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A Multi Item Integrated Inventory Model with Reparability and Manufacturing of Fresh Products

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Abstract

In this paper a multi item integrated inventory model is presented with reparability of returned items. It is assumed here that only a certain ratio of returned items can be repaired and the remaining stock of returned items is salvaged. By using these returned items, the waste can be reduced, which pollute the environment. This is a green supply chain where the demand for the products is selling price dependent and production rate is taken as a function of demand rate. The shortages are allowed here. A numerical example and sensitivity analysis are also presented to illustrate the model.

Keywords: integrated inventory, reparability, multi items, deterioration, shortages

1. Introduction

In so many industrial systems, there is more than one manufacturing plant and they produce different type of items. Most of the classical inventory system tries to calculate the economic production quantity by which the total cost of the system can be optimized. But these models are developed only for single item. In this model we have presented an integrated inventory model for multi items. Considering multi items in inventory model, (Ben-Daya & Raouf, 1993) have developed an approach for more realistic and general concept of budgetary and floor constraints, where the demand of items follows a uniform probability distribution function. A multi item inventory model for deteriorating items is developed by (Bhattacharya, 2005). (Roy et al., 1995) developed multi deteriorating items with constraint space and investment and obtained some interesting results.(Balkhi ,2009) has introduced a general model for multi item production inventory system in which the cost parameters are treated as an arbitrary function of time. (Yadav et al., 2011) developed a fuzzy multi item production inventory model with reliability and flexibility under limited storage capacity with deterioration via geometric programming. (Singhal & Singh, 2013) introduced a volume flexible multi items inventory system with imprecise environment. (Tayal et al., 2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages.

However, in most of the classical production inventory models even concerned with multi items the attention is given only for the optimality of separate member of the integrated system. For any successful supply chain coordination between the vendor and the buyer is required. This close relationship between vendor and buyer is a key of success for any business organization. Then, a new approach of integration of all the functions in a supply chain was identified.(Singh & Diksha ,2009) developed an integrated cooperative inventory model for vendor and buyer under progressive credit period in which demand is assumed to be a multivariate function. (Hadidi et al., 2011) developed an integrated production inventory model for scheduling and perfect maintenance. This work integrates, simultaneously, the decisions of preventive maintenance and job order sequencing for a single machine.(Soni & Patel,2014) presented an optimal decision policy for integrated vendor-buyer inventory system concerning defective items with variable lead time and service level constraint. (Tayal et al, 2016) introduced an integrated production inventory model for perishable products with trade credit period and investment in preservation technology.

Concerning the deterioration and reparability in an inventory model is more realistic and general concept. (Park,

1983) studied a production inventory system for a single product with deteriorating raw materials. (Yan & Cheng, 1998) introduced an optimal production stopping and restarting times for an EOQ model with deteriorating items.

(Maity & Maiti, 2009) developed optimal inventory policies for deteriorating complementary and substitute items. (Singh et al., 2010) presented an EOQ model with Pareto distribution for deterioration, Trapezoidal type demand and backlogging under trade credit policy. Singh et al. (2013) introduced an EOQ model with volume agility, variable demand rate, Weibull deterioration rate and inflation. (Tayal et al.,2014) developed a two echelon supply chain model for deteriorating items with effective investment in preservation technology after that (Tayal et al. ,2014) studied an inventory model for deteriorating items with seasonal products and an option of an alternative market. (Singh et al., 2015) presented an EPQ inventory model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate. (Singh et al., 2016) developed an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. One of the first authors was (Schrady, 1967) who developed a simple heuristic procedure for determining the lot sizes of remanufacturing and manufacturing lots. He proposed a simple EOQ-technique that optimizes the sum of fixed and holding costs per time unit. (Teunter.2001) generalized the results of Schrady in a way that he examined different structures of a remanufacturing cycle. His analysis concludes that it is not efficient if more than one remanufacturing lot and more than one manufacturing lot are established in the same repair cycle. (Saxena et.al. 2013) presented two-warehouse production inventory model with variable demand and permissible delay in payment under inflation. (Saxena et.al.2014) generalized production model under fuzzy environment. They compared both the results obtained considering crisp data as well as fuzzy data. (Singh & Singh, 2013) presented green supply chain model with product remanufacturing under volume flexible environment. (Singh & Prasher, 2014) introduced a production inventory model with flexible manufacturing, random machine breakdown and stochastic repair time.

In this paper we have presented an integrated inventory model for multi items in a closed loop supply chain with reparability of returned and collected items. The shortages are allowed for retailer and assumed to be completely lost. The repaired items are assumed to be equivalent to fresh products.

2. Assumptions and Notations

2.1 Assumptions

1. This is a multi item inventory model, presented for the integrated production of new products and reparability of collected items.
2. The demand for the products is a function of selling price.
3. The products after reparability are assumed to be equivalent to new products.
4. The production rate is assumed to be a function of demand rate.
5. The used items are collected at a rate of $b(\alpha_i - \beta_i s_i)$.
6. A certain ratio γ_i of collected items, whose quality level is acceptable for reparability, is used for production and rest of the material is salvaged.
7. The items are assumed to be deteriorating in nature.
8. The shortages are allowed for retailer and occurring shortages are assumed to be completely lost.

2.2 Notations

θ deterioration rate

α_i, β_i demand parameters for i^{th} item

s_{i1}, s_{i2} selling price per unit for the producer and the retailer for i^{th} item

a_i production parameter for i^{th} item, $a \geq 1$

b_i collection parameter for i^{th} item, $b<1$

t_{1i} time for remanufacturing for i^{th} item

t_{2i} time at which inventory level for remanufactured i^{th} item becomes zero

t_{3i} time up to which production of fresh i^{th} items occur

T_i length of the complete cycle for i^{th} item

$I_{ri}(t)$ inventory level for remanufactured i^{th} item at any time t

$I_{mi}(t)$ inventory level for fresh produced i^{th} item at any time t

$I_{Ri}(t)$ inventory level of collected i^{th} item at any time t

$I_{si}(t)$ retailer's inventory level for i^{th} item at any time t

v_i the time at which the inventory level of i^{th} item becomes zero for retailer

n number of replenishment cycles for the retailer

c_{mi} procurement cost per unit for i^{th} item

c_{Ri} acquisition cost per unit for i^{th} item

P_{mi} production cost per unit for ith item

P_{ri} remanufacturing cost per unit for i^{th} item

$h_{ri}, h_{mi}, h_{Ri}, h_{si}$ holding cost per unit for i^{th} item for remanufactured items, produced items, collective items and for the retailer

O_i ordering cost per order

c_{1i} purchasing cost per unit for i^{th} item for the retailer

c_{2i} lost sale cost cost per unit for i^{th} item for the retailer

K_{1i}, K_{2i}, K_{3i} set up cost for remanufacturing, fresh production and for the collection

$I_{1i}(0)$ initial inventory level for the retailer for i^{th} item

Q_{2i} shortage amount for i^{th} item

3. Mathematical Model

In this the used items are collected from the market and a certain ratio of these items is repaired. In the above mentioned figure the remanufacturing graph is shown. During $[0, t_{1i}]$ remanufacturing occurs and the inventory level of collected items decreases and becomes zero at time t_{1i}. During $[t_{1i}, t_{2i}]$ the inventory level of repaired items depletes due to combined effect of demand and deterioration. After $t = t_{1i}$ the inventory level of collected items again piled up. At $t = t_{2i}$ the manufacturing of fresh products start and occurs up to $t = t_{3i}$. The below mentioned figure (1) shows the inventory level of this system.

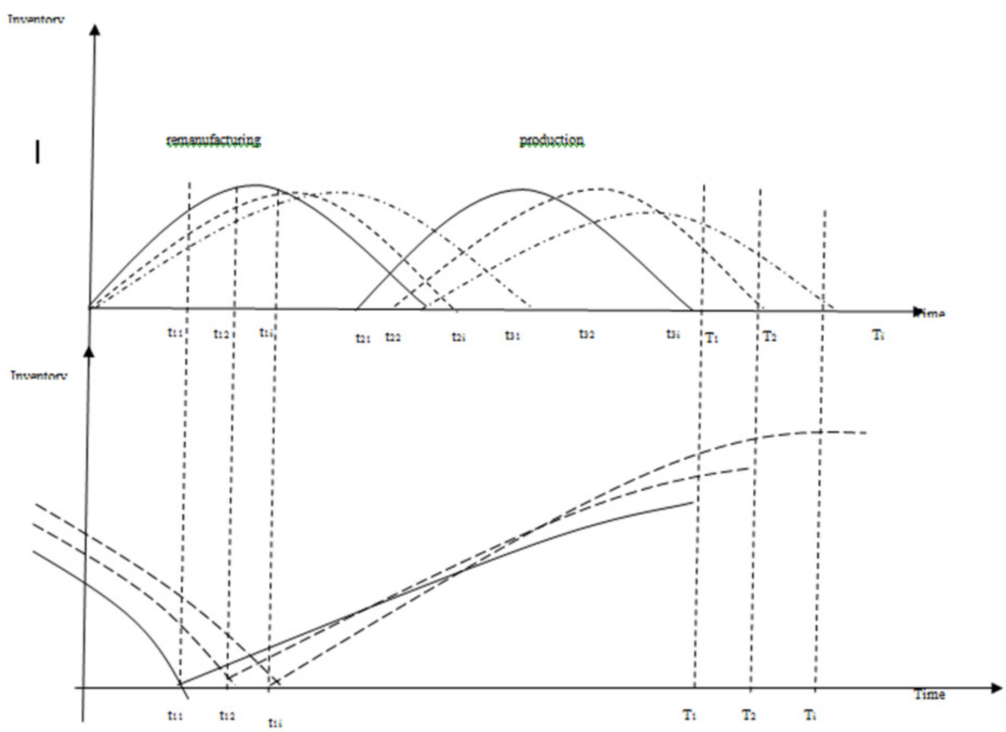

Figure 1. Inventory time behavior for repaired items, fresh produced items and returned items

These are the differential equations showing the inventory with the variation in time for repaired items, fresh production and collected items.

$$\frac{dI_{ri}(t)}{dt}+\theta I_{ri}(t)=(a-1)(\alpha_i-\beta_i s_{i1}) \qquad 0\le t\le t_{1i} \tag{1}$$

$$\frac{dI_{ri}(t)}{dt}+\theta I_{ri}(t)=(\alpha_i-\beta_i s_{i1}) \qquad t_{1i}\le t\le t_{2i} \tag{2}$$

$$\frac{dI_{mi}(t)}{dt}+\theta I_{mi}(t)=(a-1)(\alpha_i-\beta_i s_{i1}) \qquad t_{2i}\le t\le t_{3i} \tag{3}$$

$$\frac{dI_{mi}(t)}{dt}+\theta I_{mi}(t)=(\alpha_i-\beta_i s_{i1}) \qquad t_{3i}\le t\le T_i \tag{4}$$

$$\frac{dI_{Ri}(t)}{dt}+\theta I_{Ri}(t)=(b-1)(\alpha_i-\beta_i s_{i1}) \qquad 0\le t\le t_{1i} \tag{5}$$

$$\frac{dI_{Ri}(t)}{dt}+\theta I_{Ri}(t)=b(\alpha_i-\beta_i s_{i1}) \qquad t_{1i}\le t\le T_i \tag{6}$$

with boundary conditions:

$$I_{ri}(0)=0,\ I_{ri}(t_{2i})=0,\ I_{mi}(t_{2i})=0,\ I_{mi}(T_i)=0,\ I_{Ri}(t_{1i})=0,\ I_{Ri}(0)=I_{Ri}(T_i) \tag{7}$$

The solution of these above mentioned equations are given as follow:

$$I_{ri}(t)=\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})(1-e^{-\theta t}) \qquad 0\le t\le t_{1i} \tag{8}$$

$$I_{ri}(t)=\frac{(\alpha_i-\beta_i s_{i1})}{\theta}(e^{\theta(t_{2i}-t)}-1) \qquad t_{1i}\le t\le t_{2i} \tag{9}$$

$$I_{mi}(t)=\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})(1-e^{\theta(t_{2i}-t)}) \qquad t_{2i}\le t\le t_{3i} \tag{10}$$

$$I_{mi}(t)=\frac{(\alpha_i-\beta_i s_{i1})}{\theta}(e^{\theta(T_i-t)}-1) \qquad t_{3i}\le t\le T_i \tag{11}$$

$$I_{Ri}(t)=\frac{(\alpha_i-\beta_i s_{i1})}{\theta}(b-a)(1-e^{\theta(t_{1i}-t)}) \qquad 0\le t\le t_{1i} \tag{12}$$

$$I_{Ri}(t)=\frac{b}{\theta}(\alpha_i-\beta_i s_{i1})(1-e^{\theta(t_{1i}-t)}) \qquad t_{1i}\le t\le T_i \tag{13}$$

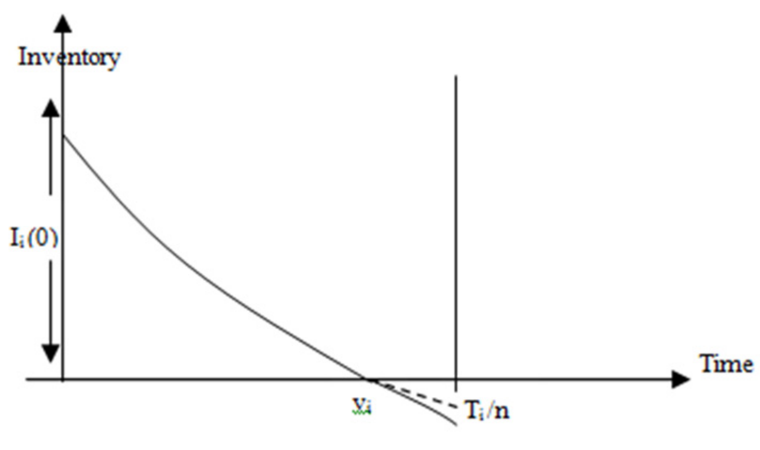

Figure 2. Retailer’s inventory

If $I_{si}(t)$ denotes the retailer's inventory level for ith item at any time t, then the differential equations showing the inventory level at any time t are given as follow:

$$\frac{dI_{si}(t)}{dt} + \theta I_{si}(t) = -(\alpha_i - \beta_i s_{i2}) \qquad 0 \le t \le v_i \tag{14}$$

$$\frac{dI_{si}(t)}{dt} = -(\alpha_i - \beta_i s_{i2}) \qquad v_i \le t \le \frac{T_i}{n} \tag{15}$$

$$\text{with boundary condition } I_{si}(v_i) = 0 \tag{16}$$

The solution of these above mentioned equations are given as follow:

$$I_{si}(t) = \frac{(\alpha_i - \beta_i s_{i2})}{\theta}(e^{\theta(v_i - t)} - 1) \qquad 0 \le t \le v_i \tag{17}$$

$$I_{si}(t) = (\alpha_i - \beta_i s_{i2})(v_i - t) \qquad v_i \le t \le \frac{T_i}{n} \tag{18}$$

The total average cost for the ith item for the manufacturer is given by:

$$T.A.C_{mi} = \frac{1}{T_i}\text{[Procurement cost + Acquisition cost + Production cost + Remanufacturing cost + Holding cost + Salvage cost + Set up cost]} \tag{19}$$

The total average cost for the ith item for the retailer is given by:

$$T.A.C_{si} = \frac{n}{T_i}\text{[Purchasing cost + Holding cost + ordering cost + lost sale cost]} \tag{20}$$

Different associated costs for manufacturer:

$$\text{Procurement cost} = c_{mi}\int_{t_{2i}}^{t_{3i}} a(\alpha_i - \beta_i s_{i1})\,dt$$

$$\text{Procurement cost} = c_{mi}a(\alpha_i - \beta_i s_{i1})(t_{3i} - t_{2i}) \tag{21}$$

$$\text{Acquisition cost} = c_{Ri}\int_0^{T_i} b(\alpha_i - \beta_i s_{i1})\,dt$$

$$\text{Acquisition cost} = c_{Ri}bT_i(\alpha_i - \beta_i s_{i1}) \tag{22}$$

$$\text{Production cost} = P_{mi}a\int_{t_{2i}}^{t_{3i}} (\alpha_i - \beta_i s_{i1})dt$$

$$\text{Production cost} = P_{mi}a(t_{3i} - t_{2i})(\alpha_i - \beta_i s_{i1}) \tag{23}$$

$$\text{Remanufacturing cost} = P_{ri}a\int_0^{t_{1i}} (\alpha_i - \beta_i s_{i1})dt$$

$$\text{Remanufacturing cost} = P_{ri}at_{1i}(\alpha_i - \beta_i s_{i1}) \tag{24}$$

$$\text{Set up cost} = K_{1i} + K_{2i} + K_{3i} \tag{25}$$

$$\text{Salvage cost} = s_{vi}\{(1-\gamma_i)b(\alpha_i - \beta_i s_{i1})\}T_i \tag{26}$$

Holding cost =

$$h_{ri}\int_0^{t_{1i}} I_{ri}(t)\,dt + h_{ri}\int_{t_{1i}}^{t_{2i}} I_{ri}(t)\,dt + h_{mi}\int_{t_{2i}}^{t_{3i}} I_{mi}(t)\,dt + h_{mi}\int_{t_{3i}}^{T_i} I_{mi}(t)\,dt + h_{Ri}\int_0^{t_{1i}} I_{Ri}(t)\,dt + h_{Ri}\int_{t_{1i}}^{T_i} I_{Ri}(t)\,dt$$

Holding cost =

$$h_{ri}\{\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})(t_{1i}+\frac{e^{-\theta t_{1i}}-1}{\theta})+\frac{(\alpha_i-\beta_i s_{i1})}{\theta}(\frac{e^{\theta(t_{2i}-t_{1i})}-1}{\theta})+(t_{1i}-t_{2i})\}$$
$$+h_{mi}\{\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})\{(t_{3i}-t_{2i})+\frac{e^{\theta(t_{2i}-t_{3i})}-1}{\theta}\}+\frac{(\alpha_i-\beta_i s_{i1})}{\theta}\{(\frac{e^{\theta(T_i-t_{3i})}-1}{\theta})-(T_i-t_{3i})\}\}$$
$$+h_{Ri}\{\frac{(b-a)}{\theta}(\alpha_i-\beta_i s_{i1})(t_{1i}-\frac{1-e^{\theta t_{1i}}}{\theta})+b\frac{(\alpha_i-\beta_i s_{i1})}{\theta}\{(\frac{e^{\theta(t_{1i}-T_i)}-1}{\theta})+(T_i-t_{1i})\}\}\quad(27)$$

Different associated costs for retailer:

Purchasing cost = $(I_{1i}(0)+Q_{2i})c_{1i}$

where

$$I_{1i}(0)=\frac{(\alpha_i-\beta_i s_{i2})}{\theta}(e^{\theta v_i}-1)$$

$$Q_2=\int_{v_i}^{\frac{T_i}{n}}(\alpha_i-\beta_i s_{i2})\eta\,dt$$

$$\text{P.C.}=\{\frac{(\alpha_i-\beta_i s_{i2})}{\theta}(e^{\theta v_i}-1)+(\alpha_i-\beta_i s_{i2})\eta(\frac{T_i}{n}-v_i)\}c_{1i}\quad(28)$$

$$\text{Holding cost} = h_{si}\int_0^{v_i}I_{si}(t)\,dt$$

$$H.C_s=h_{si}\frac{(\alpha_i-\beta_i s_{i2})}{\theta}(\frac{e^{\theta v_i}-1}{\theta}-v_i)\quad(29)$$

$$\text{Ordering cost} = O_i\quad(30)$$

$$\text{Lost sale cost} = c_{2i}\int_{v_i}^{T_i/n}(\alpha_i-\beta_i s_{i2})\,dt$$

$$L.S.C_i=c_{2i}(\alpha_i-\beta_i s_{i2})(\frac{T_i}{n}-v_i)\quad(31)$$

Hence the total cost per unit time of the given inventory model as a function of t_{1i}, t_{2i}, t_{3i}, v_i and T_i say T.A.C.(t_{1i}, t_{2i}, t_{3i}, v_i, T_i) is given by

$$T.A.C.=\frac{1}{T}\{c_{mi}a(\alpha_i-\beta_i s_{i1})(t_{3i}-t_{2i})+c_{Ri}bT_i(\alpha_i-\beta_i s_{i1})+P_{mi}a(t_{3i}-t_{2i})(\alpha_i-\beta_i s_{i1})+$$
$$P_{ri}at_{1i}(\alpha_i-\beta_i s_{i1})+K_{1i}+K_{2i}+K_{3i}+s_{vi}\{(1-\gamma_i)b(\alpha_i-\beta_i s_{i1})\}T_i+$$
$$h_{ri}\{\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})(t_{1i}+\frac{e^{-\theta t_{1i}}-1}{\theta})+\frac{(\alpha_i-\beta_i s_{i1})}{\theta}(\frac{e^{\theta(t_{2i}-t_{1i})}-1}{\theta})+(t_{1i}-t_{2i})\}+h_{mi}\{\frac{(a-1)}{\theta}(\alpha_i-\beta_i s_{i1})\{(t_{3i}-t_{2i})+\frac{e^{\theta(t_{2i}-t_{3i})}-1}{\theta}\}+$$
$$\frac{(\alpha_i-\beta_i s_{i1})}{\theta}\{(\frac{e^{\theta(T_i-t_{3i})}-1}{\theta})-(T_i-t_{3i})\}\}+h_{Ri}\{\frac{(b-a)}{\theta}(\alpha_i-\beta_i s_{i1})(t_{1i}-\frac{1-e^{\theta t_{1i}}}{\theta})+$$
$$b\frac{(\alpha_i-\beta_i s_{i1})}{\theta}\{(\frac{e^{\theta(t_{1i}-T_i)}-1}{\theta})+(T_i-t_{1i})\}\}+\frac{T_i}{n}[\{\frac{(\alpha_i-\beta_i s_{i2})}{\theta}(e^{\theta v_i}-1)+(\alpha_i-\beta_i s_{i2})\eta(\frac{T_i}{n}-v_i)\}c_{1i}$$
$$h_{si}\frac{(\alpha_i-\beta_i s_{i2})}{\theta}(\frac{e^{\theta v_i}-1}{\theta}-v_i)+O_i+c_{2i}(\alpha_i-\beta_i s_{i2})(\frac{T_i}{n}-v_i)]\quad(32)$$

Equation (32) denotes the cost function of the system in terms of t_{1i}, t_{2i}, t_{3i}, v_i and T_i. To find out the optimal solution of this system we have to find out the optimal values of t_{1i}, t_{2i}, t_{3i}, v_i and T_i. We have some relations between these variables.

$$0 \le t_{1i} \le t_{2i} \le t_{3i} \le T_i \tag{33}$$

$$0 \le v_i \le T_i \tag{34}$$

$$(a-1)(1-e^{-\theta t_{1i}}) = (e^{\theta(t_{2i}-t_{1i})}-1) \tag{35}$$

$$(a-1)(1-e^{\theta(t_{2i}-t_{3i})}) = (e^{\theta(T_i-t_{3i})}-1) \tag{36}$$

$$(b-a)(1-e^{\theta t_{1i}}) = b(1-e^{\theta(t_{1i}-T_i)}) \tag{37}$$

Equations (33) and (34) are the essential conditions for the existence of this model. Equations (35) and (36) show the inventory level $I_r(t)$ and $I_m(t)$ at $t=t_1$ and $t=t_3$. Equation (37) demonstrates that the inventory level of collected and returned items will be the same at t=0 and t=T.

Using the equations (35) - (37) the values of t_{1i}, t_{2i} and t_{3i} can be find in the form of T_i, It can be said that

$$t_{1i} = f_1(T_i), \qquad t_{2i} = f_2(T_i), \qquad t_{3i} = f_3(T_i)$$

Therefore the total average cost will be the function of variables T_i and v_i.

4. Numerical Example

A numerical example is carried out to illustrate the model. Corresponding to the below mentioned parametric values the optimal value of time v_i, T_i and $T.A.C_i$ are obtained for different three products.

Table 1.

Parameters	Product 1	Product 2	Product 3
α_i	50	55	45
γ_i	0.4	0.45	0.35
s_{i1}	30	32	25
s_{i2}	40	41	43
a	1.5	1.6	1.4
b	0.4	0.45	0.35
β_i	0.05	0.06	0.04
h_{ri}	0.5	0.55	0.4
h_{mi}	0.5	0.55	0.45
h_{Ri}	0.5	0.4	0.4
θ	0.01	0.01	0.01
c_{mi}	10	11	9
c_{Ri}	12	12.5	11.5
P_{mi}	14	14.5	13.5
P_{ri}	16	16.5	15.5
S_{vi}	4	4.5	3.5
K_{1i}	500	500	500
K_{2i}	600	600	600
K_{3i}	700	700	700
c_{1i}	20	20	20
h_{is}	0.4	0.45	0.35
O_i	250	300	200
n	5	5	5
v_i	**20.1567**	**19.8956**	**20.2844**
T_i	**23.7439**	**23.4203**	**23.9021**
$T.A.C_i$	**1487.61**	**1898.66**	**1642.7**

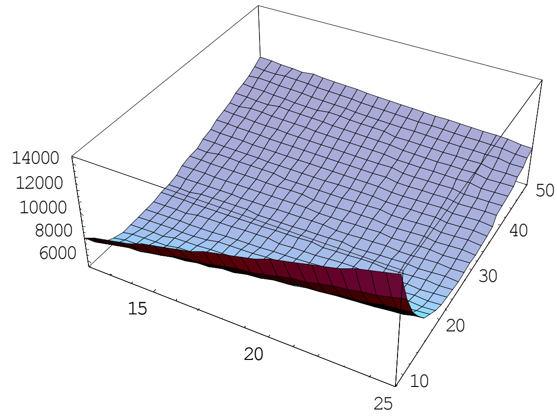

Figure 3. Convexity of the $T.A.C_1$ function

5. Sensitivity Analysis

Corresponding to different associated parameters, a sensitivity analysis is carried out to check the stability of the model. The analysis has been done with the parameters θ, b, β_1, γ_1, α_1, s_1 and a_1 taking one parameter at a time and other variables unchanged and is shown in below mentioned tables.

Table 2. Sensitivity analysis with respect to deterioration parameter (θ):

% variation in θ	θ	v_1	T_1	$T.A.C_1$
-20%	0.008	20.0962	23.6689	1128.13
-15%	0.0085	20.1113	23.6876	1233.69
-10%	0.009	20.1264	23.7063	1327.62
-5%	0.0095	20.1416	23.7251	1411.78
0%	0.01	20.1567	23.7439	1487.61
5%	0.0105	20.1719	23.7627	1556.31
10%	0.011	20.1872	23.7816	1618.86
15%	0.0115	20.2024	23.8005	1676.05
20%	0.012	20.2177	23.8194	1728.56

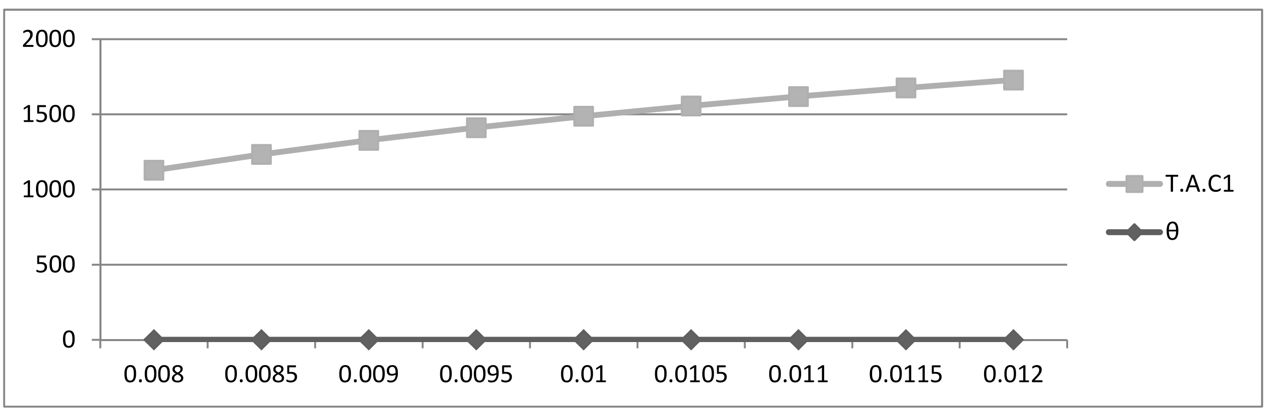

Figure 4. Variation in $T.A.C_1$ with the variation in θ

Table 3. Sensitivity analysis with respect to parameter b:

% variation in b	**b**	**v_1**	**T_1**	**$T.A.C_1$**
-20%	0.32	19.9468	23.4837	1739.47
-15%	0.34	20.0018	23.5518	1672.5
-10%	0.36	20.0552	23.618	1608.2
-5%	0.38	20.1069	23.6821	1546.57
0%	0.4	20.1567	23.7439	1487.61
5%	0.42	20.2046	23.8032	1431.33
10%	0.44	20.2503	23.8598	1377.75
15%	0.46	20.2937	23.9136	1326.86
20%	0.48	20.3346	23.9643	1278.67

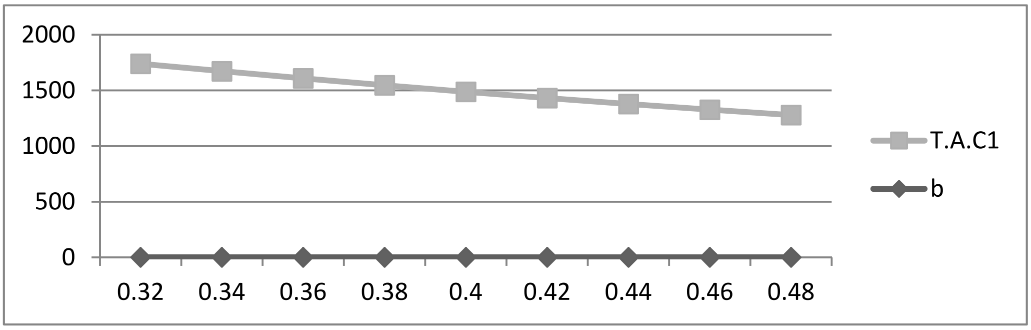

Figure 5. Variation in $T.A.C_1$ with the variation in b

Table 4. Sensitivity analysis with respect to demand parameter (β_1):

% variation in β_1	β_1	v_1	T_1	$T.A.C_1$
-20%	0.04	20.148	23.7331	1494.73
-15%	0.0425	20.1502	23.7358	1492.95
-10%	0.045	20.1523	23.7385	1491.17
-5%	0.0475	20.1545	23.7412	1489.39
0%	0.05	20.1567	23.7439	1487.61
5%	0.0525	20.1589	23.7466	1485.83
10%	0.055	20.1611	23.7494	1484.05
15%	0.0575	20.1634	23.7521	1482.27
20%	0.06	20.1656	23.7549	1480.48

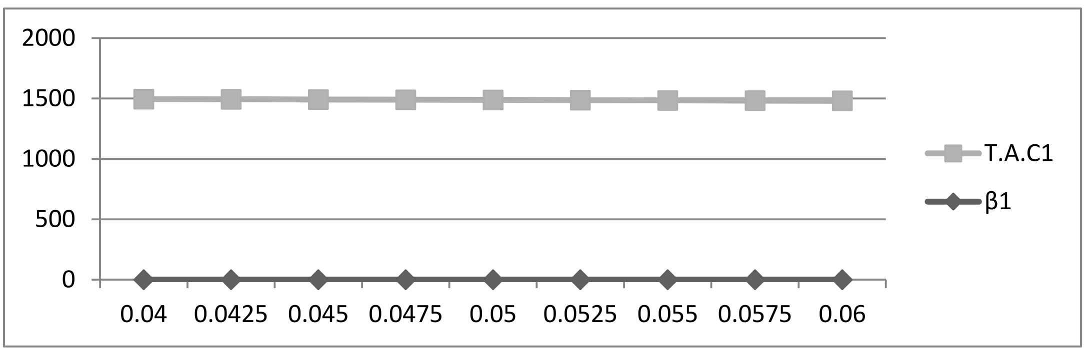

Figure 6. Variation in $T.A.C_1$ with the variation in β_1

Table 5. Sensitivity analysis with respect to parameter γ_1:

% variation in γ_1	γ_1	v_1	T_1	$T.A.C_1$
-20%	0.32	20.1567	23.7439	1481.4
-15%	0.34	20.1567	23.7439	1482.95
-10%	0.36	20.1567	23.7439	1484.5
-5%	0.38	20.1567	23.7439	1486.06
0%	0.4	20.1567	23.7439	1487.61
5%	0.42	20.1567	23.7439	1489.16
10%	0.44	20.1567	23.7439	1490.71
15%	0.46	20.1567	23.7439	1492.26
20%	0.48	20.1567	23.7439	1493.82

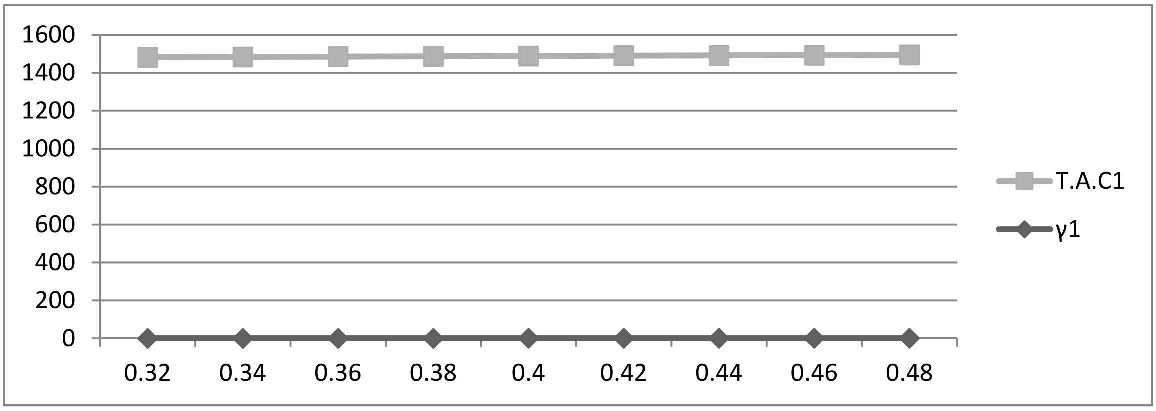

Figure 7. Variation in T.A.C1 with the variation in γ1

Table 6. Sensitivity analysis with respect to demand parameter (α1):

% variation in α_1	α_1	v_1	T_1	$T.A.C_1$
-20%	40	20.5201	24.1943	1249.82
-15%	42.5	20.4139	24.0627	1309.33
-10%	45	20.319	23.9451	1368.8
-5%	47.5	20.2338	23.8394	1428.22
0%	50	20.1567	23.7439	1487.61
5%	52.5	20.0868	23.6572	1546.97
10%	55	20.023	23.5781	1606.3
15%	57.5	19.9645	23.5056	1665.61
20%	60	19.9108	23.439	1724.9

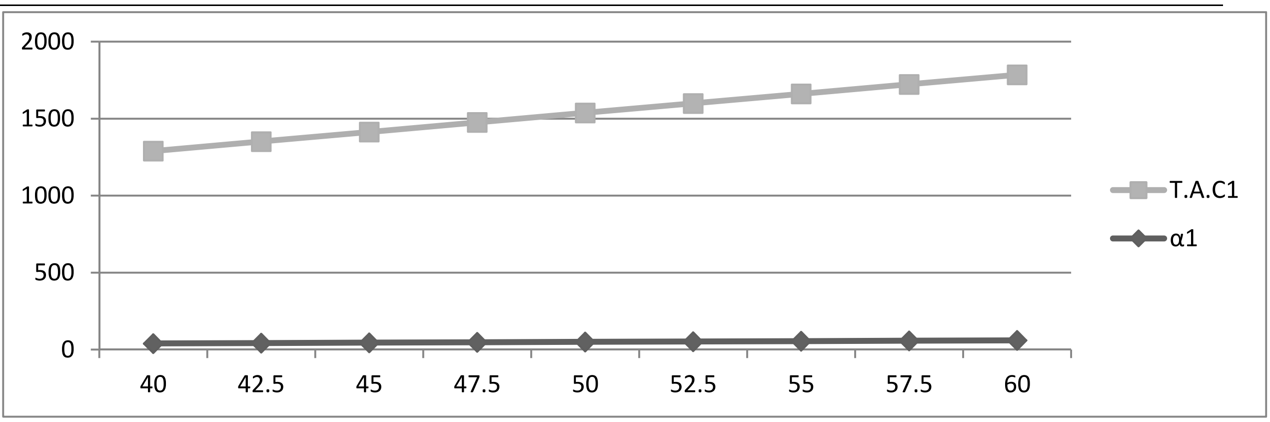

Figure 8. Variation in T.A.C1 with the variation in α1

Table 7. Sensitivity analysis with respect to selling price (s11)

% variation in s_{11}	s_{11}	v_1	T_1	$T.A.C_1$
-20%	24	20.148	23.7331	1494.73
-15%	25.5	20.1502	23.7358	1492.95
-10%	27	20.1523	23.7385	1491.17
-5%	28.5	20.1545	23.7412	1489.39
0%	30	20.1567	23.7439	1487.61
5%	31.5	20.1589	23.7466	1485.83
10%	33	20.1611	23.7494	1484.05
15%	34.5	20.1634	23.7521	1482.27
20%	36	20.1656	23.7549	1480.48

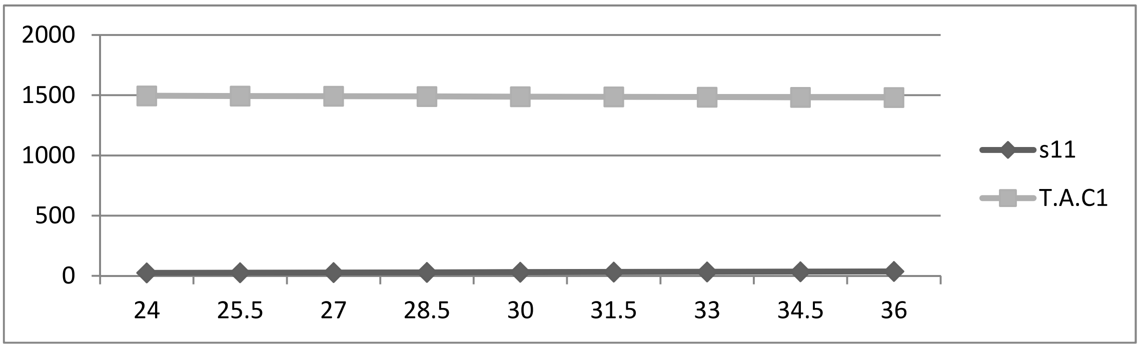

Figure 9. Variation in $T.A.C_1$ with the variation in s_1

Table 8. Sensitivity analysis with respect to production parameter (a1):

% variation in a_1	a_1	v_1	T_1	$T.A.C_1$
-20%	1.2	20.6989	24.4159	1592.02
-15%	1.275	20.5379	24.2164	1561.34
-10%	1.35	20.3959	24.0404	1534.98
-5%	1.425	20.2697	23.8839	1509.62
0%	1.5	20.1567	23.7439	1487.61
5%	1.575	20.0551	23.6179	1467.68
10%	1.65	19.9632	23.504	1449.56
15%	1.725	19.8797	23.4005	1433.01
20%	1.8	19.8034	23.306	1417.83

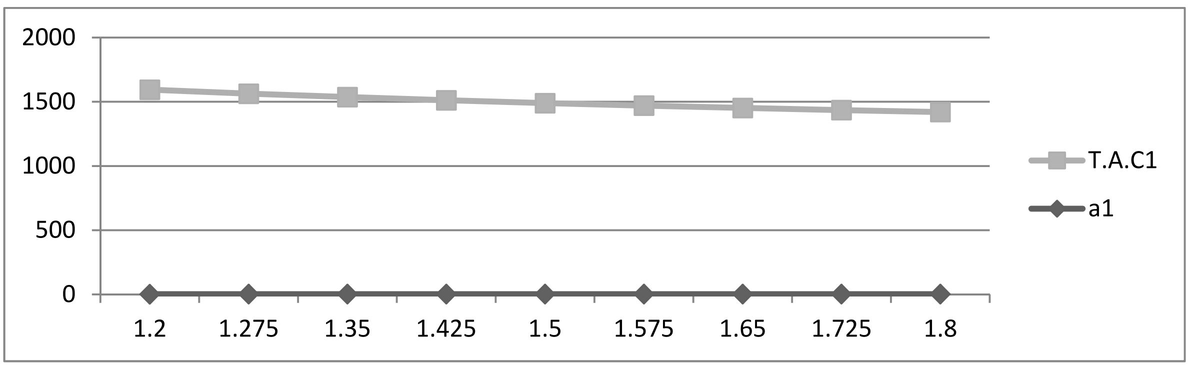

Figure 10. Variation in T.A.C1 with the variation in a1

5.1 Observations

Table 2 shows the variation in T.A.C. with the variation in deterioration parameter ‘θ’. From this table it is observed that with the increment in deterioration rate the T.A.C. of the system increases.

1. From table 3 it is observed that an increment in parameter 'b' results a decrease in T.A.C. of the system.
2. Table 4 lists the variation in demand parameter (β_1). It is observed from this table that as the value of 'β_1' increases, the T.A.C. of the system shows the reverse effect.
3. Table 5 and table 6 show the variation in parameter 'γ_i' and in demand parameter 'α_i' and from these it is observed that an increment in both the parameters result an increment in T.A.C.
4. Table 7and table 8 show the variation in T.A.C. with the changes in selling price 's_1' and production parameter 'a' respectively. It is observed that an increment in selling price and production parameter result a decline in T.A.C.

6. Conclusion

In this paper we have presented an integrated production inventory model for multi items. This is a closed loop supply chain, introduced with the production of new items and remanufacturing of collected and returned items with deterioration. The demand rate is taken as a function of selling price which shows a very realistic phenomenon. A numerical example is shown to illustrate the model. The model is optimized and the convexity of the model is shown. A sensitivity analysis is also performed to check the stability of the model. For future scope the model can be extended for stochastic demand rate and with learning and forgetting effects for production and manufacturing.

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