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New Approach of Multistage Model in Supply Chain with Game Theory

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Abstract

This paper researches the relationships between seller and buyer with regard to game theory. The research continues by assuming an indirectly managing by an Intermediation. The intermediation is considered as third party who tried to decrease the distance between seller and buyer willing. In our proposed methodology, Bi-level programming is used for modeling the decision making between seller and buyer in supply chain, and then extend the model in Multi-level decision making. In the presented solution, the third part offers a price to each of the seller and buyer individually and supposed as leader. Final answers of described algorithms are Nash equilibrium point for supply chain. The object of seller and buyer are considered as a follower in each stage. Profits maximization for sellers and buyer are calculated by considering their own constraints.

Keywords: Intermediation, Multi-level programming, Nash equilibrium, seller- buyer, supply chain

1. Introduction

In this section first the basic concepts of game theory and bi (multi)-level programming is mentioned. Then the brief considerations of related researches are proposed.

1.1 Basic Concepts

In this part basic concept of game theory with regard to Nash equilibrium and bi-level programming and multi-level programming are explained respectively.

1.1.1 Game Theory

Game theory is science of studying decision making behavior of multi-person or multi-firm. There are three significant mathematical structures for model in the game theory problems. The first one is strategic form, the second one is extensive form and the third one is coalitional form (Chiang et al, 1994).

These three types are different in amount of detail which is use to model the game theory problem. In this paper strategic approach is used for modeling the seller buyer supply chain in presence of intermediary. Hence, strategic structure is explained here. Strategic structure is base on these basics:

a. Set of players: $i \in \{1, \dots, n\}$, note that n is a finite number.

b. Pure strategy space: S_i and contains s_{ij} .

s_{ij} : The available individual strategies for i th player.

c. Pay off function: $u_i: S \to R(\text{Real set})$ for each player.

Cartesian set of all sets is: $S=S_1\times S_2\times\ldots\times S_n$.

Nash equilibrium is the most significant and applied solution finding method for game theory problems. Nash equilibrium for players is a point that each of the players will decrease its profit or payoff if it faced with any deviates from the strategy which results to Nash equilibrium point. This definition is right when the other players' strategy is fixed and they do not change the existing strategies. Consequently, Nash equilibrium is given the best strategies for all players if all of them have no deviate from the strategies which results to the Nash equilibrium point. It could be single or several Nash equilibrium point. Definition 1 defines the Nash equilibrium point.

Definition 1: suppose that there is exist n player in strategic structure of game. By considering $s_{-i}^* = (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$, the profile strategies(s_1*, . . . , s_n*) are a Nash equilibrium if and only if:(Colson et al., 2005)

$\forall i, s_i \in S_i, s_i \neq s_i^*: u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$

1.2 Bi-Level Programming

A bi-level programming problem is resulted in two successful decision makers. Also it is decentralized decision making. In bi-level decision making(BLP) there is two decision maker which both interested in optimizing their own objective function(s) with no consideration on the others goals. But the decision of them has effects to the others as well the space of decision.

General formulation of a bi-level programming problem (BLPP) is as equ (Colson, 2005; Roghanian, 2007).

$$\min_x F(x,y)$$
$$\text{s.t.}\, G(x,y) \leq 0,$$
$$\min_y f(x,y)$$
$$\text{st.}\, g(x,y) \leq 0,$$

Where $x \in R^{n1}$ and $y \in R^{n2}$. The variables of bi-level programming are divided into two categorize. First the upper-level variables: $x \in R^{n1}$ and second one is the lower-level $y \in R^{n2}$. Also the functions are categorized in to two classes: $F: R^{n1} \times R^{n2} \to R$ is the upper-level objective functions and lower-level constraints $f: R^{n1} \times R^{n2} \to R$ is the lower-level objective function. The vector-valued functions of the upper-level is $G: R^{n1} \times R^{n2} \to R^{m1}$and $g: R^{n1} \times R^{n2} \to R^{m1}$ is lower-level constraints. Both the objective functions and constraints could be linear, fractional, etc (Roghanian 2007).

Each level of bi-level programming can contain a bi-level programming. The result is a multi-level programming.

2. Problem Formulation

In this section objective function, notation and formulation of seller buyer model are explained. We extend Esmaeili et al (2009) model and some of its notation and parts of formulation is used. In that model they consider only seller and buyer, but we propose a model with regarding to intermediary role in supply chain and use the Constatino et al (2009) models.

Objective function of seller is calculated as bellow:

Seller's profit = Sales revenue - Production cost - Setup cost.

And also the objective function of buyer is calculated as bellow:

Buyer's profit = Sales revenue - Purchase cost - Market cost - Ordering cost -Holding cost.

Therefore, our function is formulated as follow:

2.1 Object Functions

π:total profit

π_s: the seller profit.

π_b: the buyer profit.

2.2 Decision Variables

Vs: the price offered by the seller to the buyer. ($/unit)

Vs': the price offered by the intermediary to the seller.

Vb: the price offered by the buyer to the seller.

Vb': the price offered by the intermediary to the buyer.

Q: lot size (units) determined by the seller.

P: selling price charged by the buyer. ($/unit)

M: marketing expenditure incurred by the buyer. ($/unit) (Esmaeili et al, 2009).

T: transaction generates costs

Ys: the seller value of equilibrium that the buyer accepts when offered.

Yb: the buyer value of equilibrium that the seller accepts when offered.

Us: the exit value of the seller.

Ub: the exit value of the buyer.

λ: coefficient of buyer profit from whole profit, and $\lambda \in [0,1]$.

ρ: the probability of continuing and is the break down probability

2.3 Input Parameters

k: scaling constant for demand function. (k>0)

u: scaling constant for production function. (u≥0)

i: percent inventory holding cost per unit per year.

α: price elasticity of demand function.(α≥1)

β : marketing expenditure elasticity of demand. (2<α-β, 0<β<1,μ(α-β)<1)

A_b: buyer's ordering cost. ($/order)

A_S: seller's setup (ordering cost) ($/setup)

C_S: seller's production cost including purchasing cost. ($/unit)

D(P,M):total annual demand; for notational simplicity we let. D=D(P,M)

2.4 Seller-buyer problem

The seller problem is formulated as follow:

$$Max\pi s = Vs.D - C_S.D - A_S.\frac{D}{Q}$$
$$st:$$
$$Vs = R.(C_S + A_S.Q^{-1}) \tag{1}$$

The buyer's objective is determining the selling price (P) and marketing expenditure (M) for maximizing her owns profit. These two mentioned variables are important in defining the total market demand and so on the lot size of seller, Esmaeili et al, (2009). Esmaeili et al (2009) consider the model proposed by Abad (1988) with adding the marketing expenditure. Here we consider their model by adding the third party to model. Hence, the buyer's annual profit function is as follow:

$$Max\pi b = P.D - M.D - A_b.\frac{D}{Q} - 0.5i.Vb.D - Vb.D$$
$$st:$$
$$P = \frac{\alpha(Vb + A_b Q^{-1})}{\alpha - \beta - 1}$$
$$M = \frac{\beta(Vb + A_b Q^{-1})}{\alpha - \beta - 1}$$
$$Q = \sqrt{\frac{2A_b D}{iVb}}$$
$$Q \geq \frac{A_b}{Vb}\left(\frac{\alpha - \beta - 2}{2}\right) \tag{2}$$

2.5 Bilateral Model

At first we consider a simple structure of supply chain with one seller and one buyer for illustrating the whole procedure of model such as Costantino et al (2009). Seller and buyer both are willing to obtain more benefit from their mutual contract. This mentioned contract is about sailing an especial production which is produced by seller and is wholesaled to the buyer and in next stage it would be sold to the final customer. This contract would divide the extra profit made from cooperation of supply chain member such as seller and buyer in best mode.

Generally, seller and buyer interested in an individual price setting. Suppose that seller selects "s" for selling its production and buyer selects "b". In this stage for simplifying the model it is assumed that both "Vs" and "Vb" are known to seller and buyer. It means that the information is perfect. Fig.1

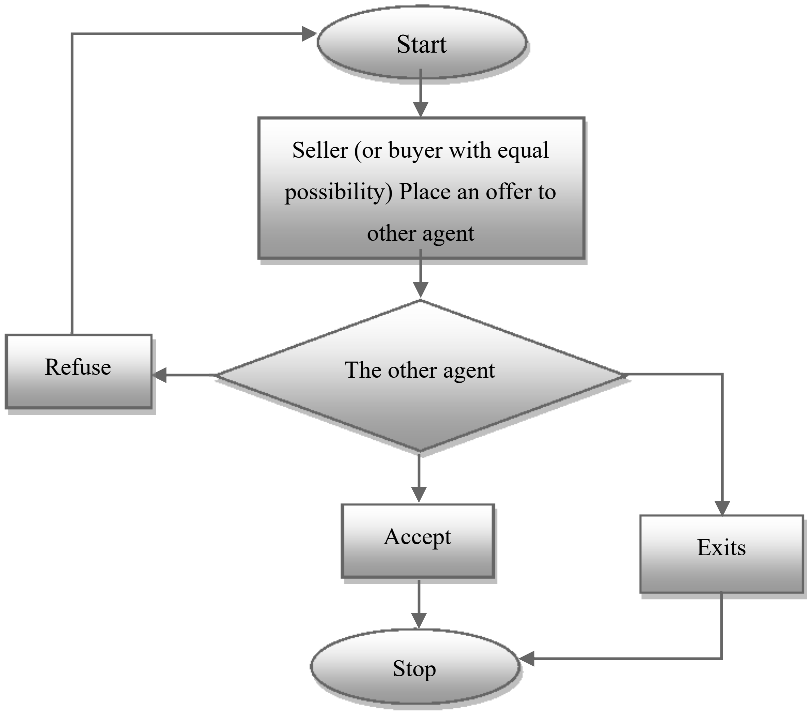

Figure 1. Seller-buyer bargaining process

If the supply chain members agreed to use help of third party for nearing the other define amount, another negotiation would be made. The intermediary or third party offers another price to each member. The intermediary ask seller "Vs'" and offered buyer "Vb'". Notice that, $(Vb' - Vs') \geq 0$.

By considering the transaction cost equal to T and also 1-λ as the coefficient of neat profit (π) for seller and λ for the buyer and also 0≤λ ≤1 we have:

$$\pi b = \lambda \pi$$

$$\pi s = (1 - \lambda)\pi$$

πb is the buyer profit and πs is the seller's one. Also T is calculated as follow:

$$\pi = Vb - Vs - T > 0 \quad \text{and,}$$

$$\pi = \pi b + \pi s$$

The role of third party is to define Vb' and Vs' as the supply chain members prefer to accept the intermediaries' offers. So these two follow equation should be exist:

$$Vb - Vb' = \lambda \pi$$

$$Vs' - Vs = (1 - \lambda)\pi$$

Anyway, it has to reduce transaction costs, improving the efficiency (e.g. Q≤T), otherwise its income, equal to spread, is negative as

$$Vb' - \mathrm{Vs}' = \mathrm{T}$$

A mutual game would be made with sequential choices between seller and buyer. In each round, both the seller and buyer have the choice to begin the game. At each round, the offered of each agent could be accepted, rejected or result to exist the game by the other agent. Notice that if one rejects the others offer, it could request a counteroffer. The game continues while seller and buyer prefer to cooperate with each other. If they reaches the agreement or one or both of them wants to exit because the transaction is not profitable for them, the game would be end its repeats.

The buyer wouldn't accept the offers which are worse than the predefined value. This predefined value is a limitation for not going below that. Also defining this minimum wage is showing the power of the buyer.

Ertogral and Wu (2001) explained an analytical description for game theory that is showing the existence of unique perfect equilibrium in sub-games problem instead of Nash equilibrium strategy. Suppose these assumptions:

π: Transaction surplus

Us: The exit value for seller.

Ub: The exit value for buyer.

$1-\rho$: break down probability.

So the buyer and seller value of equilibrium, which the other party would accepts when one offered that is displayed by ys and yb respectively, are calculated as follow:

$$ys = \pi - \mathrm{Ub} - \frac{\rho^2}{2(2-\rho)}(\pi - Ub - Us) \tag{3}$$

$$yb = \pi - \mathrm{Us} - \frac{\rho^2}{2(2-\rho)}(\pi - Ub - Us) \tag{4}$$

The negotiation power of each payer and the games results is directly depends on two main concepts:

1. Their alternative of gaining profit outside the transaction.
2. Their ability for effecting on breakdown probability.

3. Intermediation

In this section the especial conditions for accepting the existence of intermediary in the proposed game model is explained. This conditions is produced when the players gain an advantageous not lower than when they negotiate to each other directly, Costantino et al (2009).

3.1 Perfect Information

Intermediary is searching an opportunity for inducing and encouraging seller and buyer to accept the intervention of it. Hence, the intermediary should offer amounts that have advantageous for both seller and buyer to accept this kind of negotiation instead of direct one. Therefore, their profit must not be lower than the benefit from bilateral negotiation between seller and buyer.

Consequently, $\pi - ys$ and are $\pi - yb$ the lower expectation limitation of the buyer and seller respectively. So,

$$vb - vb' \geq \pi - ys$$

$$vs' - vs \geq \pi - yb$$

To economically justify its intervention, the intermediary has to play with transaction costs not to pass a limit M, through a price offer (vb' and vs')

So, the higher the generable spread is, in particular its difference with the limit M, the more probable is the existence on the market of a regulator agent.

By substituting upper Eqs. We have:

$$vb' - vs' \leq (vb - vs) - Ub - Us - \frac{\rho^2}{2(2-\rho)}(\pi - Ub - Us) \tag{5}$$

By adding Eqs.(1) and Eqs.(2) we have:

$$\begin{aligned} &Max\ \pi s = Vs.D - C_S.D - A_S \cdot \frac{D}{Q} \\ &st: \\ &Vs = R.(C_S + A_S.Q^{-1}) \end{aligned}$$

$$\begin{aligned} &Max\pi b = P.D - M.D - A_b \cdot \frac{D}{Q} - 0.5i.Vb.D - Vb.D \\ &st: \\ &P = \frac{\alpha Vb + A_b Q^{-1})}{\alpha - \beta - 1} \\ &M = \frac{\beta Vb + A_b Q^{-1})}{\alpha - \beta - 1} \\ &Q = \sqrt{\frac{2A_b D}{iVb}} \\ &Q \geq \frac{A_b}{Vb}(\frac{\alpha - \beta - 2}{2}) \end{aligned}$$

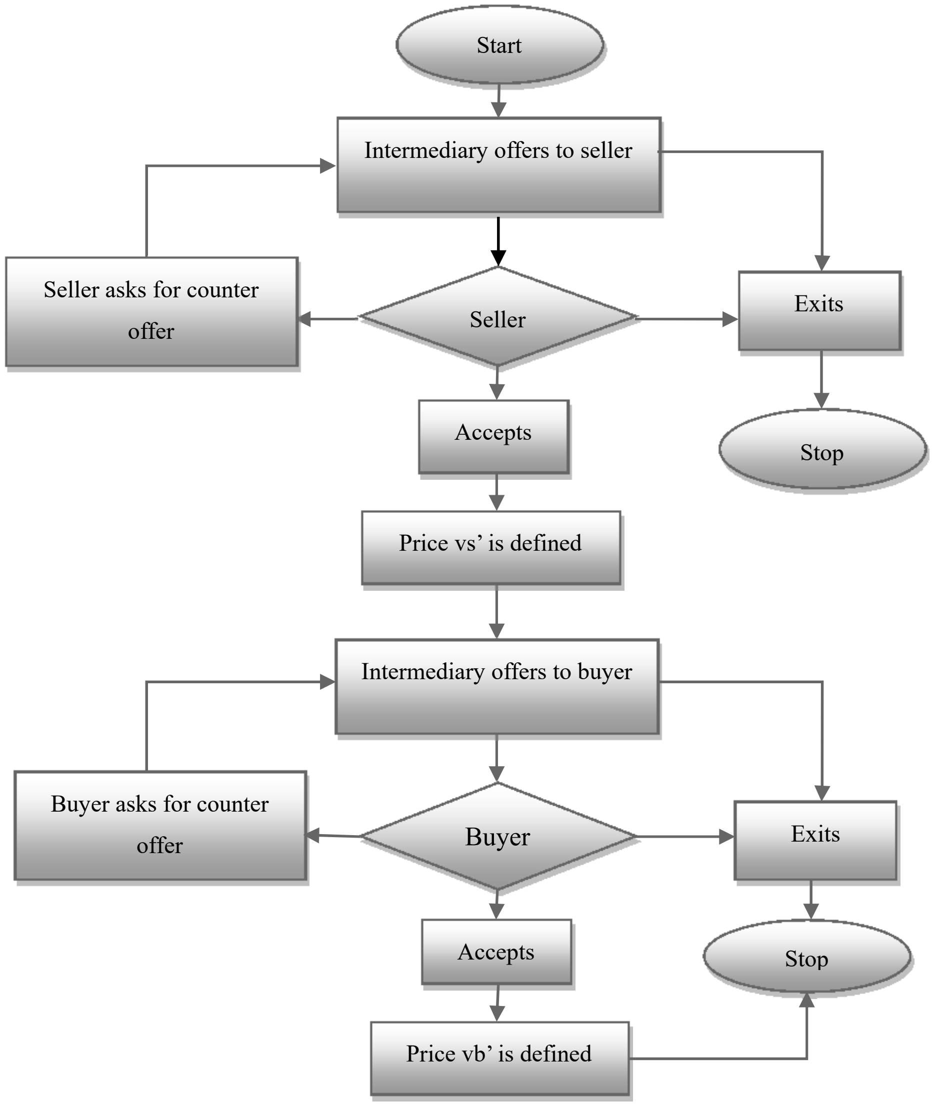

Figure 2. Intermediary and seller-buyer bargaining process

3.2 Imperfect Information

Suppose that in a negotiation with a seller and buyer and intermediary, exist two stages. The initial offer of each stage defines the best desired profit for each player. Fig.2. illustrate this concept. The game is contains these stages:

1. In first stage the intermediary start a direct transaction with the seller and vs'^{*} is defined. vs'^{*} is in relation to an opportunity cost $vs \in [vs_1, vs_2]$ not revealed. The presented game is a dynamic form with sequential choices. It would be continued until the parties reach the agreement by continued offering.
2. In second stage the similar game would be applied for the intermediary and the buyer. But in this stage they negotiate on vb'^{*} in relation to $\in [vb_1, vb_2]$. The sequence of offers tends to find a trade-off between the lower possible price for the customer and the maximum spread for the intermediary, Costantino et al (2009).

Assume that the customer displays its vb_1 as its best choices. If $vb_1$is higher than the presumable exit value vs_1, there would be an ability of reaching to agreement.
The introduction of the intermediary generates:

1. A set of possible offers to the seller in $[vs'_1, vs'_2]$ and $vs'_1 = vs_1$and $vs'_2 = vb_1 - T$.
2. A set of possible offers to the buyer in $[vb'_1, vb'_2]$ and $vb'_1 = vs^{*} + T$ and$vb'_2 = vb_2$.

According to the perceptions of intermediary and depends on $vs \in [vs_1, vs_2]$, any $vs' \in [vs'_1, vs'_2]$ represents an offer the supplier can accept, refuse or ask for a counter offer, with different probabilities of occurrence.

At the same time, depending on$vb \in [vb_1, vb_2]$, any $vb' \in [vb'_1, vb'_2]$ represents a price the customer can accept, refuse or ask for a counter offer.

4. **Conclusion**

This paper presented the implementation of an intermediation model in supply chains. We represent the bilateral bargaining model between seller and buyer where their relationships are indirectly managed by intermediator. Bi-level programming is used for modeling the decision making between seller and buyer in supply chain, and then we extend the model in Multi-level decision making. Then, the third part offers a price to each of the seller and buyer individually as leader. Seller and buyer are considered as a follower in each stage. Final answers are Nash equilibrium point for supply chain.

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