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Analysis in Large Deformation of a Rigid Plastic Prestressed Beam in Ultra-High Performance Fiber-Reinforced Concrete

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Abstract

One of the major concerns in designing of prestressed beams in ultra-high performance fiber- reinforced concrete (UHPFRC**)** is improvement of their ductility fracture due to the nature of the materials used in their manufacture. This can induce plastic behaviours which it is necessary to take into account by designing of such structures, especially when they are of large spans. In the present work, it is proposed an analytical model in large deformation of a rigid plastic prestressed UHPFRC beam embedded at one end and having at other end rolled support. She is approached by a local uniform load and external moments to supports. The proposed non-linear model can find exact analytical solutions for the determination of the local arrows and the associated charge by the technique of Lagrange multiplier which allows finding the stationary points of differentiable function of one or several variables under constraints. The results of this work can be useful in designing and calculation of long span prestressed structures with plastic rigid behavior.

Keywords: lagrange multiplier, ductility fracture, rigid plastic

1. Introduction

Ultra-High Performance Fiber Reinforced Concrete (UHPFRC) is a new material which the mechanical characteristics are far superior to those of traditional concrete. Its resistance to compression is 6 times higher than concrete while resistance tensile strength is approximately 4 times higher than traditional concrete. The first application of the UHPFRC to the prestressed beams was made at the end of the 1990s on elements of conventional form with sizes considerably smaller than reinforced concrete elements, in order to replace the corroded steel beams in the aggressive environment of Cattenom and Civaux nuclear cooling towers in France (Resplendino, 2004; Acker and Behloul, 2004). A few bridges have been built so far, mainly in France (Thibaux and Tanner, 2002; Resplendino and Bouteille, 2006), United States (Park et al., 2003; Graybeal and Hartmann, 2005; Naaman and Chandrangsu, 2004), Canada and Australia (Cavill and Chirgwin, 2003).and Italy (Meda and Rosati, 2003). Number of studies have been carried out to characterize the behaviour of this material under preload (AFGC, 2002; Curbach et al., 2008; Pansuk et al., 2008) for prestressed beam sizing. As known the preload is used to cross large spans. However due to the nature of the materials involved in their manufacture the UHPFRC can induce plastic behaviours which is obvious for taking into account for long span prestressed beam designing. Data concerning the problems in large deformation of the prestressed high performance concretes are rare in the technical literature. Some models of deformability of the UHPFRC prestressed by fiber reinforced polymers (FRP) have been proposed by (Abdelrahman et al., 1997; Naaman et al., 1995; Dohan et al., 1996; Mufti et al., 1996). However one of the main concerns in the designing of UHPFRC beams prestressed by fiber reinforcements is the lack of ductility due to their linear elastic behaviour up to failure. Ductility index cannot be used effectively for elements only using FRP fibers because FRP reinforcements are not plastic properties. Ductility is required to support large inelastic deformations before fracture.

Despite the interesting features that have the UHPFRC, the application of this material for new structures remains limited. This is because considerable cost of the material. The structural solutions adapted to its specific characteristics are still partially to develop. Classic reinforced concrete designing methods don't apply necessarily.

The aim of this work is to propose a nonlinear analytical model in large deformation for prestressed rigid plastic beam in ultra-high performance fiber reinforced concrete subjected to a local distributed load with external moments to supports.

2. Materials and Method

The purpose of the present study is a prestressed beam in UHPFRC, rectangular section, embedded at one end and having fixed rolled support at the other end. It is subject to a local charge evenly distributed on different sections with bending external moments to supports (Figure 1). The beam undergoes a preload by a longitudinal force n_1.

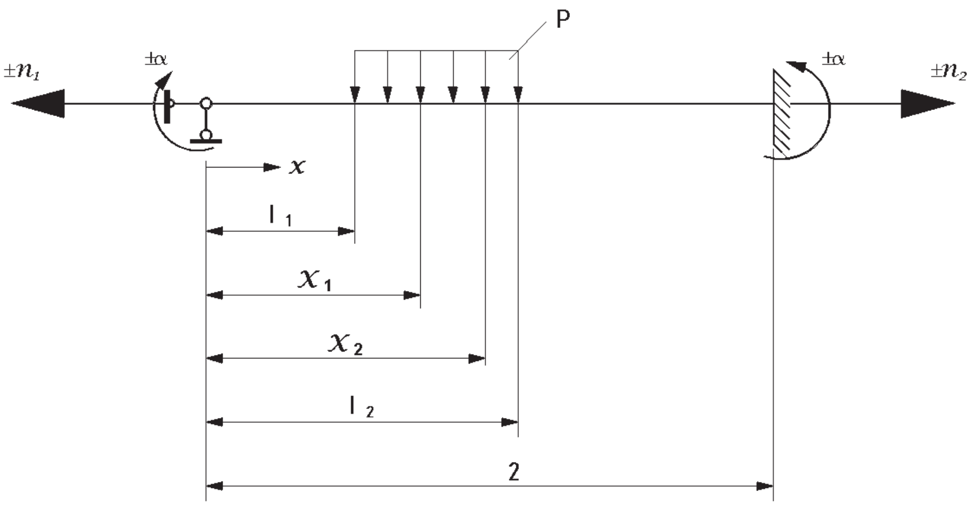

Figure 1. Prestressed beam

Because the UHPFRC has a ductile behaviour plasticity criteria are from (Chen et al., 2007; Jirásek and Bazant, 2002).

The rigid plastic model assumes that the plastic strains are more important than the elastic strains that could be overlooked. However given that strains are functions of the displacements the need for taking account of the large displacements in plastic rigid systems is obvious. In these conditions the beam equilibrium equations in large deformations will have following expressions:

$$\frac{d^2m}{dx^2} + (n \pm n_1)\frac{d^2w}{dx^2} + p = 0, \quad \frac{dn}{dx} = 0, \tag{1}$$

Here

$$x = \frac{\bar{x}}{\bar{l}}\ ; \ w = \frac{\overline{2w}}{h}\ ; \ p = \frac{\bar{p}\bar{l}^2}{\sigma_s bh^2}\ ; \ m = \frac{M}{\sigma_s\, bh^2}\ ; \ n = \frac{N}{2\sigma_s bh}\ ; \ n_1 = \frac{N_1}{2\sigma_s bh}\ ;$$

In this work the following notations are adopted:

N and m - respectively the inner normal force and the bending moment

$\overline{p}$ -uniformly distributed transverse load,

$\overline{w}$ -arrow,

$\bar{x}$ -longitudinal coordinated (the origin of the coordinates is left support),

2 h - height of the cross section,

$\overline{2l}$ -span of the beam,

σ_s - flow limit of the material

The line on the symbols indicates values with units.

When N_1 is given, N follows from the action of $\overline{p}$ corresponding to the arrows.

The load is applied on the segment $l_1 \le x \le l_2$, l_1 and l_2 being the segment boundaries.

The deformation can be divided into two stages: in 'small' and 'big' arrows. For the 'small' arrows (equal to zero) there occurs plastic sections on span $x=x_2$ as well as the right support.

On section x=x_2 the moment is:

$$m = 1 - n_1^2, \tag{2}$$

At the built-in, we have:

m = $|1 - n_1^2| \mp \propto$, $\propto$ is the value of the external moments to support (the signs "-" and "+" correspond to the positive and negative values).

The speed variation of curvature $\dot{k}$ is zero:

$$\dot{k} = -\frac{d^2w}{dx^2} = 0, \tag{3}$$

Where $w = \left\{\frac{w_0}{x_2}\right\}^{\bullet} \cdot x$, with $x \geq x_2$,

w_0 - Arrow value for $x = x_2$.

3. Results and Discussion

From the equilibrium equations (1) we have expression of m for each segment:

$$m = \left(-pl_1 + \frac{pl_1^2}{4} + pl_2 - \frac{pl_2^2}{4} - \frac{1-n_1^2}{2}\right) \cdot x \mp \propto \ , (0 \leq x \leq l_1) \tag{4}$$

$$m = -\frac{px_2^2}{2} + \left(-\frac{pl_1^2}{4} + pl_2 - \frac{1-n_1^2}{2}\right) \cdot x \mp \propto - \frac{pl_1^2}{2}, \ (l_1 \leq x \leq l_2) \tag{5}$$

$$m = \left(-\frac{1-n_1^2}{2} - \frac{pl_2^2}{4} + \frac{pl_1^2}{4}\right) \cdot x + \frac{pl_2^2}{2} - \frac{pl_1^2}{2} \mp \propto , (l_2 \leq x \leq 2). \tag{6}$$

Where as $\frac{dm}{dx}$ = 0 for $x = x_2$, we get:

$$x_2 = l_2 - \frac{l_2^2}{4} + \frac{l_1^2}{4} - \frac{1-n_1^2}{2p} \tag{7}$$

Considering that $m = 1 - n_1^2$ when $x = x_2$ we get:

$$x_2 = l_2 - \frac{l_2^2}{4} + \frac{l_1^2}{4} - \frac{1-n_1^2}{2p} \tag{8}$$

Considering that $m = 1 - n_1^2$ when $x = x_2$, we get :

$$p = \frac{\left[(1-n_1^2)\left(l_2 - \frac{l_2^2}{4} + \frac{l_1^2}{4} + 2\right) \mp 2\propto\right] + \sqrt{p}}{2\left[\left(l_2 - \frac{l_2^2}{4} + \frac{l_1^2}{4}\right) - l_2^2\right]} \ . \tag{9}$$

For p values greater than expression (9) in vicinity of x $= x_2$ it forms a plastic zone $x_1 \leq x \leq x_3$ followed by a second stage of beam deformation but $w \neq 0$, $n = const. \neq 0$. Taking into account the preload in traction-compression, the longitudinal force is equal to $n \mp n_1$, with $n \mp n_1 \leq l$, $|n_1| \leq l$.

In the plastic zone $x_1 \leq x \leq x_3$, the condition of plasticity is:

$$m = 1 - (n \mp n_1)^2, \tag{10}$$

as m = const., $\frac{dm}{dx}$ = 0.

From (1), it follows the value of arrows in this area:

$$w = w_0 - \frac{p}{2(n \mp n_1)}(x - x_2)^2, \tag{11}$$

and the arrow speeds will be:

$$\dot{w} = \dot{w}_0 - \left(\frac{p}{2(n \mp n_1)}\right)^{\bullet} \cdot (x - x_2)^2 + \frac{p}{(n \mp n_1)} \cdot (x - x_2) \cdot \dot{x_2} \ . \tag{12}$$

The zones $0 \leq x \leq x_1$ and $x_3 \leq x \leq 2$ are rigid areas. In these areas arrows have as expressions:

$$w = w_1 \cdot \frac{x}{x_1} \ , \ \dot{w} = w_3 \cdot \frac{2-x}{2-x_3} \tag{13}$$

where $\dot{w}_1$ and $\dot{w}_3$ are arrows for $x = x_1, x = x_2$ *and* $x = x_3$.

Arrow speeds in these areas are equal to:

$$\dot{w}=\left\{\frac{w_1}{x_1}\right\}^{\bullet} \text{ for } 0 \le x \le x_1 \ , \ \dot{w}=\left\{\frac{w_3}{2-x_3}\right\}^{\bullet} \cdot (2-x) \text{ for } x_3 \le x \le 2 \ . \tag{14}$$

When $x = x_1$ and $x = x_3$ it occur very small failures $[\dot{w}_x], [w_{xx}], [\dot{w}]$ which must satisfy two conditions for $x = x_1$:

$$[\dot{w}_x] + \dot{x}_1 \cdot [w_{xx}] = 0, \ [\dot{w}] + \dot{x}_1 \cdot [\dot{w}_x] = 0, \tag{15}$$

It is the same for $x = x_3$, by swapping index "1" to "3".

According to (11), (13), (4) failures $[\dot{w}_x], [w_{xx}]$will have the following expressions for $x = x_1$:

$$[\dot{w}_x] = -\left\{\frac{p}{n\mp n_1}\right\}^{\bullet} \cdot (x_1 - x_2) + \frac{p}{n\mp n_1} \cdot \dot{x}_2 - \left(\frac{w_1}{x_1}\right)^{\bullet}, \quad [w_{xx}] = -\frac{\mathrm{p}}{\mathrm{n}\mp\mathrm{n}_1} \ , \tag{16}$$

whereas in general case that x_2 is mobile.

Considering the first condition of (15) and taking into account (16), we obtain following equation:

$$-\left\{\frac{p}{n\mp n_1}\right\}^{\bullet} \cdot (x_1 - x_2) + \frac{p}{n\mp n_1} \cdot \dot{x}_2 - \left(\frac{w_1}{x_1}\right)^{\bullet} - \frac{p}{n\mp n_1} \cdot \dot{x}_1 = 0, \tag{17}$$

$$\text{or } \left\{\frac{p}{n\mp n_1} \cdot (x_2 - x_1)\right\} = \left\{\frac{w_1}{x_1}\right\}. \tag{18}$$

Taking into account the initial condition w_1=0 for x_1=x_2, integration of this expression gives:

$$w_1 = \frac{px_1}{n\mp n_1} \cdot (x_2 - x_1). \tag{19}$$

Similarly for $x = x_3$ can we obtain w_3 :

$$w_3 = \frac{p}{n\mp n_1} \cdot (x_3 - x_2) \cdot (2 - x_3). \tag{20}$$

Using expressions (19) and (20), from (11) we find the following two equivalent expressions:

$$w_0 = \frac{p}{2(n\mp n_1)}(x_2^2 - x_1^2), \quad w_0 = \frac{p}{2(n\mp n_1)}(4x_3 - 4x_2 - x_3^2 + x_2^2). \tag{21}$$

In reality, the stiffening effect of tense UHPFRC contributes to reduce the beam arrow. This stiffening effect is further enhanced with the use of steel fibers which restrict the cracks partially taking the tensile stresses.

The bending moments in areas $0 \le x \le l_1$, $l_1 \le x \le x_1$, $x_3 \le x \le l_2$, $l_2 \le x \le 2$, according to (1) have the following expressions:

:

$$m = (px_1 - pl_1) \cdot x \mp \propto, 0 \le x \le l_1 \tag{22}$$

$$m = -\frac{px_2^2}{2} + p \cdot x \cdot x_1 - \frac{pl_1^2}{2} \mp \propto, l_1 \le x \le x_1 \tag{23}$$

$$m = 1 - (n \mp n_1)^2 - \frac{p}{2}(x - x_3)^2 \ , \ x_3 \le x \le l_2 \tag{24}$$

$$m = (px_3 - pl_2)x + 1 - (n \mp n_1)^2 + pl_2^2 - px_3 l_2 \ , \ l_2 \le x \le 2. \tag{25}$$

When $m = +1 - (n \mp n_1)^2$ for $x = x_1$ and $m = -[1 - (n \mp n_1)^2] \mp \propto$ for $x = 2$ we get the following two equivalent expressions of p:

$$p = \frac{2[1-(n\mp n_1)^2 \mp \propto]}{x_1^2 - l_1^2} \ , \ p = \frac{4[1-(n\mp n_1)^2 \mp 2\propto]}{4l_2 - 4x_3 + x_3^2 - l_2^2} \ . \tag{26}$$

By comparing these two expressions we find that:

$$x_2 = -\frac{x_1^2}{4} + \frac{l_1^2}{2} + l_2 - \frac{l_2^2}{4}\ . \quad (27)$$

It is thus seen that x_2 dependent x_1.

The value of n is determined from the condition of p maximum according to (26) taking into account the first condition of (21). We get a problem to maximum under condition of p that can be reducing to a problem at absolute maximum using the Lagrange multiplier.

The principle of maximum dissipation allows us to obtain the normality laws in which a single Lagrange multiplier intervenes. To determine this Lagrange multiplier the complementarity conditions must be considered (Besson et al. 2001).

Using the first expressions of (26) and (21), the unconditional function will have the following form:

$$\emptyset = \frac{2[1-(n\mp n_1)^2 \mp \propto]}{x_1^2-l_1^2} + \lambda\frac{[1-(n\mp n)^2(x_2^2-x_1^2)]}{(x_1^2-l_1^2)(n\mp n_1)} - \lambda w_0 \quad (28)$$

λ - The Lagrange multiplier.

By deriving (28) compared to x_1 and n, we get:

$$\frac{\partial\Phi}{\partial x_1} = 2 + \frac{\lambda}{n+n_1}(x_2^2 - l_1^2) = 0\ , \quad (29)$$

$$\frac{\partial\Phi}{\partial n} = 2[-2(n\mp n_1)] + \lambda(x_2^2 - x_1^2)\frac{(x_2^2-x_1^2)[1+(n+n_1)^2\mp\propto]}{(x_2^2-l_1^2)(n\mp n_1)} = 0\ , \quad (30)$$

We obtain:

$$\lambda = -\frac{2(n\mp n_1)}{x_2^2-l_1^2}\ , \quad (31)$$

$$(n\mp n_1)^2 = \frac{(1\mp\propto)(x_2^2-x_1^2)}{x_2^2+x_1^2-2l_1^2}. \quad (32)$$

Thus the p value will be:

$$p = \frac{4(1-\propto)}{x_2^2+x_1^2-2l_1^2}. \quad (33)$$

We thus obtain the analytical solution to the problem: it can determine the p and w values knowing n and $\propto$.

4. Conclusion

In this work, it is proposed a method for analytical solving a problem in large deformation for a rigid plastic prestressed UHPFRC beam in which we are interested specifically the arrow and associated loading. The following results were obtained:

1) An exact solution to non-linear differential equation for a problem in large deformations of rigid plastic prestressed beam
2) Could find exact analytical solutions for the determination of arrows in large deformation for this model of prestressed UHPFRC beam.
3) Using the Lagrange multiplier technique it is determined expression of the service loading for this material.
4) From the equilibrium equations is established expression of the moments by zone.
5) Could identify the rupture areas and expressions of ultimate deformations.

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