# Analysis of a Continuously Variable Transmission in which Four-Bar Linkages Are Arranged in Parallel

Toshihiro Yukawa[1], Syoma Kumagai[2], Taiyo Fujisawa[2], Yoshiaki Oshida[3], Youichi Takeda[3] & Kazuyuki Hanahara[1]

[1] Faculty of Science and Engineering, Iwate University, Iwate, Japan

[2] Graduate School of Engineering, Iwate University, Iwate, Japan

[3] Division of Technical Support, Iwate University, Iwate, Japan

Correspondence: Toshihiro Yukawa, Faculty of Science and Engineering, Iwate University, 4-3-5 Ueda, Morioka, Iwate 020-8551 Japan. Tel: +81-19-621-6403. E-mail: yukawat@iwate-u.ac.jp

## Abstract

This paper describes a development of a new structural type of continuously variable transmission (CVT). We here propose a CVT with linkages and irreversible mechanisms which does not positively depend upon frictional conduction force between conduction components. In the proposed CVT, four lever-crank units are connected in parallel mechanically with the cranks at an input shaft, and the output shaft is also connected mechanically via an irreversible mechanism installed at the fulcrum of the lever. In the experiment, we confirm that the continuous control of gear ratios in real time by realizing high-precision control for expansion and contraction of the links using electric linear actuators. As a merit of the developed linkage type of CVT, it reduces power consumption, compared to other conventional CVTs.

**Keywords:** continuously variable transmission, CVT, closed link, four-bar linkage, lever-crank mechanism, slider-crank mechanism, one-way clutch, transmission efficiency, frictional force

## 1. Introduction

### 1.1 Historical Background

The first commercial belt type CVT was introduced in 1958 (Liebrand, 1996, p. 33). Its rubber V-belt CVT was restricted to 42 kW given a center distance of 520 mm (double belt application in 1972 with a ratio coverage of 3.9) (Keff, 1974). As for these CVT installed into most automotive applications, a significant increase in the power capacity combined with a reduced transmission size was needed (Brandsma, 1999). As one of the belt type CVT, Arjen Brandsma et al. have developed a push belt type CVT. The power capacity of the push belt CVT is mainly determined by ring stress. The ring stress level in the belt is not only influenced by load specification and belt design, but also by CVT design. Push belts are scalable with respect to power capacity. Also, they have discussed three parameters suitable for scaling, e.g., number of rings, element width and cone angle (Brandsma, 1999). As for the conventional developments for another type CVTs, many researchers have been developing the mechanisms, and studying the mechanics theories and control methods. For instance, A.J. Lemanski et al. have proposed a pericyclic mechanical transmission (PMT) (Lemanski, 2006). The PMT is a nutating and rotating gear mechanism that incorporates the positive engagement of meshing face-gear pairs. Zihni B. Saribay, et al. have investigated a pericyclic mechanical transmission (PMT) system by considering kinematics, tooth load capacity, bearing load capacity, and mesh efficiency (Saribay & Bill, 2013). Brad Pohl et al. have studied a continuously variable planetary traction drive under development by Fallbrook Technologies, Inc., which is kinematically equivalent to a variable planetary gear set, and can be configured as an infinitely variable transmission (IVT). Power density of a spherical CVT makes it scalable to several applications (Pohl, Simister, Smithson, & Miller, 2004). Wassif Shabbir et al. have investigated the performance of a CVT using an intuitive control in a hybrid electric vehicle (HEV) of series architecture while changing driving conditions. Also, they have employed the detailed dynamic vehicle simulation model in order to realize an in-depth evaluation of the CVT, as compared to a fixed transmission (FT) that utilizes a fixed final drive ratio between motor and wheels (Wassif & Simos, 2014).

*1.2 Previous Prototype Model*

In this paper, the previous prototype model (Kumada & Yukawa, 2010; Yukawa, Kumada, & Obinata, 2010; Yukawa, 2011) of CVT uses link mechanisms consisting of two sets of lever-cranks which are four-node link mechanisms and arranged symmetrically. We call this CVT as "L-CVT", and L- stands for "Linkage type". By adjusting the lengths of a crank and a connecting-rod in real time, the rocking angular velocity in the output side can be changed continuously with rotational speed of the crank at the input side. The driving force can be transmitted through an irreversible mechanism due to oscillation, leads an intermittent rotational movement in only one direction around the output shaft. In order to compensate for the intermittent force during driving transmission, continuous rotational movement of the output shaft is necessary to realize by the coordinated operation of multiple lever-cranks. Compared to our conventional L-CVT (Kumada & Yukawa, 2010; Yukawa, Kumada, & Obinata, 2010; Yukawa, 2011), a new prototype of L-CVT developed in this paper has four sets of lever-cranks arranged in parallel. The up-to-date developed L-CVT adopts a method in which the rotational angular velocity of the output shaft can be steplessly adjusted by expanding and contracting the lever instead of changing the lengths of the crank and the connecting-rod.

## 2. Mechanical Theory

*2.1 L-CVT using Four-Bar Linkages*

As an analysis of the basic geometric structure of a four-bar linkage mechanism, C. G. Gibson et al. have examined the geometry of a simple engineering mechanism, comprising four bars smoothly jointed together to form a movable quadrilateral with one fixed side (Gibson & Newstead, 1986). The configuration of the mechanism corresponding to the points of an elliptic curve is associated geometry and Morse theory. Figure 1 shows the basic structure of the four-bar linkage, and the situation where four sets are being installed in parallel. Link a is a fixed link installed on the base of L-CVT. The link is integrated with the base of the L-CVT, or is fixed to the base. The shortest link b is a crank which rotates around the input shaft. Link c is a connecting-rod to transmit the rotational force of the link b to link d. The link d is a lever that swings around the output shaft. As the input link b rotates, the output link d swings. In order to transmit this oscillation in only one direction, irreversible mechanisms such as a one-way clutch or a ratchet are used.

The irreversible mechanism is installed between a fulcrum of the output link d and an output shaft. In order for L-CVT using lever-crank and irreversible mechanism to have the function as a CVT, it is first necessary to realize the basic operation of lever-crank mechanism.
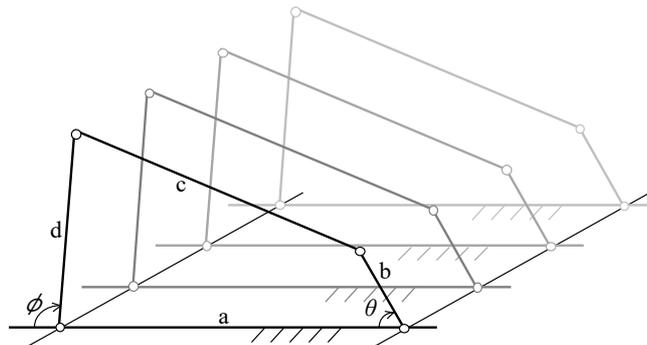


Figure 1. Arrangement of links in four-bar linkage, and L-CVT in which four units of four-bar linkages are arranged in parallel

*2.2 Necessary Condition for Lever-Crank Mechanism*

It is a necessary condition that the shortest link b can rotate around the end of the fixed link a as the fulcrum with regard to the length relationship between the four links in the lever-crank mechanism. Thus, in order to satisfy this condition on four links, the sum of the shortest link b and another link must always be smaller than the sum of the remaining two links (Kumada & Yukawa, 2010).

*2.3 Link Length*

Next, we consider a case where the movement of the tip of link b is in a uniform circular motion. When the length of each link is constant during swing motion period, the angular velocity of link d is not always constant. As the lever-crank (crank-rocker) is called a quick return mechanism from its movement, the period when the link d is

pulled by the link c is shorter than the one when the link d is pressed by the link c. In order to function as a continuously variable transmission for L-CVT, link d has to move at a constant angular velocity in both back and forth paths when link b moves at a constant angular velocity around the input shaft. In order for the L-CVT to satisfy this condition, one of the lengths of the links must be expanded and contracted.

*2.4 Continuous Control of Link Length*

Furthermore, when changing steplessly the gear ratio which corresponds to the input / output rotational speed ratio, it is required to control continuously one of the lengths of the links. The method to derive specific gear ratio and link length were described in the development of previous prototype L-CVT model (Kumada & Yukawa, 2010; Yukawa, Kumada, & Obinata, 2010; Yukawa, 2011).

## 3. Relationship between the Number of Lever-crank Units and the Output of L-CVT

*3.1 The Two Lever-crank Units*

We analyze the motion of the output link d under the condition that the length of each link is constant ($a = 150$, $b = 10$, $c = 150$, $d = 30$ ([mm]), while assuming that the number of lever-crank units mounted on the L-CVT is two. The response of the angular velocity of link d when the angular velocity of the input link b is constant ($2\pi$ [rad/s]) is shown in Figure 2. The horizontal axis $T$ represents the oscillation cycle at the input axis. The link d swings at an angular velocity of 0 to 0.022 [rad/s], and the each link d of the two lever-crank units is alternately driven by half oscillation cycles in order to compensate for full oscillation cycle of rotation at the output shaft. Due to the effect of the irreversible mechanism, the angular velocity of the output shaft is limited to the value with either positive or negative sign.
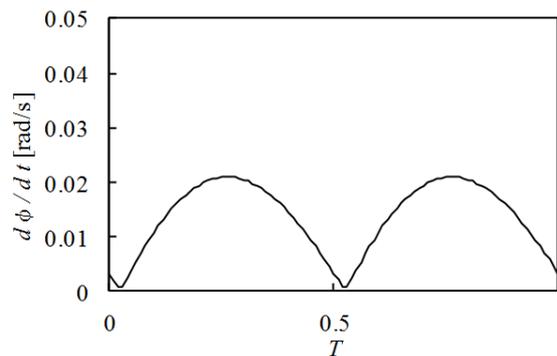


Figure 2. Response of the angle velocity of link d in the case when a couple of four-bar linkage units are combined

*3.2 The Four Lever-crank Units*

Next, we assume that the number of lever-crank units mounted on L-CVT is set to four, and analyze the motion of output link d under the condition that each link length is constant. The phases of each input link in the four lever-crank units are set to be delayed by every 90 °. For this situation, these links are combined mechanically at the crankshaft. The conditions for the angular velocity of the input link b and the length of each link are same as the ones when using the two lever-crank units described above. The angular velocity response of the output link d is shown in Figure 3. The link d swings at an angular velocity of 0.016 to 0.021 [rad/s]. At this time, the swing width of link d is smaller than the one of link d when using two lever-crank units. Each output link d in the four units is alternately driven within the section corresponding to quarter cycle in order to compensate for full one cycle of the output shaft. Judging from the above results of the angular velocity response in the output link d in two or four lever-crank units, it can be seen that the output angular velocity increases and the swing width decreases as the number of lever-crank units increases. Thus, if the number of lever-crank units is proportional to the output angular velocity, then the output angular velocity is smoothed and suppressed the fluctuation. It should also be noted that only the link moves with the maximum rocking angular velocity among the four links d in each unit can transmit the rotational force corresponding to the driving force as the output with the irreversible mechanism in the L-CVT.
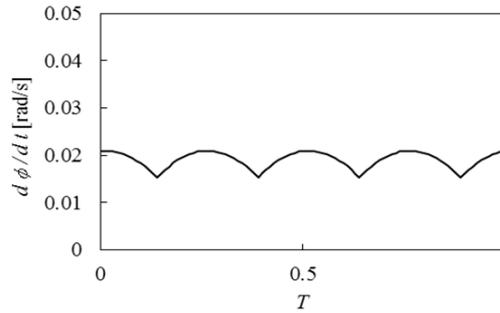
Figure 3. Response of the angle velocity of link d when four sets of four-bar linkage are combined

## 4. Expansion and Contraction of Link

In the former prototype models (Kumada & Yukawa, 2010; Yukawa, Kumada, & Obinata, 2010; Yukawa, 2011) of L-CVT, the angular velocity of the output link d can be controlled by extending and contracting the link b for shifting and the link c for satisfying its predefined length in order to realize several gear ratios. As a result, the input/output rotational speed ratio ralationship is maintained. These telescopic links can be controlled by independent drive with each motor using an external electric power supply. In contrast to L-CVTs that require such external power supply, L-CVTs whose links are expanded and contracted with three-dimensional cam are composed of mechanism without any power supplies.

At a stage prior to this development, we have installed a disk cam in advance into the prototype L-CVT model which is a reducer for a certain gear ratio. In this L-CVT as well, the expansion and contraction mechanism installed in the input shaft and link c is able to be interlocked mechanically via the planetary gear mechanism, and the plane cam. A part of the driving force applied to the input shaft is transmitted to the output side through another by-path in parallel, separately from the original driving force in order to rotate the flat cam. By this rotational force, expansion and contraction of the link c mounted with the cam follower which is interlocked with the flat cam is realized. Furthermore, in the mechanical L-CVT which is an improvement of the L-CVT, the plain cam was changed to a solid cam (3D cam). In the developed L-CVT, the amount of expansion and contraction of the link according to the cam displacement designed for each gear ratio can be mechanically controlled by the translational movement of the cam in the direction along the rotational axis of the solid cam. As mentioned above, the previous prototype L-CVT can shift the gear ratio by expansion and contraction of the links b and c.

While on the other hand, we examine the relationship between link length and gear ratio in expansion and contraction of link b and link d or either one of two links in this paper. We assume that the lengths of link a and link c are equal. Link b is the shortest link, and the relationships in the inequality of $b < a = c$, $b < d$ are satisfied. Set the link lengths as $b = 10$ and $a = c = 150$ (Set the link length as $a = 150$ to make a comparison with the L-CVT of the previous developed model (Kumada & Yukawa, 2010; Yukawa, Kumada, & Obinata, 2010)). Figure 4 shows the gear ratio when link d is being expanded and contracted. If link d is longer than link b, and shorter than link a or link c, that is, the relationship of $b = 10 < d < a = c = 150$ is satisfied, then the length of link d is proportional to the gear ratio. If the length of link d becomes longer than those of link a or link c, the gear ratio becomes comparatively small. Under the constraint condition of the link length, the speed of link d can be changed while expanding and contracting its link length. If the length of link d is longer than the one of link b, and is expanded and contracted within the range shorter than the lengths of link a or link c, then a gear ratio that proportional to its length can be obtained.

Next, we set the length of link d as d = 150. Figure 5 shows the gear ratio when link b expands and contracts within the range satisfying $10 < b < a = c = d = 150$ under the condition of $a = c = d = 150$. It can be seen that the length of link b is inversely proportional to the transmission gear ratio. Assuming that link b can be expanded and contracted within the range of $0 < b < 10$, and the length $b$ of link b approaches zero, then the transmission gear ratio increases to infinity.

As described above, the method of shifting the gear ratio by expand and contraction of either link b or link d was explained. That is to say, the link d was able to be expanded and contracted in order to obtain a gear ratio proportional to the amount of its expansion and contraction. In addition, it was understood that the gear ratio was increased by expanding and contracting the link b. As a point to be noted when designing the L-CVT, in order to prevent damage to main body when a large force is applied to the body, the dimensions for the rotary shafts and

links, the allowable torque, the interference between parts, and the accuracy of the extension control for the link must be carefully considered.
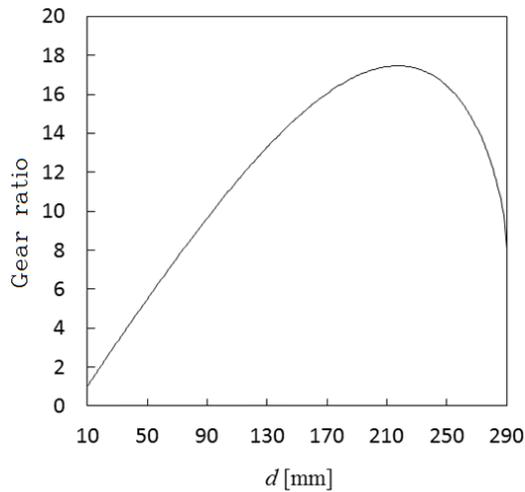


Figure 4. Link d length vs. the gear ratio
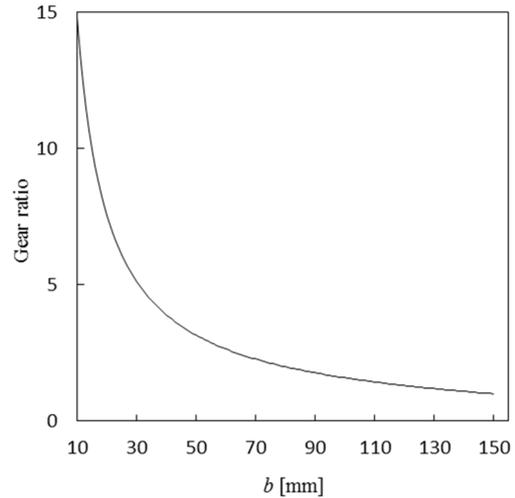


Figure 5. Link b length vs. the gear ratio

Next, we explain a method how to transmit conduction force and torque using a simultaneous expansion and contraction with both link b and link d. Setting as $a = c = 150$, link b can be expanded and contracted in the range of $1 < b < 24$. The link b expands and contracts in the range of $1 < b < 24$, and the link d expands and contracts in the range of $(b <) 25 < d < 125 (< a = c = 150)$. Figure 6 shows the relationship between the length of link b and the gear ratio when gear ratio is changed under the above conditions. The length of link d is controlled so that the gear ratio becomes larger with the length of link b. Comparing Figure 5 and Figure 6, the length of link b and the gear ratio are in inverse and direct proportional relationships, respectively.

## 5. L-CVT Equipped with Telescopic Lever and Crank Mechanisms

Based on the relationship between the links' expansion technique and the gear ratio described in the previous section, we have developed the L-CVT equipped with expansion and contraction mechanism of the link d. Instead of expanding and contracting the link b and the link c, the only link d can expand and contract by the motor (linear actuator (XA-42L-100EW, online, 2019)).
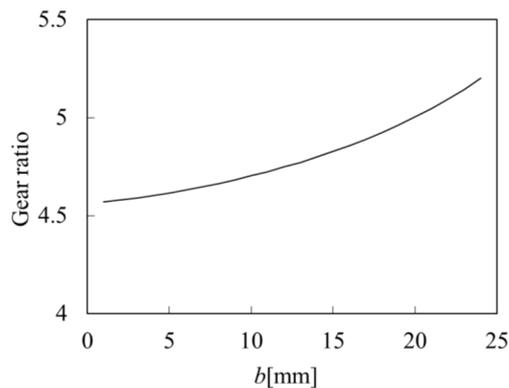


Figure 6. Link b length vs. the gear ratio

In contrast to the original L-CVT model with two degrees of freedom system in which two links (b and c) expand and contract, the new type of L-CVT has a single degree of freedom system as a simple reduced model in which the only a link d expands and contracts. Therefore, it is impossible for the new developed L-CVT to realize a precise control of the angular velocity of the link d in the output side. However, as described above, the time response curve

of the output angular velocity of the lever-crank can be smoothed with increasing the number of units consisting of lever-crank mechanism mounted on the L-CVT.

The reduction of the degree of freedom in the series of L-CVT model reduces the energy loss associated with the expansion and contraction control for the link, then it can be expected to improve the transmission efficiency in the L-CVT (Lemanski, 2006). As an advantage in the development of the L-CVT's model redution, since the link d does not rotate completely, a rotary electrode (slip ring) is not needed as a wiring means for the motor drive system necessary for controlling expansion and contraction of the link.

Considering the interference of each link, the length of link b is set to b = 10. The input shaft consists of four cranks connected as each phase of crank (link b) is delayed by 90 [°] (= $\pi$ / 2 [rad]). The length of link d can be changed in the range satisfying $b = 10 < d < a = c = 150$, and it is actually expanded and contracted in the range of 25 < d <125 ([mm])). In this situation, the gear ratio is in the range from 2.74 to 12.9, and then the gear ratio width becomes 4.71 (= 12.9 / 2.74). A one-way clutch which is an irreversible mechanism is attached at the fulcrum point of link d in order to transmit the driving force in the clockwise or counter-clockwise direction (see Figure 1).

The L-CVT designed using 3D CAD software (SOLIDWORKS) is shown in Figure 7. Four units consisting of lever-crank are installed in parallel in the L-CVT device. The command about the length of link d for arbitrary gear ratios is transmitted to the motor driver in real time by a C programming on the PC. Based on this command, prismatic motion of the electric linear actuator (SUS Co., Ltd., XA-42L-100EW (XA-42L-100EW, online, 2019)) can be realized, and expansion and contraction of the link d can also be controlled. The repeat positioning accuracy of the actuator to control expansion and contraction of the link d is ± 0.02 [mm]. Accordingly, the link d can be expanded and contracted at a maximum speed of 200 mm/s. In order to measure the number of rotations at the input and output axes, a rotary encoder (Line Seiki Co., Ltd., resolution: 2,500 [ppr]) and a counter board (Interface Co., Ltd) which connected to a backplane board on the rear side of PC are used. The encoder which can quadruple the number of pulses per revolution makes the rotation speed of the input / output axis measure with 10,000 [ppr] resolution. Figure 8 shows a photograph of the L-CVT when the length of link is $d = 25$ [mm]. Table 1 shows the specifications of prototype model of the L-CVT. Figure 9 shows the configuration of the control system for the L-CVT prototype model.

Table 1. Specifications of L-CVT

| Gear ratio range | 2.74~12.9 |
| --- | --- |
| Permission torque [Nm] | 1.0 |
| Size [mm] | 200 × 400 × 375 |
| Material | SS400 |
| Mass [kg] | 35.0 |



Figure 7. Photo of L-CVT



Figure 8. Photo of L-CVT (front view), ($d = 25$)

In the experiment where the gear ratio was changed by expansion and contraction of the link d, we changed the length of link d in the range of $25 < d < 125$ [mm]. The input / output rotational speed was measured in the situation when the length $d$ of the link d was changed at a rate of change of 1.0 mm / s. The relationship between the speed ratio calculated based on interaction of the input / output rotational speed, and the length of link d is shown in Figure 10. It can be seen that the transmission ratio and the length of link d are in a proportional relationship.
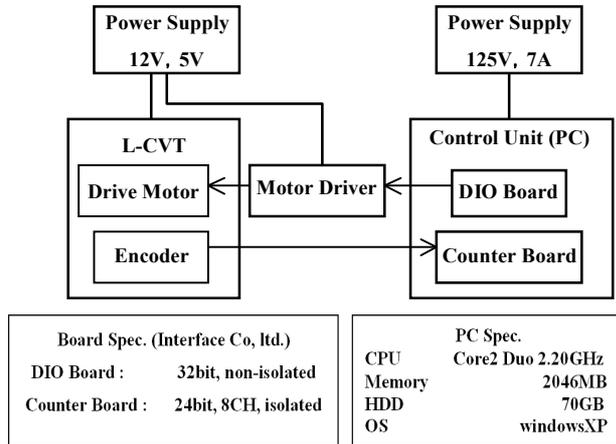


Figure 9. The configuration of L-CVT
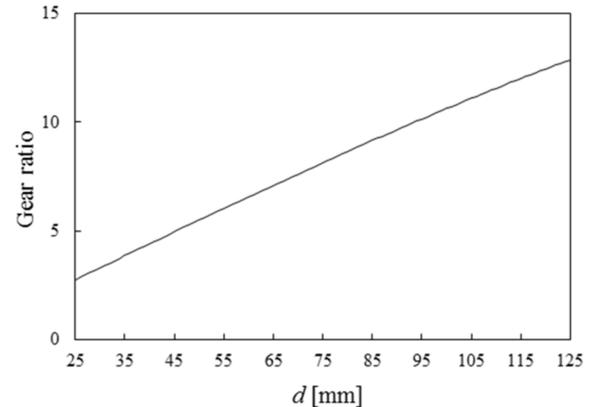
Figure 10. Link d length vs. the gear ratio

## 6. Conclusions

We have proposed a continuously variable transmission (L-CVT) using a four-bar linkage mechanism and an irreversible mechanism, which does not positively depend upon frictional conduction due to frictional force between conduction components in the transmission mechanism. In order to realize smoothing the temporal variation of the output rotational speed in the L-CVT, we have designed the conduction mechanism which is composed of four lever-crank units arranged in parallel using extension link mechanism with one degree of freedom instead of another mechanism such as a planetary gear. In the developed L-CVT, four lever-crank units were connected in parallel mechanically with the cranks which were the input shaft, and the output shaft was also connected mechanically via the irreversible mechanism installed at the fulcrum of the lever. In the experiment, it was confirmed that the gear ratio was controlled continuously in real time by realizing high-precision control for the expansion and contraction of the link using an electric linear actuator. As a merit of the developed L-CVT, the mechanism equipped with a link extension mechanism which has only one degree of freedom reduces power consumption, compared to other conventional L-CVTs.

## Acknowledgments

## References

Brandsma, A., van Lith, J., & Hendriks, E. (1999). Push belt CVT developments for high power applications. *Proc. of CVT99, Eindhoven University of Technology*, 142-147.

Gibson, C. G., & Newstead, P. E. (1986). On the geometry of the planar 4-bar mechanism. *Acta Applicandae Mathematica*, *7*(2), 113-135.

Keff, L. N. (1974). *Vraagbaak voor uw DAF, Kluwer Technische boeken B.V.* Deventer.

Kumada,T. & Yukawa, T. (2010). Development of a continuously variable transmission using lever-crank mechanism, one-way clutch, and cam. *The Japan Society of Mechanical Engineers (OPTIS 2010), 103*, 95-98. https://doi.org/10.1299/jsmeoptis.2010.9.App1

Lemanski, A. J. (2006). *Variable Speed Power Transmission System, U.S. Patent No. 7,147,583*. Washington, DC: U.S. Patent and Trademark Office.

Liebrand, N. (1996). Future potential for CVT technology. *Proc. CVT'96*, *33*.

Pohl, B., Simister, M., Smithson, R., & Miller, D. (2004). *Configuration analysis of a spherical traction drive CVT/IVT* (No. 2004-40-0009). SAE Technical Paper.

Saribay, Z. B. & Bill, R. C. (2013). Design analysis of Pericyclic Mechanical Transmission system. *Mechanism and Machine Theory, 61*, 102-122.

Shabbir, W., & Evangelou, S. A. (2014). Efficiency analysis of a continuously variable transmission with linear control for a series hybrid electric vehicle. *IFAC Proceedings Volumes*, *47*(3), 6264-6269.

XA-42L-100EW. (2019). Retrieved from http://fa.sus.co.jp/service/catalog/08P822_883.pdf

Yukawa, T. (2011). Continuously variable transmission using quadric crank chains and ratchets. In *Proc. of Int. Conf. on Renewable Energies and Power Quality (ICREPQ'11)* (Vol. 264).

Yukawa, T., & Kumada, T. (2010, July). Continuously variable transmission using quadric crank chains. In *2010 8th IEEE International Conference on Industrial Informatics* (pp. 1043-1048). IEEE.

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