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A Heuristic and Exact Method: Integrated Aircraft Routing and Crew Pairing Problem

Nurul Farihan Mohamed[1], Zaitul Marlizawati Zainuddin[1,2], Said Salhi[3], Nurul Huda Mohamed[4] & Nurul Akmal Mohamed[4]

[1] Department of Mathematical Sciences, Faculty of Science ,81310 Universiti Teknologi Malaysia, Johor Bahru, Johor, Malaysia

[2] UTM Centre for Industrial and Applied Mathematics (UTM-CIAM), 81310 Universiti Teknologi Malaysia, Johor Bahru, Johor, Malaysia

[3] Centre for Logistics and Heuristic Optimisation, Kent Business School, University of Kent, Canterbury CT2 7PE, UK

[4] Mathematics Department, Faculty of Science & Mathematics, 35900 Universiti Pendidikan Sultan Idris, Proton City, Tanjung Malim, Perak, Malaysia

Correspondence: Nurul Farihan Mohamed, Department of Mathematical Sciences, Faculty of Science, 81310 Universiti Teknologi Malaysia, Johor Bahru, Johor, Malaysia. Tel: 6017-7143-610. E-mail: farry89@gmail.com

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Abstract

In airline operations planning, there are four problems which are schedule design, fleet assignment, aircraft routing and crew pairing problem. Those problems are sequentially and interdependent. Aircraft routing and crew pairing problem are hard to solve and normally crew pairing problem dependent to the aircraft routing problem which gives the suboptimal solutions. As minimizing the costs is important in the airline system, so in order to tackle suboptimal solutions, aircraft routing problem and crew pairing problem are being integrated in one model. For solving the integrated model, the feasible aircraft routes and crew pairs are required. Because of that, a method is being proposed in this work for generating the feasible aircraft routes and crew pairs which is the constructive heuristic method. By using the generic aircraft routes and crew pairs, the integrated model then being solve by two approaches. The first approach is the exact method called the integer linear programming (ILP) while the second approach is from the heuristic method called particle swarm optimization. Encouraging results are encountered by testing on four types of aircrafts for one week flight cycle from local flights in Malaysia.

Keywords: aircraft routing problem, crew pairing problem, constructive heuristic method, integer linear programming, particle swarm optimization

1. Introduction

The aim of solving aircraft routing problem is to find the minimal costs that generates the sequence of routes for flight legs in the schedules so that each flight leg is covered exactly once while satisfy the maintenance checks. While the aim of crew pairing problem is to find the minimal costs that assigns the crew for each flight leg exactly once while fulfil some rules. Normally, the crew pairing problem is solved based on the given aircraft routes but sometimes, the results are suboptimal. Correspondingly, the integrated model of aircraft routing and crew pairing problem is being proposed for encounter the sub optimality of results.

Since the aircraft routing and crew problem have been widely known for academia and researchers, there are many works that related to individually problems and also the integrated model of those problems. For the individually problem, those works can be discovered in the (Gopalakrishnan & Johnson, 2005; Muter, Birbil, Bulbul, Sahin, & Yenigun, 2013; Saddoune, Desaulniers, Elhallaoui, & Soumis, 2011; Schaefer, Johnson, Kleywegt, & Nemhauser, 2005; Yen & Birge, 2006; Shebalov & Klabjan, 2006). While for solving the integrated aircraft routing and crew pairing problem, Benders decomposition method has been used extensively. The first work that integrates aircraft routing and crew pairing is Cordeau, Stojkovic, Soumis and Desrosiers (2001). They solve the integrated model by using Benders decomposition method as they used aircraft routing problem as the master problem while crew pairing problem as the sub problem. In order to solve the integrated problem, they used the branch and bound method for getting the integer solutions.

As for Klabjan, Johnson, Nemhauser, Gelman and Ramaswamy (2002), they presented time window and plane count constraints in the problems. Differently with Cordeau et al. (2001) and Klabjan et al. (2002) used crew pairing problem as master problem and aircraft routing problem as sub problem. In Cohn and Barnhart (2003), they introduced aircraft routing and crew pairing problem that do not have any short connection. Followed by this, Mercier, Cordeau and Soumis (2005) proposed the method by using Benders decomposition method and Pareto optimal cut that can be used to discover the speed of convergence of Benders decomposition method. The other work is (Papadakos, 2009) that integrates fleet assignment, aircraft routing and crew scheduling in one model then solved it by using Benders decomposition method and column generation. Latterly, an iterative approach was proposed by Weide, Ryan and Ehrgott (2010) in solving integrated aircraft routing and crew scheduling problem.

The integration of aircraft routing, crew pairing and flight retiming problem has been proposed in (Mercier & Soumis, 2007) and solve it by using Benders decomposition method and the main contribution of this work is the introduction of time windows that can be vary for departure time of flight legs in order to decrease the delay flights. The most recent work for this field is by Dunbar, Froyland and Wu (2014) which they introduced the heuristic approach in two algorithms that had been used in solving the integrated aircraft routing, crew pairing and flight retiming problem.

In our study, the contributions are (i) a new formulation based on the constructive heuristic method in yielding aircraft routes and crew pairs, (ii) an ILP formulation for the integrated aircraft routing and crew pairing problem, (iii) a new approach based on heuristic method for solving the integrated model which is the particle swarm optimization and (iv) the experimental results of integrated model in discovering the skilfulness of the proposed methods. Our work is differ from past researches as there is no work that has been used heuristic method, particle swarm optimization for solving the integrated model of aircraft routing and crew pairing problem.

A description of the problem is provided in the next section with the details of constructive heuristic method in generating aircraft routes and crew pairs. This is followed by the formulation of integrated model which are ILP formulations and also particle swarm optimization approach. Latter section is the experimental results and the discussions of our discovering.

2. Problem Descriptions

In this section, an outline of aircraft routing problem and crew pairing problem are being discussed followed by the explanation of the constructive heuristic method for generating aircraft routes and crew pairs. Then, the model formulations of the integrated aircraft routing and crew pairing problem is presented. The problems solved in this work are for one week schedule flight legs and only involved the domestic flights.

2.1 Aircraft Routing Problem

The objective of aircraft routing problem is to find a solution with minimal costs that determine the sequence of flight legs to be flown for each type of aircraft and while assured that covered each flight once only.

Let $N^A = (D^A, R^A)$ be a network of aircraft routes where D^A is a set of nodes while R^A is a set of arcs. A set of flight legs as F described for each type of aircraft type. Each node $i \in D^A$ stands for the departure time or the arrival time of a flight leg $f_i \in F$. The maintenance stations for aircraft is describe as $m \in M$ and m as the starting stations for the aircrafts. The source and sink nodes are described as p_m^A and q_m^A which are the starting and the ending of a route.

2.2 Crew Pairing Problem

The aim of crew pairing problem is to find a set of pairings for each flight legs which ensure that each flight is assign to one crew only while minimizing the costs involved. The definition of a pairing is a sequence of flight legs that starts and ends at the same crew base where crew is stationed (Mercier et al., 2005).

Let $N^C = (D^C, R^C)$ be a network of crew pairs where D^C is a set of nodes while R^C is a set of arcs. Each node $i \in D^C$ represents the departure time of the arrival time of a flight leg $f_i \in F$. The crew base for crews is describe as B. The source and sink nodes of a pairing for each crew base $b \in B$ are p_b^C and q_b^C.

2.3 Constructive Heuristic Method for Feasible Aircraft Routes and Crew Pairs

In this work, the constructive heuristic method had been used as the approach to generate aircraft routes and crew pairs. A brief explanation of the algorithm for individually aircraft routes and crew pairs is provided in Figure 1 below.

```
Phase 1
Step 1: Set the number of aircraft route $g$, the number of flight legs $k$, the flight leg number as in the flight schedule $f = 1,2,3,...,n$, the departure node of flight leg $f$, $p = 1,2,3,...,t$, the arrival node of flight leg $f$, $q = t+1, t+2, t+3,...,t+u$, the maintenance station $m$, the departure time of flight leg $f$ as $v$, the arrival time of flight leg $f$ as $w$, the departure station of flight leg $f$ as $r$, the arrival station of flight leg $f$ as $s$.
Step 2: Choose a maintenance station, $m$, $g = 1, k = 0$.
Phase 2
Step 1: Choose a flight leg $f$ that has a departure station at the maintenance station $m$.
Step 2: Add it to the current route, $k = k+1$. Delete $f$ from the flight schedule.
Step 3: Choose a flight leg $f$ that has these criteria:
    i.   The $r$ of $k$ is the same with $s$ of $k-1$.
    ii.  The $v$ of $k$ minus the $w$ of $k-1$ is bigger or equal to 20 minutes.
    iii. The arrival station of $f$ equals to $m$, add the pairing into the current route. Retrieve all flight legs $f$ except for flight leg $f$ in Step 1, Phase 2 into the list. $g = g+1$.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 2 until there is no more route incurred.
Phase 3
Step 1: Use $f$ in the Step 1 in Phase 2, go to Step 2 in Phase 2.
Step 2: Choose a flight leg $f$ that has these criteria:
    i.   The $r$ of $k$ is the same with $s$ of $k-1$.
    ii.  The $v$ of $k$ minus the $w$ of $k-1$ is bigger or equal to 20 minutes.
    iii. Add it to the current route, $k = k+1$. Delete $f$ from the flight schedule.
Step 3: Choose a flight leg $f$ that has these criteria:
    i.   The $r$ of $k$ is the same with $s$ of $k-1$.
    ii.  The $v$ of $k$ minus the $w$ of $k-1$ is bigger or equal to 20 minutes.
    iii. If the arrival station of $f$ equals to $m$, stop. Add the flight leg into the current route. Retrieve all the flight legs $f$ into the list. $g = g+1$.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 3 until there is no more route incurred.
Phase 4
Step 1: Choose another flight leg $f$ that has a departure station at the home base $m$ but omit $f$ that had been used. Go to Phase 2 and 3. Stop when all $f$ that has a departure station at the home base $m$ is utilized.
```

Figure 1. The steps in producing aircraft routes

```
Phase 1
Step 1: Set the number of crew pair $h$, the number of flight legs $k$, the flight leg number defined in the flight schedule $f = 1,2,3,...,n$, the departure node of flight leg $f$, $p = 1,2,3,...,t$, the arrival node of flight leg $f$, $q = t+1, t+2, t+3,...,t+u$, the initial crew base $b$, the departure time of flight leg $f$ as $v$, the arrival time of flight leg $f$ as $w$, the departure station of flight leg $f$ as $r$, the arrival station of flight leg $f$ as $s$, length of duty period, $dp$, the sit time, $st$, the initial crew base $x$ and the total amount time of duty period and sit time, $dpst$.
Step 2: $h = 1, k = 0, dp = 0, st = 0$. Choose an initial crew base $b$.
Phase 2
Step 1: Choose a flight leg $f$ that has a departure station at the crew base $b$.
Step 2: Add it to the current pairing, $k = k+1$ and $dp = st + ft$. Record the duty period of $f$, amount of sit time of $f$, number of flight leg of $f$ and $dpst$. Erase $f$ from the flight schedule.
Step 3: If $dpst \le 480$, choose a flight leg $f$ that has these criteria:
    i.   The $r$ of $k$ is the same with $s$ of $k-1$.
    ii.  The $v$ of $k$ minus the $w$ of $k-1$ is bigger or equal to 20 minutes.
    iii. The arrival station of $f$ equals to $b$, add pairing into the current pairing. Recover all the flight
```

legs f except for flight leg f in Step 1 of Phase 2 into the list. $h = h + 1$. Record the duty period of f, amount of sit time of f, $dpst$ in minutes and number of flight leg f.

Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 2 until there is no more pair incurred

Phase 3

Step 1: Use f in the Step 1 in Phase 2, go to Step 2 in Phase 2.

Step 2: If $dpst \le 480$, choose a flight leg f that has these criteria:

i. The r of k is the same with s of $k-1$.

ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.

iii. Add it to the current pairing, $k = k + 1$. Delete f from the flight schedule. Record the duty period of f, amount of sit time of f, $dpst$ in minutes and number of flight leg of f.

Step 3: If $dpst \le 480$, choose a flight leg f that has these criteria:

i. The r of k is the same with s of $k-1$.

ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.

iii. If the arrival station of k equals to b, stop. Add the pairing into the current pairing. Retrieve all of the flight legs f except for flight leg f in Step 1 of Phase 2 into the list. $h = h + 1$.

Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 3 until there is no more pair incurred

Phase 4

Step 1: Choose other flight leg f that has a departure station at the home base b but exclude f that had been used. Go to Phase 2 and 3. Stop when all the f that has a departure station at the crew base b is used and the $dpst > 480$.

Figure 2. The steps in producing crew pairs

2.5 Model Formulations

In this section, the description of the model of integrated aircraft routing and crew pairing problem has been provided. A restricted connection is occurred when the time between two sequential flight legs is between 60 minutes to 90 minutes while a short connection happened when the time between those two flight legs is between 20 minutes and 59 minutes. The overview of the notations used for integrated model is as in Table 1.

Table 1. The notation

Notation	Description
M	Set of maintenance stations
B	Set of crew bases
R	Set of two flight legs that has a restricted connection
S	Set of two flight legs that has a short connection
p_m^A	The source nodes for aircraft paths
q_m^A	The sink nodes for aircraft paths
p_b^C	The source nodes for crew paths
q_b^C	The sink nodes for crew paths
α^m	Set of possible aircraft routes from the source node p_m^A to a sink node q_m^A in N_m^A
α^b	Set of possible crew pairs from the source node p_b^C to the sink node q_b^C in N_b^A
w_μ^f	Equal to 1 if leg f belongs to path μ, and 0 otherwise
η_μ	Binary constant that represents the flow on the crew path μ
ξ_μ	Binary constants that represents the flow on the aircraft path μ
R_{ij}	Binary constant will be 1 if connection $(f_i, f_j) \in R$ is operated by the same crew but not same aircraft, 0 otherwise
c_μ	The cost of using the path μ
n_μ^{ij}	Equal to 1 if leg i and j are operated sequentially in path μ, and 0 otherwise
l_μ	The number of required aircrafts in the path μ

ω^A	The number of available aircrafts
ω^B	The number of duty periods allowed in a crew pairing
ω^C	The number of short connections allowed in one path
v_μ	The number of duties in path μ
s_μ	The number of short connections in path μ
z_{ij}	Penalty cost associated with $(f_i, f_j) \in R$

2.5.1 The Integrated Model

The integrated model of aircraft routing and crew pairing problem that we refer as (N1) is as follows:

Minimize

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu \eta_\mu + \sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu \xi_\mu + \sum_{(f_i,f_j)\in R} z_{ij} R_{ij} \tag{1}$$

Subject to

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f \xi_\mu = 1, (f \in F) \tag{2}$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f \eta_\mu = 1, (f \in F) \tag{3}$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu \xi_\mu \le \omega^A \tag{4}$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu \xi_\mu \le \omega^C \tag{5}$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu \eta_\mu \le \omega^B \tag{6}$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij} \xi_\mu \le 0, ((f_i, f_j) \in S) \tag{7}$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij} \xi_\mu - R_{ij} \le 0, ((f_i, f_j) \in R) \tag{8}$$

$$R_{ij} \in \{0,1\}, ((f_i, f_j) \in R) \tag{9}$$

$$\xi_\mu \in \{0,1\}, (m \in M; \mu \in \alpha^m) \tag{10}$$

$$\eta_\mu \in \{0,1\}, (b \in B; \mu \in \alpha^b). \tag{11}$$

The objective function (1) is to find the minimal costs of aircraft routing, crew pairing and penalty costs. While for the equation (2) and (3) is to assure that each flight leg is covered with one aircraft and one crew pair only. Equation (4) is to ensure that all flight legs operated at one time do not exceed the available aircraft. As for equation (5) is to ensure that the short connection in the path μ does not exceed ω^C. Equation (6) is to guarantee that the number of duty allowed is not being exceeded by each crew pairing. Equation (7) assure that each crew does not change the aircraft when short connection happened. Penalty costs is imposed when the second flight use the same crew but not the same aircraft that ensured by equation (8). Next, equation (9), (10) and (11) express the binary decision variables.

2.5.2 The Particle Swarm Optimization

The proposed particle swarm optimization which we refer as (N2) for integrated aircraft routing and crew pairing problem is based on the chiastic randomness and opposition based learning. In order to generate the chiastic numbers, the logistic equation is employed. There are two algorithms for particle swarm optimization for integrated model which are the step 1 in Algorithm (1) is identified in Algorithm (2). Figure 3 show the outline of the particle swarm optimization for integrated model.

```
Input: The parameters of the algorithm that consists of aircraft routes and crew pairs and the objective function.
Output: The most beneficial solution found.
Step 1: Initialize population of trial solutions, the global best, and local best using Algorithm (2).
Step 2: Calculate the velocity using equation (12) and (13).
Step 3: Calculate the next step by equation (14).
Step 4: Evaluate the current solution; update the global best, and local best.
Step 5: If the criterion of the algorithm has been fulfilled stop, otherwise go to step 2.
```

Figure 3. Outline of particle swarm optimization (Algorithm 1)

Figure 4 below describes the step 1 of Algorithm (1) which is notate as Algorithm (2). In Algorithm (2), the number of population is notate as N while the dimension of the problem is set down as D. the objective function is used to calculate the representing fitness function t for each trial function. In the other hand, the function v is computed based on the violation from constraints of the problem. Then, the evaluation of the solutions is done by using the values from those functions and also applying the Deb's rule (Deb, 2000). In the process, the feasible solutions with lower fitness are preferable. Between feasible solution and the infeasible solution, feasible solution is preferred while if both of the solutions obtained are infeasible, then the solution with the lower violation is chosen.

```
Input: The parameters of the algorithm and the objective function.
Output: A population of trial solutions, the global best, and local best.
Step 1: Initialize a random uniform number $r \in [0,1]-\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\}$ .
Step 2: For $i=1$ to $N/2$
            For $j=1$ to $D$
                $r=4r(1-\mathrm{r})$, and $x_{ij}=round(r)$
        End for
    End for
Step 3: For $i=N/2$ to $N$
            For $j=1$ to $D$
                $x_{ij}=1-x_{ij}$
        End for
    End for
Step 4: Evaluate $x_i$ for $i=1,2,...,N$. Calculate the global best, and local best.
```

Figure 4. The initialization of the Algorithm 1 (Algorithm 2)

In the next move, the velocity in Algorithm (1) is simply is set to be zero. By using the parameters α and β, the velocity is updated iteratively. Firstly, a uniform random number r is generated and if $r<\alpha$, then velocity v is updated by using the equation (12) as below.

$$v_{ij}=v_{ij}+(x_{gbj}-x_{ij}) \tag{12}$$

where $i=1,2,...,N$ and $j=1,2,...,D$ and x_{gb} is the global best solution. Differently, if $r<\beta$ then the velocity v is modified by the equation (13) below.

$$v_{ij}=v_{ij}+(x_{lbj}-x_{ij}) \tag{13}$$

where x_{lb} is the local best solution in the current population. If there is no conditions above are satisfied, then the velocity v is set to zero. Afterwards, the current population is updated by using equation (14).

$$x_{ij}=x_{ij}+v_{ij} \tag{14}$$

3. Experimental Results

The approach by ILP was solved on an Intel Core Duo processor running at 2.10 GHz using modelling languages, Microsoft Visual Studio C++ interface with ILOG CPLEX Callable Library 2.4 while the approach by particle swarm optimization was solved by using MATLAB.

3.1 Data Sets

The data sets involve in this work is domestic flight legs for four type aircrafts in one week time by an airline in Malaysia. Table 2 displays the number of feasible aircraft routes and crew pairs obtained by using constructive heuristic method.

Table 2. Number of feasible aircraft routes and crew pairs

Aircraft Type	Number of aircraft routes	Number of crew pairs
EQV	77	70
738	112	112
AT7	882	672
734	1099	854

3.2 Solution Approaches

By using the generated feasible aircraft routes and crew pairs, the integrated model then solved by using two approaches which are ILP (N1) and particle swarm optimization (N2). The experimental results are given in the Table 3 which incorporates of the costs and computational time used in solving the integrated model by using solution approaches N1 and N2 on four instances. The costs involved are defined in Ringgit Malaysia (RM).

Table 3. Summary experimental results of two proposed approaches (N1 & N2)

Aircraft type	Model 1 (N1)		Model 2 (N2)	
(#legs, #nodes)	Costs (RM)	Time (secs)	Costs (RM)	Time (secs)
EQV (126, 252)	147746	0.00	148866	0.5
738 (70, 140)	149719	1.12	149908	0.6
AT7 (364, 728)	328030	720.6	329878	316.1
734 (546, 1092)	817604	1651.8	821685	925.6

The comparison between results from both approaches had been made. From the Table 3, it showed that the costs increase linearly with the number of flight legs involved. The higher number of flight legs, the higher number of feasible aircraft routes and crew pairs are generated. From the results, the computational time grows as the feasible aircraft routes and crew pairs are increase. In the view of computational time, N2 is found more advantageous compared to computational time used by N1 as shown in Figure 5 below.

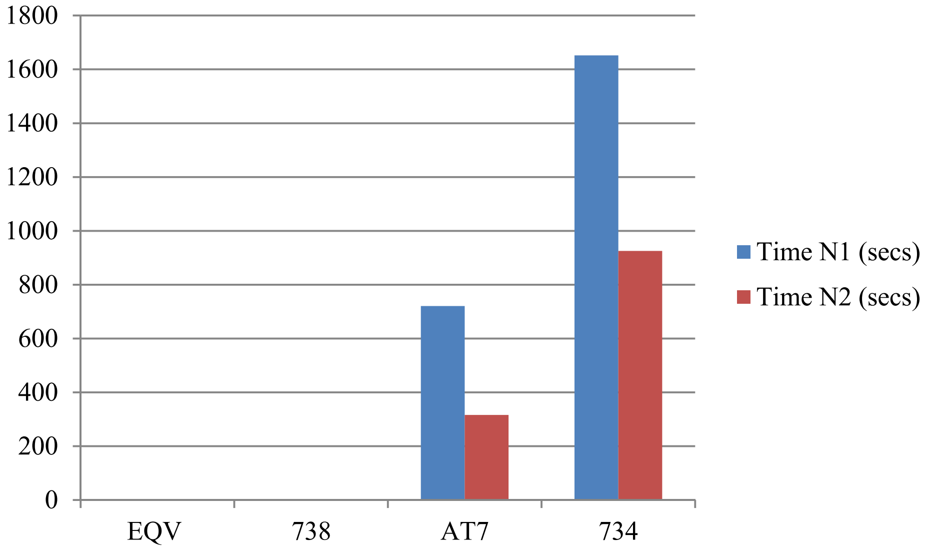

Figure 5. Comparison computational time for N1 and N2 in seconds

While in the aspect of computational costs, results from N1 are found to be lower rather than the results from N2. The comparison of computational costs for N1 and N2 are shown in the Figure 6 below.

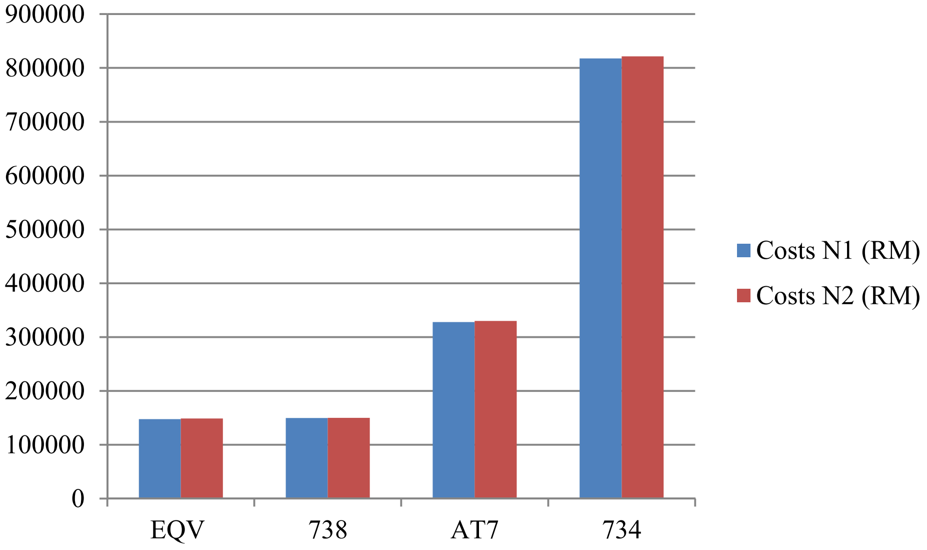

Figure 6. Comparison computational costs for N1 and N2 in Ringgit Malaysia (RM)

From the results obtained, we can conclude that N2 is more encourage as the computational time from N2 is lower than N1. Although the computational costs involved in N1 is higher that N2, but the costs between those approaches is only slightly different.

4. Conclusions

The results incurred from this work are encouraging enough in solving the complex integrated aircraft routing and crew pairing problem. One of the possible future works is to examine the much bigger data of flight legs for the larger airlines. Although the problem could be inviting competition but it would be useful for the involved airlines.

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