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Predicting Model of Traffic Volume Based on Grey-Markov

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Abstract

Grey-markov forecasting model of traffic volume was founded by applying the model of GM (1,1) and Markov random process theory. The model utilizes the advantages of Grey-markov GM (1,1) forecasting model and Markov random process in order to discover the developing and varying tendency of the forecasting data sequences of traffic volume. The analysis of an example indicates that the grey-markov model has good forecasting accuracy and excellent applicability in predicting traffic volume.

Keywords: Grey theory, Grey-markov model, Prediction of traffic volume

1. Introduction

Generally, the planning of a highway is designed on the basis of the traffic volume prediction. The so-called definition of traffic volume prediction is to study and calculate the inside increase and change of traffic, and to obtain a volume in terms of design years according to the variety of transportation capacity and the development of economy and society in the past, present and future etc. Although many learners have processed large quantity of researches for predicting the traffic volume, the result is still bad.The transportation engineering is a complicated system, which includes many factors, many structural layers and many targets. The traffic information contains the obviously layer complexity of structure, the fuzzy relation of construction, the variety of development and the indetermination of coefficients and data. Because of the influence of some artificial factors, unaffectedly environmental change and the restriction of the technique methods at present, it leads to the result that the statistic or forecast data embrace some errors, mistakes, scarcity or fakes. So the complicated system of traffic volume prediction is a representatively grey problem (Zhang and Luo, 2001).

The grey model has been applied in the traffic volume prediction, and primarily makes use of model GM (1,1) to perform the forecast (Wen et al.,2006; Xue and Zeng, 2006). Because the solution of model GM (1,1) is an exponential curve that is smooth, it doesn’t match with those data that are vibration sequences and its forecast accuracy is lower. The study object of markov transition model is a dynamic system which forecasts the future by analyzing the inside regulation of development in time to come, and it reflects the influence degree and laws which lies in the transition process of factors from one state to the other. The markov transition model is suitable for the solution to predict these stochastic data sequences that are steady, but in the realistic world these raw sequences are vibrating and changing in a certain variety trend. From the analysis above, we know that the model GM (1,1) and the markov model could be integrated with each other to forecast by their advantages. That is: model GM (1,1) can be used to forecast the change trend of data sequences, while the markov model can be used to decide the vibration regulation of their development, and both can be joined together to become a grey-markov forecast model. Since it makes full use of the old information given from these raw data and increases the forecast accuracy, the application of the grey-markov forecast model, which provides a new method to predict these greatly stochastic data sequences, has been improved further.

2. Establishment of the Mathematics Model

2.1 Model of GM(1,1)

The grey GM (1,1) model can make use of the discrete data series to establish a equation of grey continuous differential equation by adding these data from the first in Accumulating Generation Operator (AGO), and the equation can be solved to perform the forecast (Deng, 1990). Let $x^{(0)}(k)$ be a raw series, which is as follow:

$$X^{(0)}(k) = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}, k = 1, 2, \cdots, n \quad (1)$$

Let $x^{(0)}$ accumulated adding once, and the accumulated generating series is obtained:

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\} \quad (2)$$

where $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i), k=1,2,\cdots,n$

By differentiating $x^{(1)}$, a whitened differential equation is obtained

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=u \tag{3}$$

The whitened time-response of Eq. (3) is as follow (Deng 1990):

$$\hat{x}^{(1)}\left(k+1\right)=\left[x^{(0)}\left(1\right)-\frac{u}{a}\right]e^{-ak}+\frac{u}{a} \tag{4}$$

Let the solution $\hat{x}^{(1)}$ accumulate subtrating once, and the accumulated subtration series is obtained:

$$\hat{x}^{(0)}\left(k+1\right)=\hat{x}^{(1)}\left(k+1\right)-\hat{x}^{(1)}\left(k\right) \tag{5}$$

The curve of $\hat{x}^{(0)}$ reflects the vibration trend of the raw series. Finally we can adopt the method of Deng (Deng, 1990) to check the model accuracy.

2.2 Grey markov chain

Let $\{X_n, n\in T\}$ be a markov chain, where $m,n\in T(n\geq 1)$ and $i,j\in L$ (L is called status series, then the expressing (Sha, 1994)

$$p_{ij}^{(n)}=P\left(X_{m+n}=j\middle|X_m=i\right) \tag{6}$$

is called the *n* th step transition probability, and the matrix composed by $p_{ij}^{(n)}$ is called the *n*th step probability transition matrix of Markov chain, which is expressed as:

$$P^{(n)}=\left[p_{ij}^{(n)}\right]_{l\times l} \tag{7}$$

If the elements transition probabilities of markov chain are grey, it will be called a grey markov chain, and can be made up of a grey transition matrix (He and Bao, 1992). In the actual application, we know that it is difficult to make certain the values of transition probability for it lacks some information, but it is easy to have the information of grey zone $p_{ij}(\otimes)$ by studying the transition probability. When the transition matrix is a grey matrix, it is required that the elements of whitenization matrix $\tilde{P}\left(\otimes\right)=\left[\tilde{p}_{ij}\left(\otimes\right)\right]_{l\times l}$ is provided that (1) $\tilde{p}_{ij}\left(\otimes\right)\geq 0$, $i,j\in L$; (2) $\sum_{j=1}^{l}\tilde{p}_{ij}\left(\otimes\right)=1$, $i\in L$.

When the preliminary distribution of a markov limited chain is $P(0)=(p_1,p_2,\cdots,p_l)$, and the whitenization transition probability matrix is $\tilde{P}\left(\otimes\right)=\left[\tilde{p}_{ij}\left(\otimes\right)\right]_{l\times l}$, then we can get the next step distribution of the chain:

$$P(1)=P(0)\cdot\tilde{P}(\otimes) \tag{8}$$

The second step can be expressed as:

$$P\left(2\right)=P\left(1\right)\cdot\tilde{P}\left(\otimes\right)=P\left(0\right)\cdot\tilde{P}^2\left(\otimes\right) \tag{9}$$

The rest may be deduced by analogy, and the *n* th step distribution is shown as:

$$P(n)=P(0)\cdot\tilde{P}^n(\otimes) \tag{10}$$

From Eq. (10), it can be seen that we can easily forecast any future distribution of the system if we have already known the raw distribution and the grey transition probability matrix.

2.3 Grey-markov model

Let $x^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\}$ be a raw data series. After we have checked the model accuracy, we get the simulation sequence as: $\hat{x}^{(0)}=\left\{\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right\}$ by model GM(1,1). Let $\hat{y}\left(k\right)=\hat{x}^{(0)}\left(k\right)$, for a vibration sequence $\hat{Y}$ which is a markov chain, we can divide it into l states according to the concrete circumstance, and its any state $\otimes_i$ can be expressed as:

$$\otimes_i=\left[\tilde{\otimes}_{1i},\tilde{\otimes}_{2i}\right],\tilde{\otimes}_{1i}=\hat{y}\left(k\right)+A_i,\tilde{\otimes}_{2i}=\hat{y}\left(k\right)+B_i,i=1,2,\cdots,l \tag{11}$$

where A_i and B_i are constant, which can be decided by the difference between the forecast value and the raw data. $\hat{Y}$ is a function which is changed in time, and so are the grey whitened elements of $\tilde{\otimes}_{1i},\tilde{\otimes}_{2i}$.

If $N_{ij}(m)$ is the data number of the raw series which transfer m step from $\otimes_i$ to $\otimes_j$, and N_i is the number of data that are in the grey zone $\otimes_i$, then we call:

$$p_{ij}(m)=\frac{N_{ij}}{N_i},i,j=1,2,\cdots,l \tag{12}$$

the m th step transition probability. The transition matrix $R(m)$ is as follow:

$$R(m)=\begin{bmatrix} p_{11}(m) & p_{12}(m) & \cdots & p_{1l}(m) \\ p_{21}(m) & p_{22}(m) & \cdots & p_{2l}(m) \\ \vdots & \vdots & \vdots & \vdots \\ p_{l1}(m) & p_{l2}(m) & \cdots & p_{ll}(m) \end{bmatrix} \tag{13}$$

$R(m)$ reflects the transition regulation between different states and is the foundation of the forecast model of grey markov. We can predict the future trend of the system by studying the stochastic transition matrix $R(m)$.

In practical application, if the forecast values is to be placed in the zone $\otimes_k$, then investigate the k th line of the matrix $R(1)$ inside, and if $\max_j\left\{p_{kj}\left(1\right)\right\}=p_{kr}\left(1\right)$, we can conclude the next state of the system may transfer its state from $\otimes_k$ to $\otimes_r$. If $R(1)$ has more than two lines whose probability values are same alike or close to each other and it is difficult to decide the next direction of the system with certain, it is needed to study and check the matrix $R(2)$ or $R(m)$ (m$\geq$3). At the same time, it can decide the transition of the system by checking $R(1)$ or $R(m)$ (m$\geq$2), and also be made sure the forecast zone [$\tilde{\otimes}_{1i}$, $\tilde{\otimes}_{2i}$]. Finally, the eventual forecast is in the middle point of the grey zone, then got:

$$\hat{Y}'\left(k\right)=\frac{1}{2}\left(\tilde{\otimes}_{1i}+\tilde{\otimes}_{2i}\right) \tag{14}$$

which also can be expressed as:

$$\hat{Y}'\left(k\right)=\hat{y}\left(k\right)+\frac{1}{2}\left(A_i+B_i\right) \tag{15}$$

3. Example Analysis

The data of a highway's traffic volume through 11 years are listed in Tab. (1).

3.1 Establishment of GM (1,1) model

From table 1, we get $x^{(0)}$={7590, 7458, 7689, 8573, 8215, 8986, 9013, 10353, 11821, 12304, 13755}. After do them in AGO, we obtain $x^{(1)}$={7590, 15048, 22737, 31310, 39525, 48511, 57524, 67877, 79698, 92002, 105757 }. Then we can have two constants: $a=-0.0717448$, $u=6151.22$.

By combining with Eq. (3), we can establish the model GM(1,1): $\frac{dx^{(1)}}{dt}-0.0717448x^{(1)}=6151.22$.

After solving the equation, time-response function can be obtained as:

$\hat{x}^{(1)}\left(k+1\right)=93327.4e^{-0.0717448k}-85737.4$.

From Eq. (5), it can be got: $\hat{y}\left(k\right)=\hat{x}^{(0)}\left(k+1\right)=\hat{x}^{(1)}\left(k+1\right)-\hat{x}^{(1)}\left(k\right)$.

The examination result of the prediction accuracy is as follow: $\overline{x}=9614.27, S_1=2046.02, \overline{q}=22.534, S_2=447.134$.

The post-examination margin ratio is as follow: $C=S_2/S_1=0.218539<0.35$.

The probability of little error is as follow:

$P\left\{\left|q\left(k\right)-\overline{q}\right|<0.6745S_1\right\}=P\left\{\left|q\left(k\right)-\overline{q}\right|<1380.04\right\}=1>0.95$ The accuracy grade of the forecast is excellent (Deng, 1990).

3.2 Compartmentalization of the prediction

According to the raw traffic volume and for simplification, the prediction values can be divided into four states by Eq. (11) as follows:

$\otimes_1$ =[$\tilde{\otimes}_{11}$, $\tilde{\otimes}_{21}$]: $\tilde{\otimes}_{11}=\hat{y}\left(k\right)-0.12\overline{x}$, $\tilde{\otimes}_{11}=\hat{y}\left(k\right)-0.05\overline{x}$

$\otimes_2$ =[$\tilde{\otimes}_{12}$, $\tilde{\otimes}_{22}$]: $\tilde{\otimes}_{11}=\hat{y}\left(k\right)-0.05\overline{x}$, $\tilde{\otimes}_{11}=\hat{y}\left(k\right)$

$\otimes_3$ =[$\tilde{\otimes}_{13}$, $\tilde{\otimes}_{23}$]: $\tilde{\otimes}_{11}=\hat{y}\left(k\right)$, $\tilde{\otimes}_{11}=\hat{y}\left(k\right)+0.05\overline{x}$

$\otimes_4$ =[$\tilde{\otimes}_{14}$, $\tilde{\otimes}_{24}$]: $\tilde{\otimes}_{11}=\hat{y}\left(k\right)+0.05\overline{x}$, $\tilde{\otimes}_{11}=\hat{y}\left(k\right)+0.12\overline{x}$

$\otimes_4$ =[$\tilde{\otimes}_{14}$, $\tilde{\otimes}_{24}$]: $\tilde{\otimes}_{14}=\hat{y}\left(k\right)+0.05\overline{x}$, $\tilde{\otimes}_{24}=\hat{y}\left(k\right)+0.12\overline{x}$

where $\hat{y}(k)$ is the forecast traffic as model GM (1,1), and $\overline{x}$ is the annual average traffic volume. If we show the values of the fact, the prediction $\hat{y}(k)$ and four states through these years, we will obtain a diagram listed as Fig.1 in which there are four parallel and symmetry band districts form the top to the bottom.

3.3 Calculation of the transition probability

From Fig. 1, we know that the number of raw sequence, which is in the zone of $\otimes_1$, $\otimes_2$, $\otimes_3$, $\otimes_4$, is N_1 =1, N_2 =4, N_3 =3, N_4 =3, and 0, 1, 0, 0 is the number of raw data from $\otimes_1$ to $\otimes_1$, $\otimes_2$, $\otimes_3$, $\otimes_4$ respectively by a step. If the rest may be deduced in the same way, we can calculate the number of raw transition data. Finally, we have $p_{ij}(1)$ which makes up of the matrix $R(1)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/3 & 0 & 2/3 \\ 0 & 1/3 & 2/3 & 0 \end{bmatrix}$ by Eq. (12). According to $R(1)$, we can forecast the transition state of the traffic volume in the future.

3.4 Decision of the prediction and vibration zone

By studying $R(1)$,we know that the average prediction of 2001will mostly be in the vibration zone $\otimes_3$, which is [14225.1, 14705.8].Then using formula (14) or (15), we have $\hat{Y}'(2001)=\frac{1}{2}$ (14225.1+14705.8)=14465. In the same way, we can get $\hat{Y}'(2002)$ =15283, $\hat{Y}'(2003)$ =16660, $\hat{Y}'(2004)$ =17641, $\hat{Y}'(2005)$ =19194

4. Conclusion

The grey model GM (1,1) reflects the macroscopical regulation, the markov model shows the vibration development of the microcosmic system, and both not only have the mutual advantage but also can make full use of the information which is included in these raw data. Therefore, the forecasting grey-markov model has much higher accuracy, reliability and application in the traffic volume prediction. On the other hand, because the prediction accuracy is in line with the raw data series and the divided states, but there is not a given standard that can really unify and settle these problems, and the application of the model still needs a further research and improvement.

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Table 1. Historical Traffic Volume

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
AADT (n/d)	7590	7458	7689	8573	8215	8986	9013	10353	11821	12304	13755

AADT=Annual Average Daily Traffic

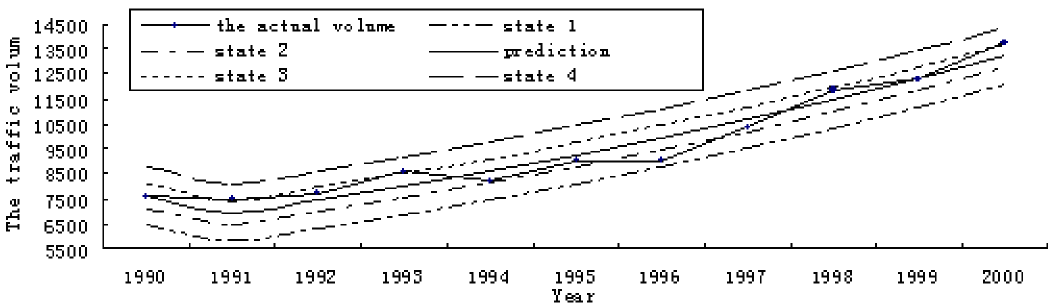

Figure 1. Annual Average Traffic Volume