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Dynamical Bayesian Significance Testing for Information on Performance Variation of Rolling Bearing for Space Applications

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Abstract

A dynamical Bayesian significance testing method is proposed to examine information on performance variation of rolling bearings for space applications under the condition of an unknown probability distribution and trend in advance. Sub-series of time series of rolling bearing performance are obtained via a regularly sampling, probability density functions of sub-series are acquired with bootstrap and maximum entropy theory, a referenced sequence from sub-series is found by minimum variance principle, posterior probability density function is established according to Bayesian theory, and mutation probability is defined in the light of fuzzy set theory. At the given significance level, dynamical Bayesian significance testing for information on performance variation of rolling bearings is put into effect with the help of mutation probability. Experimental investigation presents that the method proposed can effectively detect variation information of rolling bearing performance with unknown probability distributions and trends.

Keywords: rolling bearing, space applications, Bayesian significance testing, information analysis, performance variation

1. Introducation

With the devolopment of the fields of aeronautics and astronautics, bullet trains, and alternative energy, research of rolling bearing performance has attracted much attention, with many new findings (Randall & Antoni, 2011; Oguma, 2011; Xia, 2012; Mukhopadhyay & Bhattacharya, 2011; Sinha et al., 2010). At present, studies of rolling bearing performance mainly rely on a known probability distribution and trend in advance. For example, the probability distribution of performance is considered as a normal distribution, a Weibull distribution, or a Poisson distribution; and the trend of performance is regarded as a given potential function and kernel function and wavelet basis function, and a piecewise linearized function. However, many performance indexes are required for rolling bearings, different performance indexes for different applications (Shimizu, 2012; Siegel David et al., 2012; Yasufuku et al., 2010; Soylemezoglu et al., 2010; Arakere et al., 2010). So far, failure probability distributions and degradation trends of much performance, such as friction torque, vibration, and running accuracy, still are unknown. Particularly, degradation of rolling bearing performance belongs to a non-stationary stochastic process characterized by nonlinear dynamics, which goes through three phases, early degradation phase, gradual degradation phase, and rapid degradation phase, along with a change in failure probability distributions and trends of performance (Xia, 2012a & 2012b; Sinou, 2009; Ahmad et al., 2009; Xia & Chen, 2013). Thus, the rolling bearing performance analysis theory relied on prior information of probability distributions and trends encounters serious challenges, resulting in this hard problem to solve. For this end, under the condition of unknown probability distributions and trends in advance, a method for dynamical Bayesian significance testing is proposed to examine information on rolling bearing performance variation, for the early detection of the hidden danger of failure of rolling bearing performance, thus avoiding serious accident. Experimental investigation on vibration acceleration of rolling bearings for space applications is conducted for corroboration of the method.

2. Mathematical Model

Suppose performance data of a rolling bearing in service are sampled R times and R time series of performance data are obtained. Let X_r stand for the rth time series that is given by

$$X_r = \left(x_r(1), x_r(2), ..., x_r(h), ..., x_r(H) \right);\ r = 1,2,...,R \tag{1}$$

where $x_r(h)$ is the hth datum in X_r; h is a sequence number, h = 1, 2, …, H; and H is the number of data in X_r.

The rth time series X_r is divided into D sub-series and the dth sub-series is given by

$$X_{rd} = \left(x_{rd}(1), x_{rd}(2), ..., x_{rd}(i), ..., x_{rd}(I) \right); d = 1,2,...,D \tag{2}$$

where $x_{rd}(i)$ stands for the ith datum in X_{rd}; i for a sequence number, I = 1, 2, …, I; and I for the number of data, which is expressed as

$$I = \frac{H}{D} \tag{3}$$

According to bootstrap, an equiprobable resampling with replacement from X_{rd} is implemented by following steps:

(1) Let the constant B be equal to 500000, and let the variable b take a value 1, where B is the number of the resampling samples and b is the bth equiprobable resampling.

(2) Let one datum be drawn by an equiprobable resampling with replacement from X_{rd}.

(3) Let the step (2) be repeated I times, so that I data can be sampled.

(4) Calculate the mean $y_{rd}(b)$ of I data, which is considered as one of the data in the generated data series Y_{rd}.

(5) Add 1 to b.

(6) If $b>B$, go to the step (7); otherwise go to the step (2).

(7) Let the generated data series be of size B = 500000, so that many generated data are obtained.

Via steps (1) to (7), the generated data series Y_{rd} is gained, as follows:

$$Y_{rd} = \left(y_{rd}(1), y_{rd}(2), ..., y_{rd}(b), ..., x_{rd}(B) \right) \tag{4}$$

with

$$y_{rd}(b) = \frac{1}{I} \sum_{i=1}^{I} \theta_b(i); b = 1,2,...,B \tag{5}$$

where $\theta_b(i)$ is the ith data obtained and $y_{rd}(b)$ is the mean of I data in the bth sampling.

The origin moment of X_{rd} is as follows:

$$M_{rdm} = \frac{1}{B} \sum_{b=1}^{B} (y_{rd}(b))^m; m = 1,2,...,M_{rd} \tag{5'}$$

where M_{rd} is the highest order of the origin moments and M_{rdm} is the mth order origin moment.

Assume x is a random variable for describing rolling bearing performance data. According to maximum entropy theory, a probability density function $f_{rd}(x)$ is obtained by

$$f_{rd}(x) = \exp\left(\sum_{k=0}^{M_{rd}} c_{rdk} x^k \right) \tag{6}$$

where c_{rdk} is the kth Lagrangian multiplier about X_{rd} and k = 0,1,…, M_{rd}.

In Equation (6), the Lagrangian multiplier c_{rdk} (k = 1,2,…, M_{rd}) can be solved by

$$M_{rdm} = \frac{\int_{R_{rd}} x^m \exp\left(\sum_{k=1}^{M_{rd}} c_{rdk} x^k \right) \mathrm{d}x}{\int_{R_{rd}} \exp\left(\sum_{k=1}^{M_{rd}} c_{rdk} x^k \right) \mathrm{d}x}; m = 1,2,...,M_{rd} \tag{7}$$

The first Lagrangian multiplier c_{rd0} can be obtained by

$$c_{rd0} = -\ln\left(\int_{R_{rd}} \exp\left(\sum_{m=1}^{M_{rd}} c_{rdm} x^m \right) \mathrm{d}x \right) \tag{8}$$

where R_{rd} is the integrating range of x about X_{rd}.

Let $r = 1$ in Equation (6), then the probability density function of the dth sub-series X_{1d} in the first time series X_1 is obtained as

$$f_{1d}(x) = \exp\left(c_{1d0} + \sum_{m=1}^{M_{rd}} c_{1dm} x^m \right) \tag{9}$$

For the first time series X_1, let X_{1d} be both a prior sample and a current sample and $f_{1d}(x)$ be both a prior distribution and a current sample distribution. According to Bayesian statistics, the posterior probability density function of X_{1d} is obtained as

$$\varphi_{1d}(x) = \frac{f_{1d}(x) f_{1d}(x)}{\int_{R_{1d}} f_{1d}(x) f_{1d}(x) \mathrm{d}x} \tag{10}$$

According to statistics, the mathematical expectation E_{1d} of X_{1d} is defined as

$$E_{1d} = \int_{R_{1d}} x \varphi_{1d}(x) \mathrm{d}x \tag{11}$$

and the variance D_{1d} of X_{1d} is defined as

$$D_{1d} = \int_{R_{1d}} (x - E_{1d})^2 \varphi_{1d}(x) \mathrm{d}x \tag{12}$$

According to the minimum variance principle, the minimum variance $D_{1\min}$ is given by

$$D_{1\min} = \min(D_{1,1}, D_{1,2}, ..., D_{1d}, ..., D_{1D}) \tag{13}$$

For the first data series, suppose the sub-series with the minimum variance $D_{1\min}$ is marked by $X_{1\min}$ and the posterior probability density function of $X_{1\min}$ is marked by $\varphi_{1\min}(x)$. Define $X_{1\min}$ and $f_{1\min}(x)$ as the referenced sequence and the referenced distribution, respectively.

For the rth time series ($r = 2, 3, \ldots, R$), let X_{rd} and $f_{rd}(x)$ be the current sample and current sample distribution, respectively, then according to Bayesian statistics the posterior probability density function $\varphi_{rd}(x)$ of X_{rd} is as follows:

$$\varphi_{rd}(x) = \frac{f_{1\min}(x) f_{rd}(x)}{\int_{R_0} f_{1\min}(x) f_{rd}(x) \mathrm{d}x}; r = 2,3,..., R \tag{14}$$

where R_0 is the integrating range of x.

According to statistics, the mathematical expectation E_{rd} of X_{rd} is defined as

$$E_{rd} = \int_{R_0} x \varphi_{rd}(x) \mathrm{d}x; r = 2,3,..., R \tag{15}$$

and the variance D_{rd} is defined as

$$D_{rd} = \int_{R_0} (x - E_{rd})^2 \varphi_{rd}(x) \mathrm{d}x; r = 2,3,..., R \tag{16}$$

Variance ratio of X_{rd} to $X_{1\min}$ is defined as

$$\lambda_{1,rd} = \frac{D_{rd}}{D_{1\min}}; r = 2,3,\ldots,R \tag{17}$$

In the light of concept of intersection of fuzzy sets, a mutation probability $\alpha_{1,rd}$ is defined as

$$\alpha_{1,rd} = 1 - A(\varphi_{rd}(x) \cap \varphi_{1\min}(x)) \tag{18}$$

where $A(\varphi_{rd}(x) \cap \varphi_{1\min}(x))$ stands for the area of the intersection of $\varphi_{rd}(x)$ and $\varphi_{1\min}(x)$.

The mutation probability $\alpha_{1,rd}$ can take values in [0,1]. Let significance level be α=0.1, then significance testing for performance variation of rolling bearings can be conducted.

If

$$\alpha_{1,rd} > \alpha \tag{19}$$

then variation of X_{rd} is of significance; otherwise, variation of X_{rd} is of no significance.

3. Case Studies

This case involves with experiment on vibration acceleration of a rolling bearing for space applications. The rolling bearing that was installed on a specialized performance rig worked for 46 days (time interval: 8 November 2010 to 23 December 2010, running conditions of axial load of 49N and of rotational speed of 1000 r/min) and test data, in dB, were sampled 10 times (viz., R = 10), one time every 5 days and 4000 data (viz., H = 4000) every time, as shown in Figures 1 and 2.

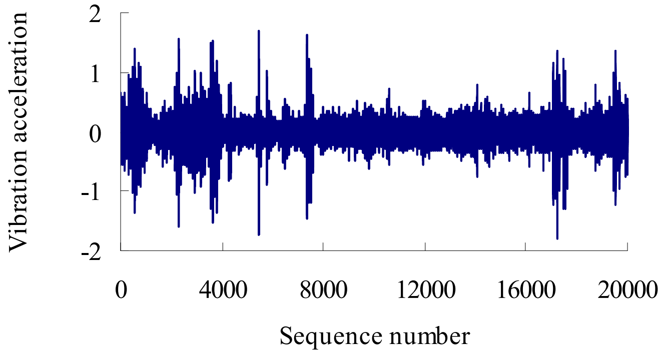

Figure 1. Experimental data of time series from X_1 to X_5

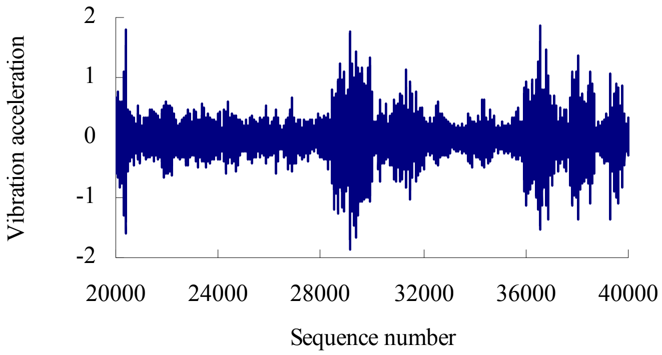

Figure 2. Experimental data of time series from X_6 to X_{10}

It is easy to see from Figures 1 and 2 that as time series, information of rolling bearing vibration acceleration presents a complex and variational status, with an unknown probability distribution and trend.

From Figures 1 and 2, every 400 data are considered as a sub-series, viz., I = 400, and 4000 data in the first sub-series X_1 are regarded as prior information that includes ten sub-series, $X_{1,1}$, $X_{1,2}$, …, X_{1d}, …, $X_{1,10}$ (including

400 data in every sub-series).

Using Equations (9) to (13), the mathematical expectation E_{1d} and the variance D_{1d} of X_{1d} are calculated for selection of the referenced sequence $X_{1\min}$ and results are listed in Table 1.

Table 1. Selection of referenced sequence

Prior sample (8 November 2010)	Current sample (8 November 2010)	Mathematical expectation	Variance×10^{-5}
The first sub-series	The first sub-series	-0.003	5.8993
The second sub-series	The second sub-series	-0.0045	16.294
The third sub-series	The third sub-series	-0.0056	3.7922
The fourth sub-series	The fourth sub-series	-0.0054	**1.4836**
The fifth sub-series	The fifth sub-series	-0.0048	2.4824
The sixth sub-series	The sixth sub-series	-0.0012	8.7588
The seventh sub-series	The seventh sub-series	-0.0070	9.4744
The eighth sub-series	The eighth sub-series	-0.0024	43.029
The ninth sub-series	The ninth sub-series	0.0005	31.533
The tenth sub-series	The tenth sub-series	-0.0035	22.149

According to Table 1, the fourth sub-series, viz., $X_{1\min} = X_{1,4}$, is selected as the referenced sequence due to its minimum variance $D_{1,4} = D_{1\min}$. Based on this, with the help of Equations (15), (17), and (18), the mathematical expectation, the variance ratio, and the mutation probability are obtained as shown in Figures (3), (4), and (5), respectively.

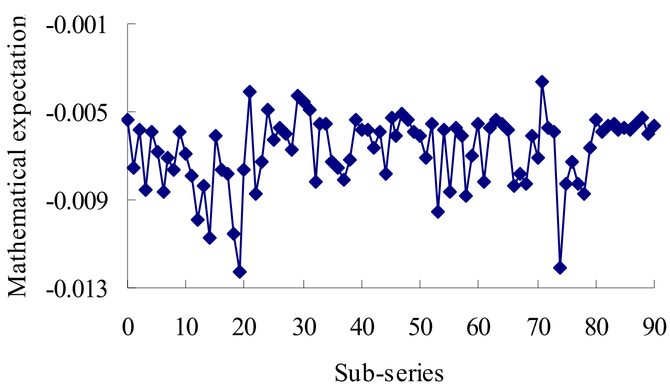

Figure 3. Mathematical expectation

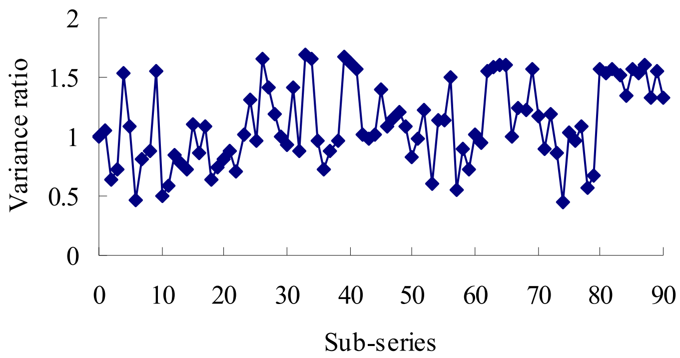

Figure 4. Variance ratio

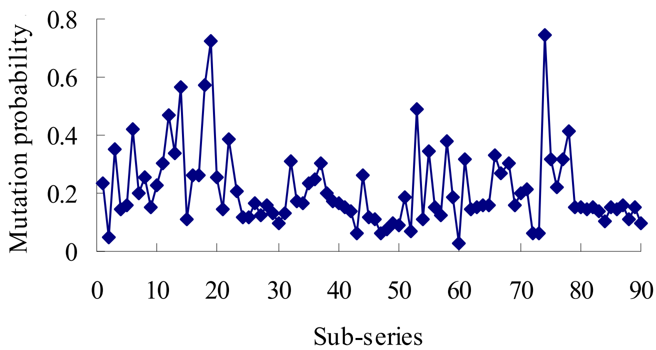

Figure 5. Mutation probability

From Figure 5, overall, from beginning of the second sub-series (corresponding to abscissa values 1 to 10 in Figure 5) to end of the third sub-series (corresponding to abscissa values 11 to 20 in Figure 5), the mutation probability is in a rising trend; from beginning of the fourth sub-series (corresponding to abscissa values 21 to 30 in Figure 5) to end of the ninth sub-series (corresponding to abscissa values 71 to 80 in Figure 5), the mutation probability that takes values in the range from 0.03 to 0.32 is in a large fluctuation; and from beginning of the tenth sub-series (corresponding to abscissa values 80 to 90 in Figure 5), the mutation probability that takes values about 0.1 is in a new stability. As a result, variation information of rolling bearing vibration acceleration is tested as follows:

(1) From 8 November to 18 November, vibration performance variation becomes gradually significant, showing an early degradation phase;

(2) From 23 November to 18 December, vibration performance variation is complex and variable, alternating significance and no significance and revealing a transitional period from early degradation phase to gradual degradation phase;

(3) On 23 December, vibration performance variation is not significant, meaning a start of gradual degradation phase.

It can be seen from the above that the method proposed is able to test information on rolling bearing performance variation.

4. Conclusions

The dynamical Bayesian significance testing method, under the condition of unknown probability distributions and trends in advance, can examine information on rolling bearing performance variation for the early detection of the hidden danger of failure of rolling bearing performance, thus avoiding serious accident. Experimental investigation on vibration acceleration of the rolling bearing for space applications shows correctness of the method.

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