

Nonlinear Damage Mechanics Approach in Investigating Cracks in Reinforced Concrete Flexural Members

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Received: April 20, 2016

Accepted: May 27, 2016

Online Published: July 4, 2016

doi:10.5539/mas.v10n10p62

URL: <http://dx.doi.org/10.5539/mas.v10n10p62>

Abstract

This paper was investigating cracks in reinforced concrete members subjected to pure bending using nonlinear damage mechanics approach. Concrete, steel and total damage variables were introduced and correlations were derived. Comparisons with different experimental and theoretical methods were executed, comparison between the proposed approach and different codes of practices were also executed. A parametric study was carried out for total damage variable considering both concrete cover and reinforcement ratio at different concrete damage variable.

Keywords: damage mechanics, damage variable, reinforced concrete, flexural, cracks

1. Introduction

Mechanical damage is the gradual process of mechanical deterioration that ultimately results in component failure. Fracture is an instance of substantial material damage typically associated with exposure to more extreme loading conditions. Damage mechanics is the study of material damage based on the introduction of damage variables and their evolution under the applied loading conditions. In the past few decades, the damage assessment in construction and industrial fields raised.

Bengt B. Broms (1964) developed a simple method for calculation of crack width and crack spacing in reinforced concrete members. The crack widths determined by this method were compared with test data obtained from flexural and tensile reinforced concrete members. Bengt B. Broms (1965) investigated theoretically and experimentally the factors affecting the formation of tensile cracks in reinforced concrete members, developed hypotheses for the prediction of crack width and crack spacing for flexural members. J. L. Lndoocons (1988) reviewed some practical damage growth equations for creep, fatigue, creep-fatigue interaction, ductile damage, and brittle damage and discussed the capabilities of Continuum Damage Mechanics to improve both the crack initiation and crack propagation predictive tools. Robins et al (2001) investigated the fracture of steel fiber reinforced sprayed concrete under flexural loading, with the aim of developing a stress-block model to predict flexural behavior in the form of a load-deflection response. Milan Jirásek (2004) explained the basic concepts of continuum damage mechanics and its nonlocal formulation with application to concrete. Using one- and two-dimensional examples, it was shown that a stress-strain law with softening postulated within the standard continuum theory. Jerzy Bobinski and Jacek Tejchman (2005) modelled concrete using a simple isotropic damage continuum model and presented finite element results on the behavior of concrete under plane strain conditions. Ziad N. Taqieddin (2008) modeled the behavior of RC members using the nonlinear finite element analysis (NFEA). A thermodynamically consistent constitutive model for concrete that incorporates concrete-plasticity and fracture-energy-based continuum damage mechanics was presented. George Z. Voyiadjis and Peter I. Kattan (2008), (2012) examined various definitions of the damage variables and compared it. They gave special emphasis to a new damage variable that is defined in terms of the elastic stiffness of the material. Said M. Allam et al (2012) developed some building code equations, which used for the calculation of the crack width in reinforced concrete flexural members. Y. S. Upadhyaya and B. K. Sridhara (2012) predicted Continuum Damage Mechanics (CDM) approach to model the crack initiation life with a damage value and damage beyond the crack initiation phase by Fracture Mechanics in terms of crack size. Kent A. Harries et al (2012) presented a study of flexural crack widths of beams reinforced with high-strength ASTM A1035 reinforcing steel. Ashkan Saboori et al. (2015) presented an anisotropic damage mechanics model to

describe the behavior and failure of concrete under biaxial fatigue loading, a strength softening function was proposed in order to address the reduction in the strength of concrete due to fatigue.

2. Method

2.1 Nonlinear Damage Analysis of Reinforced Concrete Flexural Member

Consider a beam of span L and cross sectional area $A = b \times h$ was subjected to pure bending moment M as shown in Figure (1).

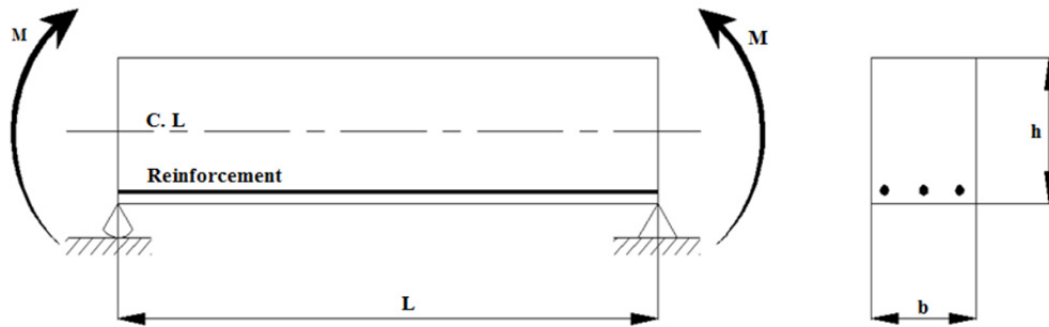


Figure (1) Geometry and Cross Section of Considered Beam

When the flexural member loaded with its bending capacity (ultimate moment), cracks will be formed and the damaged shape will be as shown in Figure (2).

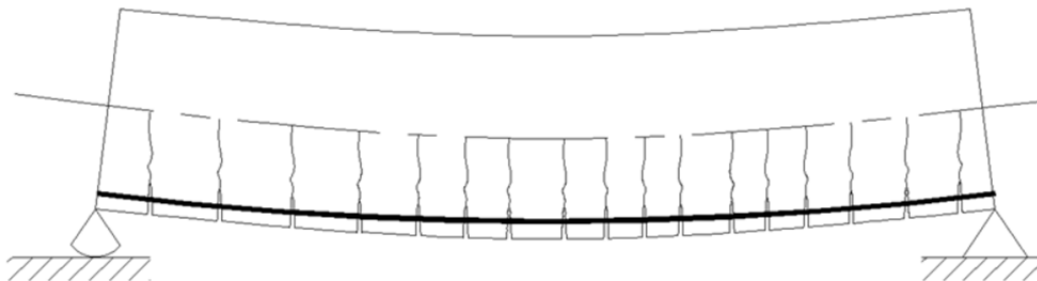


Figure 2. Damaged Shape

In order to use principle of damage mechanics, primary and secondary voids and cracks are removed to obtain the effective undamaged shape as shown in figure (3), where the cross sectional area will be $\bar{A} = b \times \bar{h}$. One should note that, for simplicity removing cracks and voids was modifying the depth only without the width.

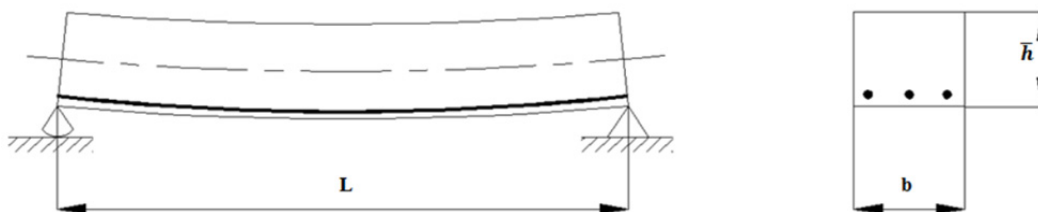


Figure 3. Effective Undamaged Shape

Bending stresses in the undamaged beam (before cracking) can be determined easily from:

$$\sigma = \frac{M}{S} \quad (1)$$

Where:

σ is the bending stress.

M is the applied bending moment.

S is the first moment of area.

Also the bending stress in the effective undamaged beam (after removing cracks and voids) can be determined from:

$$\bar{\sigma} = \frac{M}{\bar{S}} \quad (2)$$

Where:

$\bar{\sigma}$ is the bending stress in the effective undamaged beam.

\bar{S} is the first moment of area for a cross section in the effective undamaged beam.

By using the definition of damage variable (ℓ), which, depending on the reduction of the elastic modulus:

$$\ell = \frac{\bar{E} - E}{E} \quad (3)$$

Where:

$E = \frac{\sigma}{\epsilon}$ is the elastic modulus.

$\bar{E} = \frac{\bar{\sigma}}{\bar{\epsilon}}$ is the effective elastic modulus.

ϵ is the strain in damage state.

$\bar{\epsilon}$ is the effective strain in the fictitious state (state of removing cracks and voids).

From equations (1), (2) and (3), one can deduce that:

$$\ell = \frac{S\epsilon}{\bar{S}\bar{\epsilon}} - 1 \quad (4)$$

By applying the hypothesis of equal strain energy in damaged case and fictitious case, one obtains:

$$\frac{1}{2}\sigma\epsilon = \frac{1}{2}\bar{\sigma}\bar{\epsilon} \quad (5)$$

From equations (1), (2) and (5), and substitution in equation (4):

$$\ell = \frac{S^2}{\bar{S}^2} - 1 \quad (6)$$

Equation (6) is considered as a novel expression for damage variable, and can be symbolized for reinforced concrete flexural member as:

$$\ell_t = \frac{S_t^2}{\bar{S}_t^2} - 1 \quad (7)$$

Where:

ℓ_t is the reinforced concrete damage variable.

Also, in the same way, one can introduce two damage variables:

$$\ell_c = \frac{S_c^2}{\bar{S}_c^2} - 1 \quad (8)$$

$$\ell_s = \frac{S_s^2}{\bar{S}_s^2} - 1$$

Where:

ℓ_c is the concrete damage variable.

S_c is the first moment of area for concrete section.

\bar{S}_c is the effective first moment of area for concrete section in fictitious state.

ℓ_s is the reinforcement steel damage variable.

S_s is the first moment of area for reinforcement steel.

\bar{S}_s is the effective first moment of area for reinforcement steel in fictitious state.

The reinforced concrete section is treated as a composite section, so one can find that:

1 - In damaged case:

$$EI = E_c I_c + E_s I_s \quad (9)$$

2 - In fictitious case:

$$\bar{E}\bar{I} = \bar{E}_c \bar{I}_c + \bar{E}_s \bar{I}_s \quad (10)$$

Where:

E_c is the elastic modulus of concrete in the damaged case.

E_s is the elastic modulus of steel in the damaged case.

\bar{E}_c is the elastic modulus of concrete in the fictitious case.

\bar{E}_s is the elastic modulus of steel in the fictitious case.

I_c is the second moment of area for concrete section in damaged case.

I_s is the second moment of area for reinforcement in damaged case.

\bar{I}_c is the second moment of area for concrete section in fictitious case.

\bar{I}_s is the second moment of area for reinforcement in fictitious case.

Dividing equation (10) by equation (9), one can obtain:

$$(\ell_t + 1) \frac{\bar{I}}{I} = \frac{1}{\left(\frac{I_c}{I_c}\right)\left(\frac{1}{\ell_c+1}\right) + \frac{E_s I_s}{E_c I_c}} + \frac{1}{\left(\frac{I_s}{I_s}\right)\left(\frac{1}{\ell_s+1}\right) + \frac{E_c I_c}{E_s I_s}} \quad (11)$$

Assume that damage occurs only in concrete, then $\ell_s = 0$ (i. e. $\bar{E}_s = E_s$), and where the modular ratios in

damaged case and fictitious case are $n = \frac{E_s}{E_c}$, $\bar{n} = \frac{\bar{E}_s}{\bar{E}_c} = \frac{E_s}{\bar{E}_c}$, then:

$$(\ell_t + 1) \frac{\bar{I}}{I} = \frac{1}{\left(\frac{I_c}{I_c}\right)\left(\frac{1}{\ell_c+1}\right) + \frac{I_s}{I_c}} + \frac{1}{\left(\frac{I_s}{I_s}\right) + \frac{I_c}{n I_s}} \quad (12)$$

After some manipulations:

$$\sqrt{(\ell_t + 1)} = \frac{h}{\bar{h}} \left(\frac{1}{\left(\frac{h}{\bar{h}}\right)\left(\frac{1}{\sqrt{\ell_c+1}}\right) + \frac{A_s \times (d-\frac{h}{2})^2}{A_c \times \frac{\bar{h}^2}{12}}} + \frac{1}{\left(\frac{d-\frac{h}{2}}{\bar{d}-\frac{\bar{h}}{2}}\right) + \frac{1}{n} \frac{A_c \times \frac{h^2}{12}}{A_s \times (d-\frac{h}{2})^2}} \right) \quad (13)$$

$$\text{Put } \rho = \frac{A_s}{bh} \text{ and } \bar{\rho} = \frac{\bar{A}_s}{\bar{b}\bar{h}} = \frac{A_s}{\bar{b}\bar{h}} \quad (14)$$

ρ and $\bar{\rho}$ are reinforcement and effective reinforcement ratios.

Where, $A_c = bh - A_s$

Then:

$$\frac{A_c}{A_s} = \frac{1-\rho}{\rho} \quad (15)$$

In the same way:

$$\frac{\bar{A}_c}{A_s} = \frac{1-\bar{\rho}}{\bar{\rho}} \quad (16)$$

By substitution from (14), (15) and (16), one can obtain:

$$\sqrt{(\ell_t + 1)} = \frac{h}{\bar{h}} \left(\frac{1}{\left(\frac{h}{\bar{h}}\right) \left(\frac{1}{\sqrt{\ell_c+1}}\right) + \bar{n} \frac{\bar{\rho}}{1-\bar{\rho}} \frac{\left(d-\frac{h}{2}\right)^2}{\frac{h^2}{12}}} + \frac{1}{\left(\frac{d-\frac{h}{2}}{\bar{h}}\right) + \frac{1-\rho}{n \rho} \frac{\frac{h^2}{12}}{\left(d-\frac{h}{2}\right)^2}} \right) \quad (17)$$

The above relation shows that the total damage variable of reinforced concrete depends on the damage variable of concrete, reinforcement ratio and concrete cover.

2.2 Problem Solution

- 1 - Specify the compressive strength of concrete (f_c), elastic modulus of concrete in damaged case ($E_c = 4700\sqrt{f_c}$ "MPa") and elastic modulus of steel (E_s).
- 2 - Determine the dimensions of the proposed flexural member (b, h, L).
- 3 - Use practical values of concrete damage variable (ℓ_c).
- 4 - Find the modular ratio in a damaged case (n) and modular ratio in the fictitious case (\bar{n}), Also find the depth of the cross section of fictitious case (\bar{h}) in case of concrete damage only.
- 5 - Get the reinforcement ratio in a damaged case (ρ) and fictitious case ($\bar{\rho}$).
- 6 - Calculate the total damage variable for reinforced concrete (ℓ_t).
- 7 - Find the depth of the cross section of fictitious case (\bar{h}) in case of reinforced concrete damage which should be less than the depth of the cross section of fictitious case for case of concrete damage only.
- 8 - Calculate the volume of voids and cracks by subtracting the volume of effective flexural member from the volume of the damaged flexural member.

2.3 Flexural Members Cracks Propagation According to Codes of Practices

2.3.1 Eurocode2 1992-1 (2001)

For calculating the crack width in flexural member, one can follow:

$$W_k = S_{r,max}.(\varepsilon_{sm} - \varepsilon_{cm}) \quad (18)$$

Where:

W_k is the design crack width.

$S_{r,max}$ is the maximum crack spacing.

ε_{sm} is the main steel strain.

ε_{cm} is the main concrete strain.

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\left(f_s - k_t \left(\frac{f_{cteff}(1+2\rho_{eff})}{\rho_{eff}}\right)\right)}{E_s} \geq 0.6 \frac{f_s}{E_s} \quad (19)$$

f_s is the stress in tension reinforcement.

k_t is the factor expressing the duration of loading.

f_{cteff} is the mean value of effective tensile strength of the concrete.

$$\rho_{eff} = \frac{A_s}{A_{ceff}}$$

A_s is the area of steel.

A_{ceff} effective tension area of concrete.

$$S_{r,max} = 3.4c + 0.425k_1k_2\phi/\rho_{eff} \quad (20)$$

c is the concrete cover.

k_1 is the coefficient of bond properties.

k_2 is the coefficient of strain distribution.

\emptyset is the bar diameter.

2.3.2 ACI 318

ACI proposed equation for crack width of flexural member is:

$$W_{max} = 0.011\beta f_s \sqrt{d_c A_o} * 10^{-3} \quad (21)$$

$\beta = \frac{h-x}{d-x}$ is the ratio of the distance between neutral axis and extreme tension face.

$A_o = \frac{A_e}{n_b}$ is the area of the concrete surrounding reinforcement bar.

A_e is effective area of concrete in tension

n_b number of tension reinforcing bars.

d_c is the distance measured from the centroid of tensile steel to the extreme tensioned fiber.

2.3.3 British Standard BS 8110

According to BS 8110 the crack width can be calculated from:

$$W = \frac{3a_{cr}\varepsilon_m}{1+2\left(\frac{a_{cr}-c_{min}}{h-x}\right)} \quad (22)$$

Where:

a_{cr} is the distance from the point considered to the surface of the nearest longitudinal bar.

ε_m is the average strain at the level where cracking is being considered.

c_{min} is the minimum concrete cover.

x is the depth of neutral axis and h is the member depth.

2.3.4 Oh and Kang (1987) Formula For Crack Width

Oh and Kang (1987) suggested the following equation:

$$W = \emptyset a_o (\varepsilon_s - 0.0002) R \quad (23)$$

Where:

$$a_o = 159 \left(\frac{t_b}{h-x}\right)^{4.5} + 2.83 \left(\frac{A_o}{A_{s1}}\right)^{1/3} \quad (24)$$

$$R = (h-x)/(d-x) \quad (25)$$

$A_o = \frac{A_e}{n_b}$ is the area of the concrete surrounding reinforcement bar.

$A_e = bh_1$, h_1 is the depth of equivalent area.

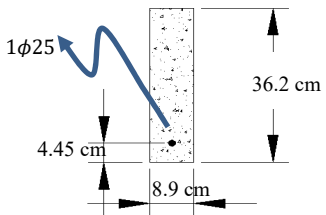
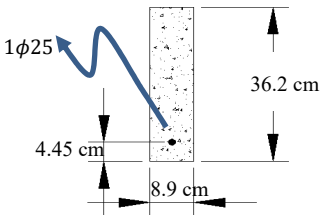
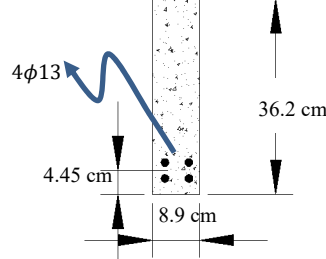
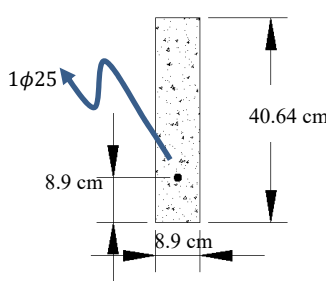
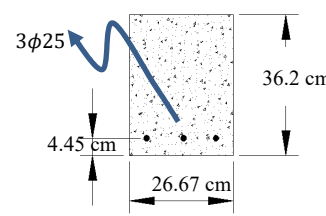
$$h_1 = \frac{(h-x)^3}{3(d-x)^2} \quad (26)$$

x is the depth of neutral axis and h is the member depth.

3. Analysis and Results

The proposed was verified depending on experimental work introduced by Bengt B. Broms (1964) and (1965), where crack width and crack spacing were measured within the pure moment region for the flexural members between the load points. Crack widths were measured to the nearest 0.001 in. by a 50-power hand microscope. The crack spacing was measured by a steel ruler to the nearest 0.1 inch. The beam specimens were tested in a standard 400,000 Ib. Baldwin Universal Testing Machine. The following Table (1) shows the detail of the cross sections of the used specimens.

Table 1. Cross Sections Details of Concerned Specimen by Bengt B. Broms

Specimen Code	Details
B-R1-1	
B-R1-2	
B-R4	
B-R5	
B-R6	

The beams were loaded either at the third or quarter points through a pair of 20 ft. reaction beams attached to the head of the testing machine. The free span length of the members was 12.0 ft. Rollers with a diameter of 2.0 in. were used at the loads and the support points. The applied load was increased in ten to fifteen increments until the maximum load was reached. The average crack width visible at the surface of flexural members can be calculated if it is assumed that the elongation (or contraction) of the concrete between two tensile cracks is negligible. If this assumption is made, then the product of the number of tensile cracks N at the level of the reinforcement within the length L times the average crack width W_{avg} (the average crack width) is equal to the total elongation of the reinforcement as expressed by the equation:

$$W_{avg} \times N = \epsilon_s \times L \tag{27}$$

Where ε_s is the average steel strain. Since the number of tensile cracks within the length L equal to L/S_{avg} , where S_{avg} is the average crack spacing, Equation (18) can be rewritten as:

$$W_{avg} = \varepsilon_s \times S_{avg} \quad (28)$$

From the observation that the average crack spacing S_{avg} is approximately equal to twice the distance t (concrete cover), the average crack width at the level of the reinforcement can be calculated as:

$$W_{avg} = \varepsilon_s \times 2t \quad (29)$$

Also, Makhlof and Malhas reported results of tests on 11 beams reinforced with high strength deformed bars. The crack width was recorded. The specimens consisted of 3 wide beams of 600 mm width, 400 mm depth and clear concrete cover = 50 mm. The experimental specimens were tested under a load level of 80–110% of service load. Group “B” aimed to assess the magnitude of crack widths in full-size beams. The following Table (2) shows the detail of the cross sections of the used specimens.

Table 2. Cross Sections Details of Concerned Specimen by Makhlof and Malhas

Specimen Code	Details
G1	<p>4φ14 + 2φ20</p> <p>60 cm</p> <p>40 cm</p> <p>5 cm</p>
G2	<p>6φ20</p> <p>60 cm</p> <p>40 cm</p> <p>5 cm</p>
G3	<p>9φ20</p> <p>60 cm</p> <p>40 cm</p> <p>5 cm</p>

Table 3 shows the obtained results of the proposed approach Considering both Bengt B. Broms and Makhlof specimens. These results can be used to calculate the crack width (W) and compare it with that obtained experimentally by Bengt B. Broms and Makhlof, as shown in Table 4 and Table 5. Also comparison with theoretical method and equations of codes of practices was made where, Bengt B. Broms's calculations were compared with those obtained from the new approach, as shown in Table 4, and comparisons with ACI 318, Eurocode 2, BS8110 and Oh and Kang method were shown in Table 6.

Table 3. Results of Proposed Approach

Specimen Code	Concrete Strength (f_c) "MPa"	Elastic Modulus of Concrete for Damaged Case (E_c) "MPa"	Damage Variable for Concrete (ℓ_c)	Elastic Modulus of Concrete for Fictitious Case (\bar{E}_c) "MPa"	Elastic Modulus of Steel (E_s) "GPa"	Modular Ratio for Damaged Case (n)	Modular Ratio for Fictitious Case (\bar{n})	Damage Variable for Reinforced Concrete (ℓ_t)	Effective Depth of Fictitious Case (\bar{h})	Volume of Voids and Cracks
		B-R1-1	31.44	26353.55	0.00425	26465.552	194.432	7.377	7.346	0.003019
B-R1-2	29.16	25380	0.00394	25479.997	194.432	7.661	7.630	0.0027657	36.1726	29.7413
B-R4	35.7	28588.98	0.005	28731.928	182.228	6.374	6.342	0.003695	36.1639	39.1339
B-R5	35.7	26640.75	0.005	26773.954	194.432	7.298	7.261	0.004001	40.5973	46.2553
B-R6	24.821	23415.72	0.0017	23455.525	194.432	8.303	8.289	0.001161	36.1820	58.4287
G1	40	29725.41	0.004375	29855.458	200	6.728	6.698	0.003935	39.9597	120.6701
G2	40	29725.41	0.004375	29855.458	200	6.728	6.698	0.003731	39.9603	118.8363
G3	40	29725.41	0.004375	29855.458	200	6.728	6.698	0.003456	39.9599	120.2194

Table 4. Comparison of Cracks Width from Proposed Approach and Brom's Work

Specimen Code	$W_{new Approach}$	$W_{Brom's Experiment}$	$W_{Brom's Calculations}$	$\frac{W_{new Approach}}{W_{Brom's Experiment}}$	$\frac{W_{Brom's Calculations}}{W_{Brom's Experiment}}$
B-R1-1	0.01586	0.01651	0.028638	0.96	1.73
B-R1-2	0.01338	0.01333	0.019177	1.00	1.43
B-R4	0.010752	0.011747	0.021717	0.92	1.84
B-R5	0.021572	0.0254	0.036271	0.85	1.42
B-R6	0.006838	0.00625	0.008564	1.09	1.37

Table 4 shows a good agreement between experimental results of Brom and the new approach results, while Brom's calculations did not give the same agreement as the experiment.

Table 5. Comparison of Cracks Width from Proposed Approach and Makloul's Work

Specimen Code	$W_{new Approach}$	$W_{Makloul's Experiment}$	$\frac{W_{new Approach}}{W_{Makloul's Experiment}}$
G1	0.0135	0.011	1.227
G2	0.0138	0.015	0.92
G3	0.01487	0.018	0.8261

Table 5 shows a good agreement between experimental results of Makloul and the new approach results.

Table 6. Comparison of Cracks Width from Some Codes of Practice and New Proposed Approach

Specimen Code	$W_{Eurocode2}$	$\frac{W_{new\ Approach}}{W_{Eurocode2}}$	$W_{BS\ 8110}$	$\frac{W_{new\ Approach}}{W_{BS\ 8110}}$	$W_{ACI\ 318}$	$\frac{W_{new\ Approach}}{W_{ACI\ 318}}$	$W_{Oh-Kang}$	$\frac{W_{new\ Approach}}{W_{Oh-Kang}}$
G1	0.018	0.75	0.009	1.5	0.02	0.675	0.015	0.9
G2	0.019	0.726	0.013	1.061	0.024	0.575	0.019	0.726316
G3	0.016	0.929	0.012	1.239	0.021	0.708	0.017	0.874706

Table 6 shows a good agreement between the new approach and codes of practice, whereas the percentage of reinforcement increases, the agreement increase, Also there is a good agreement between the new approach and Oh – Kang method.

4. Parametric Study

In this parametric study, the total damage variable of the reinforced concrete (ℓ_t) was studied with concern of many variables like concrete cover and reinforcement ratio at different concrete damage variables (ℓ_c).

Figure 4 shows the relation between concrete cover and total reinforced concrete damage variable (ℓ_t) at different concrete damage variable (ℓ_c) and states that the total reinforced concrete damage variable (ℓ_t) increases as the concrete cover increases. The rate of increase affected by the concrete damage variable, where the rate increases as the concrete damage variable increases.

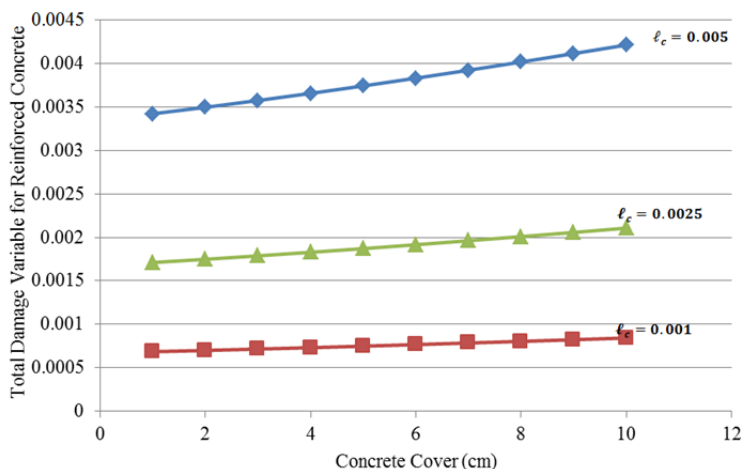


Figure 4. Effect of Concrete Cover on Total Reinforced Concrete Damage Variable

Also Figure 5 shows the relation between the reinforcement ratio and total reinforced concrete damage variable (ℓ_t) at different concrete damage variable (ℓ_c) and states that the total reinforced concrete damage variable (ℓ_t) decreases as the reinforcement ratio increases. The rate of decrease affected by the concrete damage variable, where the rate increases as the concrete damage variable increases.

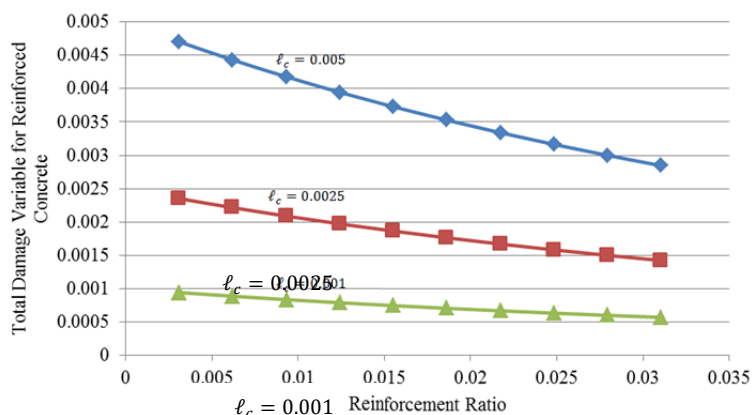


Figure 5. Effect of Reinforcement Ratio on Total Reinforced Concrete Damage Variable

5. Conclusion

From the results presented in this paper and comparison that made with both different codes of practices an experimental work and from the parametric study executed, one can conclude that:

- Proposed damage mechanics approach gives a good result in investigating cracks in flexural members, and can be used to calculate the cracks and voids volume, then crack width can be obtained.
- The total reinforced concrete damage variable increases as the concrete cover increases.
- The rate of increase of total reinforced concrete damage variable, with the increase of concrete cover, increases as the concrete damage variable increases.
- The total reinforced concrete damage variable decreases as the reinforcement ratio increases.
- The rate of decrease of total reinforced concrete damage variable, with the increase of reinforcement ratio, increases as the concrete damage variable increases.

6. Notation

a_{cr} is the distance from the point considered to the surface of the nearest longitudinal bar.

A_{ceff} effective tension area of concrete.

A_e is effective area of concrete in tension

A_o is the area of concrete surrounding reinforcement bar.

A_s is the area of steel.

β is the ratio of the distance between neutral axis and extreme tension face.

c is the concrete cover.

d_c is the distance measured from the centroid of tensile steel to the extreme tensioned fiber.

E is the elastic modulus.

E_c is the elastic modulus of concrete in the damaged case.

E_s is the elastic modulus of steel in the damaged case.

\bar{E} is the effective elastic modulus.

\bar{E}_c is the elastic modulus of concrete in the fictitious case.

\bar{E}_s is the elastic modulus of steel in the fictitious case.

ϵ is the strain in damage state.

$\bar{\epsilon}$ is the effective strain in fictitious state (state of removing cracks and voids).

ϵ_{sm} is the main steel strain.

ϵ_{cm} is the main concrete strain.

ϵ_m is the average strain at the level where cracking is being considered.

f_s is the stress in tension reinforcement.

f_{cteff} is the mean value of tensile strength of the concrete effective.

h_1 is the depth of equivalent area.

I_c is the second moment of area for concrete section in damaged case.

I_s is the second moment of area for reinforcement in damaged case.

\bar{I}_c is the second moment of area for concrete section in fictitious case.

\bar{I}_s is the second moment of area for reinforcement in fictitious case.

k_t is the factor expressing the duration of loading.

k_1 is the coefficient of bond properties.

k_2 is the coefficient of strain distribution.

ℓ_c is the concrete damage variable.

ℓ_s is the reinforcement steel damage variable.

ℓ_t is the reinforced concrete damage variable.

M is the applied bending moment.

n_b number of tension reinforcing bars.

ρ is reinforcement ratio.

$\bar{\rho}$ is the effective reinforcement ratio.

S is the first moment of area

S_c is the first moment of area for concrete section.

$S_{r,max}$ is the maximum crack spacing.

S_s is the first moment of area for reinforcement steel.

\bar{S} is the first moment of area for a cross section in the effective undamaged beam.

\bar{S}_c is the effective first moment of area for concrete section in fictitious state.

\bar{S}_s is the effective first moment of area for reinforcement steel in fictitious state.

σ is the bending stress.

$\bar{\sigma}$ is the bending stress in the effective undamaged beam.

W_k is the design crack width.

x is the depth of neutral axis and h is the member depth.

\emptyset is the bar diameter.

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