

# The Lightning Current Measurement Based on Wavelet Transform

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## Abstract

The current model of the lightning process is established and simulated in this article, and aiming at the disadvantage that the hardware filtering de-noising method adopted in the general lightning measurement system can not effectively separate signals and noises in same one frequency channel, this article uses the wavelet transform to replace the usual hardware filtering method in the lightning measurement system in order to overcome the disadvantage, and simulate this method. The result shows that this method is feasible and effective, with certain theoretical and practical values.

**Keywords:** Channel base current model, Engineering model, Noise current, Wavelet transform

## 1. Introduction

Lightning is a universal natural phenomenon, and a natural disaster, especially in the region with more lightning, it will threaten human security. A violent storm with lightning will also damage the buildings and crops to some extent. Therefore, the supervision of the lightning has important civil and military meanings. The measurement, supervision, and research of the lightning signal can not only help people to effectively warn and defend themselves, but can lead them to further explore the nature, know the nature and change the nature. At present, the researches about the lightning current mainly focus on low frequency and very low frequency (Xuan, 2009, pp. 519-525 & W. M. McRae, 2000, pp. 609-618 & M. Thèvenot, 1999, pp. 297-310), and the usual de-noising method of detecting the current signal is the hardware filtering, but this method is not ideal for the signals and noises in the same frequency range. The wavelet transform is introduced into the lightning current de-noising, and by virtue of the characteristics of the wavelet, the lightning current signals and the noise signals can be effectively distinguished.

## 2. Establishment and simulation of the lightning current model

The essential of the lightning is the strong discharge of different charge centers, so the lightning process and relative parameters should be studied deeply, and it is necessary to establish the mathematical model of the lightning current wave. The lightning current model can be divided into the engineering model and the channel base current model, and the former reflects the currents in the whole lightning return stroke channel, and its base is the channel base current model which computes the current at any height point by the current of the channel base. The general channel base current model at present has two types (M. A. Uman, 1987 & R. Thottappillil, 1997, pp. 6987-7006), and one of them is the double-index model, and its mathematical expression can be denoted as follows.

$$i(t) = I_0(e^{-\alpha t} - e^{-\beta t}) \quad (1)$$

Where,  $I_0$  is the parameter of the current intension which generally means the peak value of the channel base current,  $\alpha$  is the before-wave attenuation coefficient,  $\beta$  is the after-wave attenuation coefficient, and both of them are the important parameters to decide the current, and their concrete values are different each lightning. The simulation figure of the double-index function model is seen in Figure 1.

The other type is the Heidler model, and its mathematical expression can be denoted as follows.

$$i(0, t) = I_0 x(t) y(t) \tag{2}$$

Where,  $I_0$  is the peak return current of the channel base,  $x(t)$  represents the ascending time function of the current waveform,  $y(t)$  represents the durative time function of the current waveform, and generally,

$$x(t) = \frac{\left(\frac{t}{\tau_1}\right)^n}{1 + \left(\frac{t}{\tau_1}\right)^n}, \quad y(t) = \exp\left(-\frac{t}{\tau_2}\right)$$

,  $\tau_1$  and  $\tau_2$  respectively are the before-wave time constant and the after-wave time constant, and  $n$  is the order to describe the current gradient. The simulation figure of the Heidler function model is seen in Figure 2.

The expression of the double-index model is simple, and it can reflect the main parameters of the channel base current, convenient for the computation of integral and differential. But at the initial time, the derivative value of the current is not zero, which doesn't accord with the practical situation, so the electromagnetic field induced by the lightning computed by this model will induce certain error. But the derivative value of the current to the time at the initial time of the Heidler model is zero, which accords with the actual situation, and it is better than the double-index function model at this point, but its disadvantage is that it is the non-integral function, so it is difficult to be utilized for the direct numerical computation of the electromagnetic field produced by the lightning.

The lightning engineering model can be divided into two sorts (M. A. Uman, 1987 & R. Thottappillil, 1997, pp. 6987-7006), and the first sort of model thinks that the charge amount infusing in the lightning channel at certain time is decided by the earth, and the second sort of model thinks that the current value of the return channel base is decided by the charge amount infusing in the earth in the return, and the main engineering model usually used is the first sort. The first early engineering model is the BG model, and this model thinks that the current at the any attitude point where the return waves are transferred up in the return channel equals to the current at the channel base ground, i.e. the current is transferred without losses and delays in the channel. Its mathematical expression is

$$i(h, t) = i(0, t), h \leq vt \tag{3}$$

Where,  $h$  is the height of the point in the channel, and  $v$  is the speed that the return current transfers up. The expression of the BG model is simple, but to fulfill this model, the current at any point in the channel under the return before-wave must achieve the current value of the channel base, which denotes that the diffusion speed of the before-wave in the channel must be infinite, which obviously doesn't accord with the actual situation. The simulation figure of the BG model is seen in Figure 3 (the channel base current model is the double-index model).

The first sort of engineering model usually used is the TL model, and this model thinks that the return current in the lightning transfers up along the return channel base without losses, and the transfer speed is constant, and the return channel is thought as an ideal transfer line. Its mathematical expression is

$$i(h, t) = i(0, t - \frac{h}{v}), t \geq \frac{h}{v} \tag{4}$$

Where,  $h$  is the height of the point in the channel, and  $v$  is the transfer speed that the return channel. The simulation figure of the TL model is seen in Figure 4 (the channel base current model is the double-index model).

### 3. Advantages and de-noising principle of the wavelet transform

The concept of "wavelet" was proposed by Morlet and Arens in 1980s (Stéphane Jaffard, 2001). Define the function,  $\psi(t) \in L^2(\mathbb{R})$ , and if  $\psi(t)$  fulfills the permissibility condition,

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\widehat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \tag{5}$$

Where,  $\widehat{\psi}(\omega)$  is the Fourier transform of  $\psi(t)$ , and  $\psi(t)$  is called as the base wavelet, and by the flex and

translation of the base wavelet, a family of function can be obtained.

$$\psi_{a,b}(t) = |a|^{-1/2} \psi\left(\frac{t-b}{a}\right) \quad (6)$$

This family of function is called as the wavelet function system, and  $a$  is the flex factor,  $b$  is the translation factor, and the continual wave transform of the finite signal  $f(t)$  can be defined as

$$(W_\psi f)(a,b) = \int_{-\infty}^{\infty} f(t) \overline{\psi_{a,b}(t)} dt \quad (7)$$

Where,  $\overline{\psi_{a,b}(t)}$  is the complex conjugate of  $\psi_{a,b}(t)$ .

From the formula (3), the signal  $f(t)$  in the continual wavelet transform is transformed as the information  $(W_\psi f)(a,b)$  with two parameters,  $a$  and  $b$ , where, the change of the flex factor  $a$  represents the change of the frequency, and the change of the translation factor  $b$  represents the change of the time. So, when the base wavelet  $\psi(t)$  and its Fourier transform  $\widehat{\psi}(\omega)$  fulfill the condition of the window function, the continual wavelet transform of the signal is to offer a time-frequency window of the signal in the time-frequency plane, and its time window is  $[b + at_0 - a\Delta_\psi, b + at_0 + a\Delta_\psi]$ , where,  $t_0$  is the central time of the base wavelet  $\psi(t)$ ,  $\Delta_\psi$

is the radius of  $\psi(t)$ . The frequency window is  $[\frac{\omega_0}{a} - \frac{\Delta_{\widehat{\psi}}}{a}, \frac{\omega_0}{a} + \frac{\Delta_{\widehat{\psi}}}{a}]$ , where,  $\omega_0$  is the central

frequency of the Fourier transform  $\widehat{\psi}(\omega)$  of the base wavelet, and  $\Delta_{\widehat{\psi}}$  is the radius of  $\widehat{\psi}(\omega)$ . It can be proved that the product minimum of the time window width and the frequency window width are fixed, so when measuring the high frequency information ( $a$  is small), the frequency window will be wide but the time window is narrow, and when measuring the low frequency information ( $a$  is big), the time window will be wide but the frequency window is narrow, which can be obvious from the expression of the time-frequency window. So different values of  $a$  can be selected to adjust the proportion between the time window and the frequency window, and obtain the frequency spectrum which is interested at any time, which is also the advantage to de-noise by the wavelet transform, and it can effectively separate the signals and noises in the same frequency range. The concrete de-noising process is to first select one kind of wavelet, decompose the wavelet into  $N$  layers, and then select thresholds for the coefficients of various layers, deal with the detailed coefficients by the soft threshold, and recover the original signal to the processed coefficient by the wavelet rebuilding.

#### 4. Simulation result and analysis

In practice, the lightning current detected in the ground is the current of the return channel base, so the Heidler model introduced in this article is adopted to simulate it. Take the parameters of the lightning current as  $I_0 = 20kA$ ,  $n = 10$ ,  $\tau_1 = 10\mu s$ , and  $\tau_2 = 350\mu s$ . When introducing the Gauss white noise current into the current, the simulation waveform of the measured lightning is seen in Figure 5.

From Figure 5, in the environment that SNR is 10, the waveform of the lightning current fluctuates largely, but with the increase of the noise current, the measured effective lightning current information will be gradually drown by the noise. Therefore, how to effectively separate the measured lightning current signal with the noise current becomes the key to decide whether the lightning measurement system could work normally. In general lightning measurement system, the usual de-noising method is to de-noise by the hardware, but when the noise is the white noise, the hardware filtering can not effectively separate the signals and the noises, because the hardware filtering can not effectively separate the signal and noise in same one frequency range. Figure 6 is the simulation result of the filtering de-noising by the filter, and it is obvious that the de-noising effect is not ideal.

Aiming at the disadvantages of the hardware filtering, the method of wavelet transform is introduced to de-noise because the wavelet transform could reflect the information of the time field and the frequency field at the same time, so proper wavelet transform could effectively separate the lightning current and the noise current in same one frequency range. Figure 7 is the simulation result of the de-noising of the measured lightning current of the sym4 wavelet in the SymletsA wavelet family, and it is obvious that the effect is ideal, and it can basically recover the waveform of the lightning current signal before introducing the white noises.

#### 5. Conclusions

The current model of the lightning process is established and simulated in this article, and aiming at the

disadvantage that the hardware filtering de-noising method adopted in the general lightning measurement system can not effectively separate signals and noises in same one frequency channel, this article uses the wavelet transform to replace the usual hardware filtering method in the lightning measurement system in order to overcome the disadvantage, and the simulation result proves the feasibility and validity of this method, and this method has certain practical and extended values.

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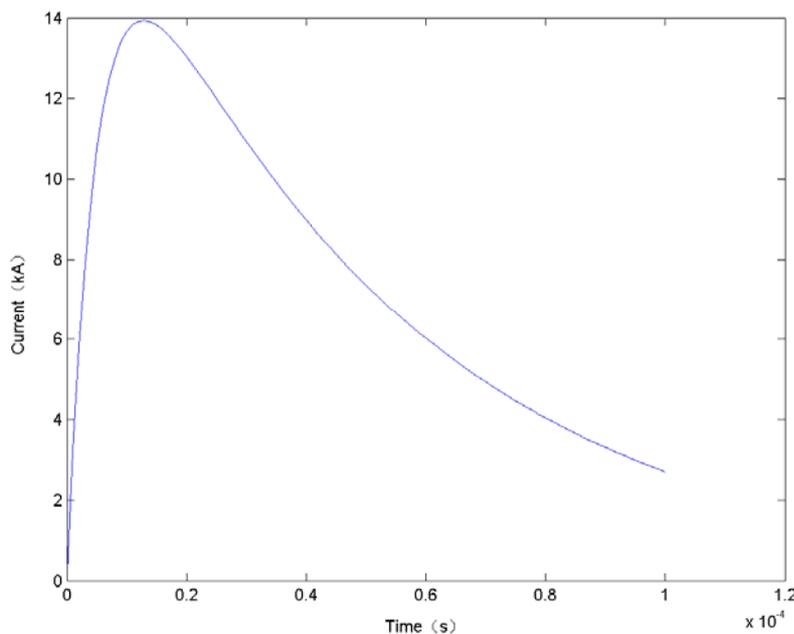


Figure 1. Double-index Model

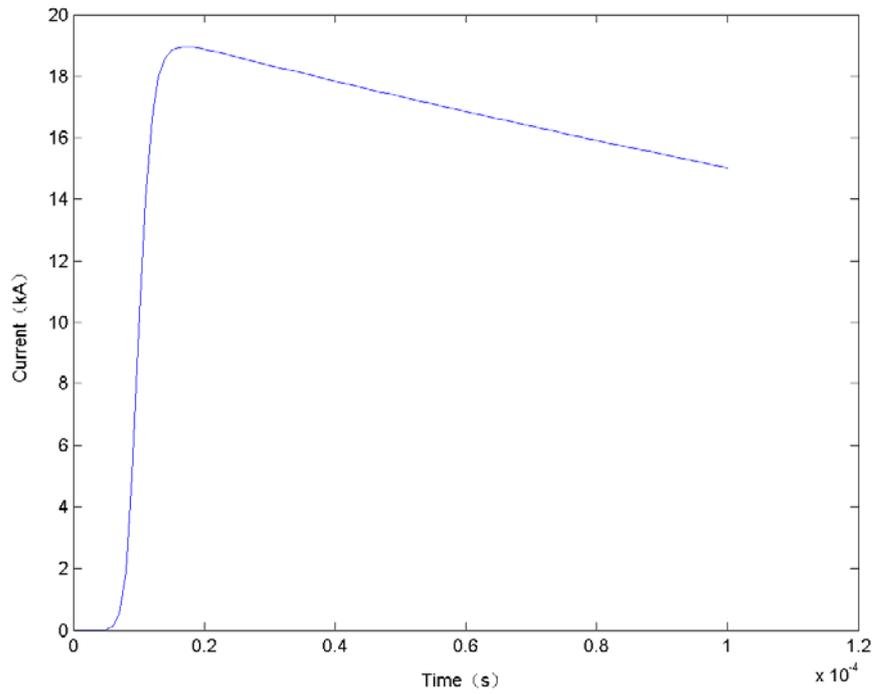


Figure 2. Heidler Model

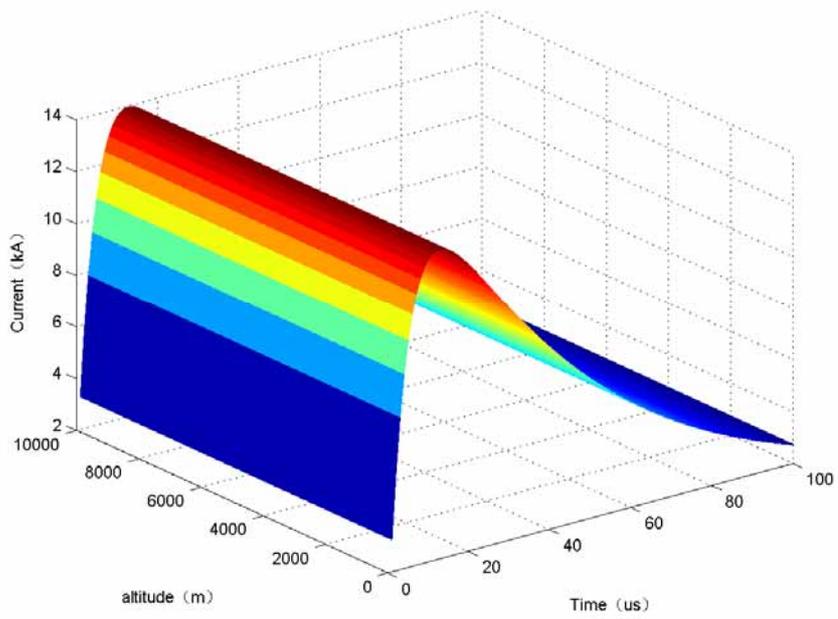


Figure 3. BG Model

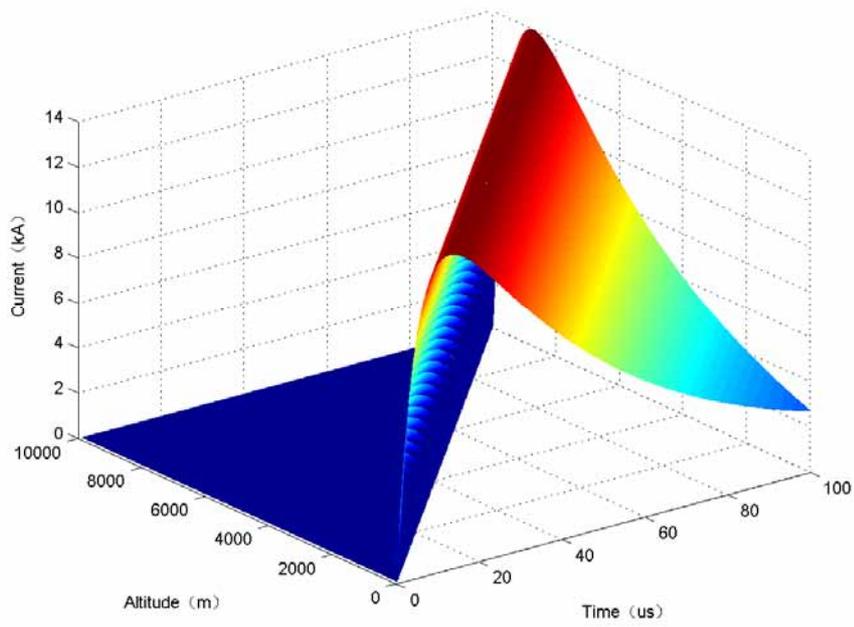


Figure 4. TL Model

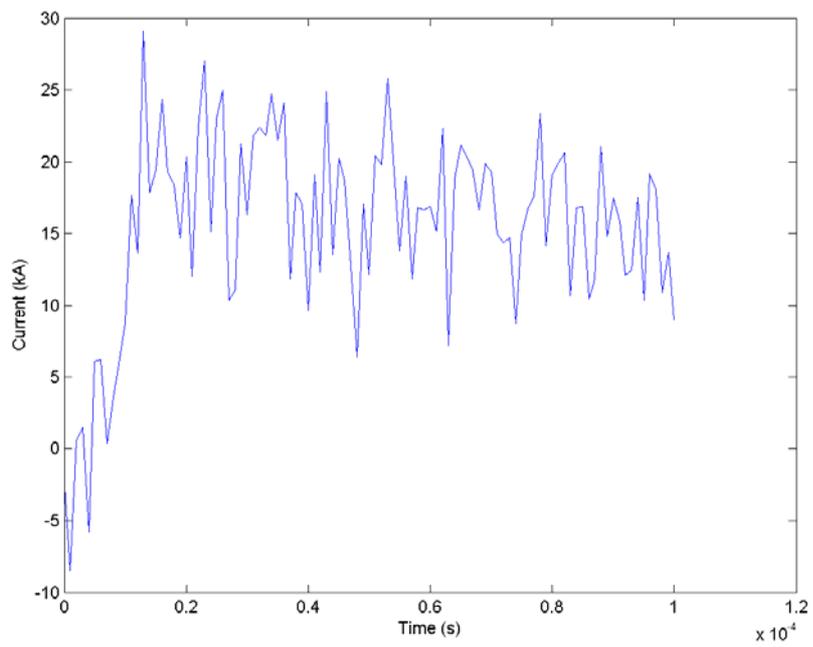


Figure 5. Lightning Current with White Noises (SNR:10)

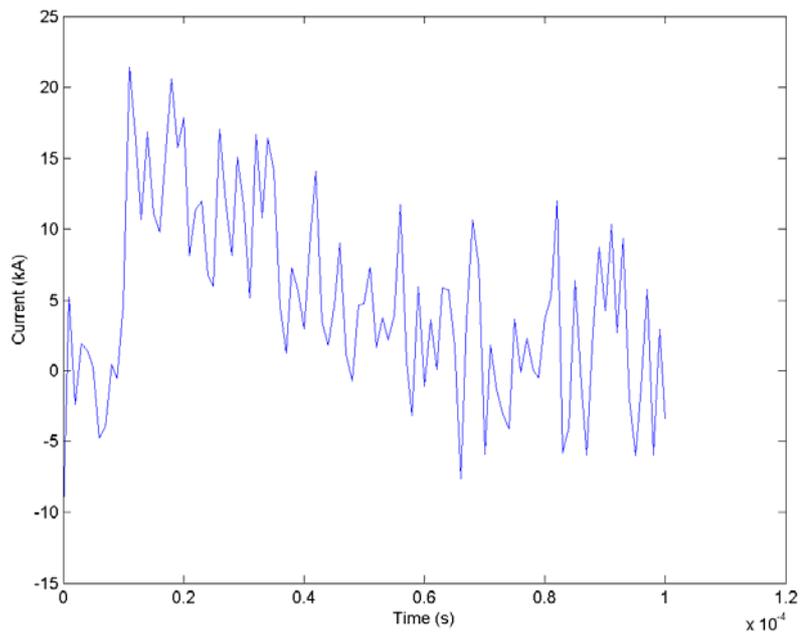


Figure 6. Filtering De-noising Result by the Filter

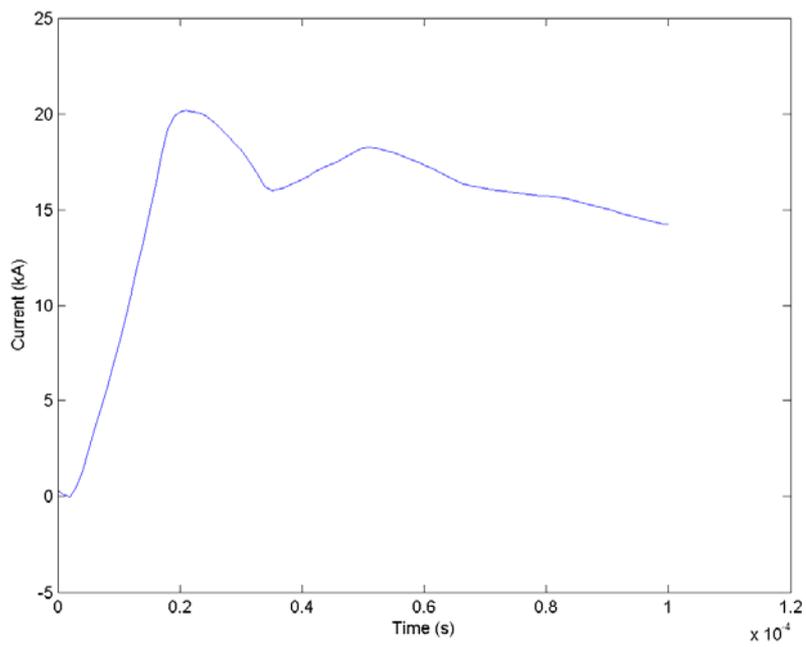


Figure 7. Filtering De-noising Result by the Wavelet Transform