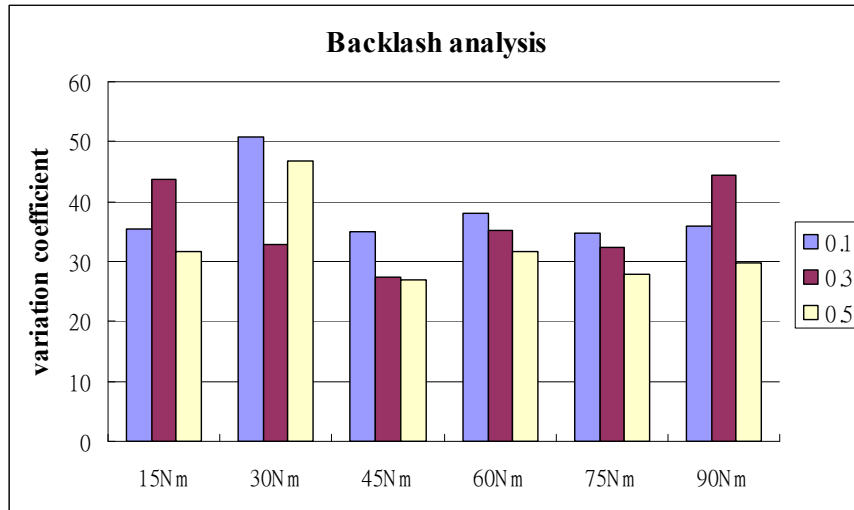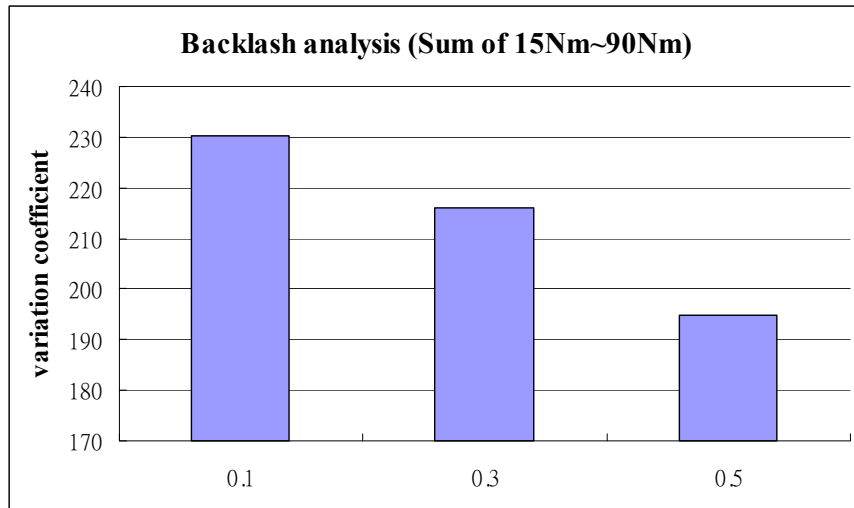
(a) Backlash
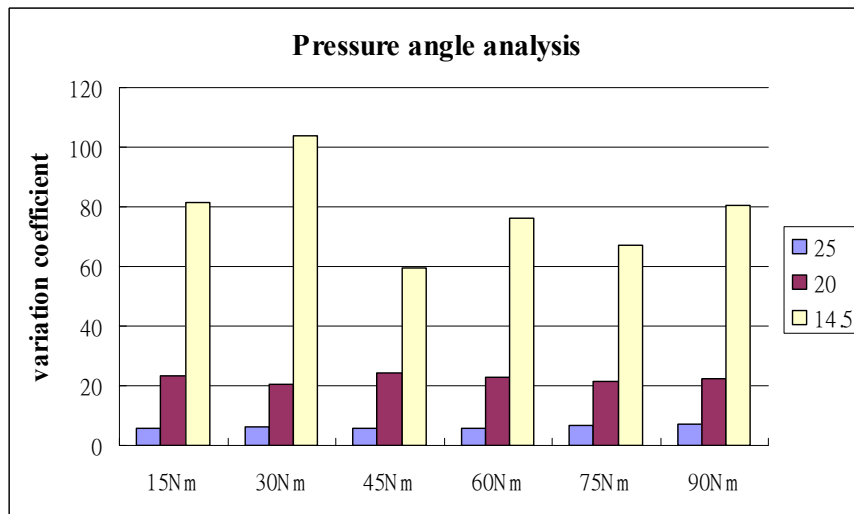
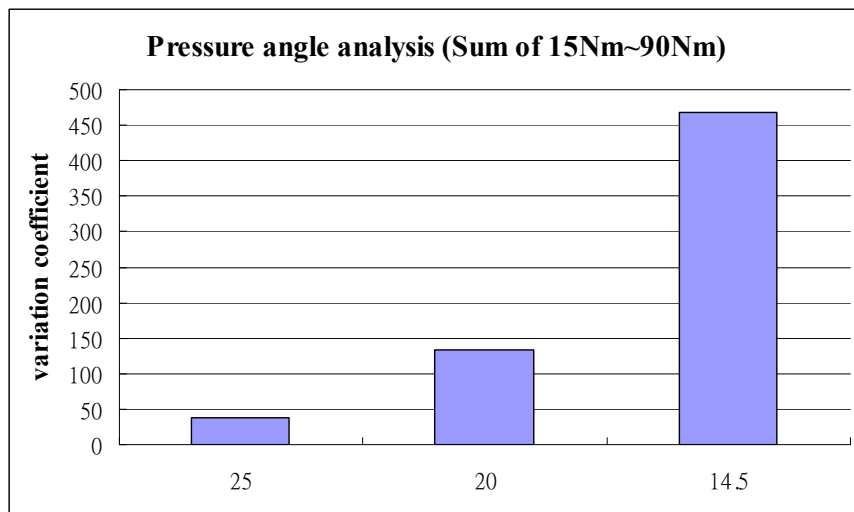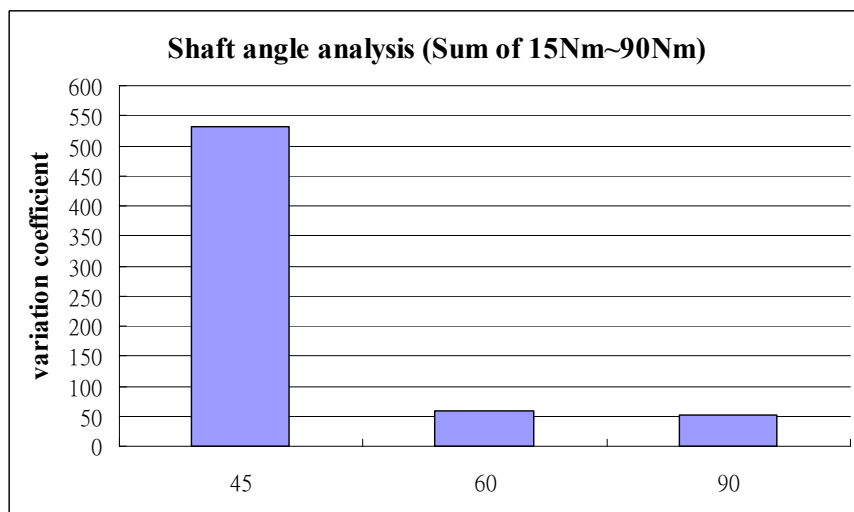
(b) Pressure angle

(c) Shaft angle

Figure 7. Statistical results in each driving torque case

(a) Backlash

(b) Pressure angle

(c) Shaft angle

Figure 8. Statistical results by sum of variation coefficient of each driving torque case

3.3 Influence on Dynamics of the Shaft Angle Parameter

Since the maximum variation coefficient in shaft angle parameter is the largest as shown in Figures 7 and 8, it places the maximum influence on the shaft angle design. The variation coefficient of shaft angle at 45° is the largest in each driving torque case (see Figure 7(c)). When the shaft angle increases to 60°, the stress fluctuation frequency would obviously reduce. The shaft angle at 90° is the most stable one and thus has the least stress variation coefficient in most driving torque cases. Furthermore, the shaft angle at 90° can obtain more stable operation than the case at 60° under a larger driving torque. Besides, Figure 8 (c) represents the whole tendency in the smaller shaft angle (like shaft angle at 45°), however, its combination with other two parameters may not be a better condition. The shaft angle at 90° can get the lowest variation coefficient. Hence, unless there are special power requirements, the shaft angle at 90° is recommended as the design basis.

4. Conclusions

By constructing a dynamics analysis model for straight bevel gears, this analysis generates accurate shaft angle, pressure angle, and backlash design parameters and then measures the influence of each parameter on the gear's dynamics. From the statistical analysis, the influence of the backlash is not obvious because the variation coefficient is quite similar in different backlash size. However, the size of the backlash still can influence the gear's dynamic properties. The analysis also reveals that the pressure angle can be designed according to load. That is, when the pressure angle is large, the stress and stress fluctuation on the gear can be reduced, but the axial component force is increased, placing a burden on the bearing. The component having the maximum influence on dynamics is the shaft angle, and the shaft angle at 90° with other two design parameters would be recommended as a better design.

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