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Laminar-to-Turbulent Transition Flow Inside a Heated Circular Tube

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Abstract

The natural transition flow and convective heat transfer inside an electrically heated circular tube are analyzed. The transition flow is treated as a composition of fully developed laminar and turbulent flows by assuming the fluctuating velocity in radial direction exists as if the flow is fully turbulent. The composite ratios are used to define the composite flow, and they fluctuate in transition flow. The criterion of minimum entropy production is used to derive an equation which can describe how transition evolves. It is pointed out that the fluctuations of the composite ratios govern the transition behavior. One fluctuation function is given to attain agreements with measurements including those obtained in heat transfer and flow experiments.

Keywords: transition flow, fluctuations, composition of flows, minimum entropy production

1. Introduction

The laminar-to-turbulent transitions of both the incompressible and compressible fluids are of practical importance for many applications. For the axisymmetric low speed flows in pipes, careful measurements have been repeatedly conducted for over one century, and some have been briefly reviewed by T. Mullin (2011).

The flow and heat transfer of hydrocarbons inside electrically heated circular tubes have attracted interests for the cooling designs of scramjets over years (Linne, Meyer, Edwards & Eitman 1997; Huang, Sobel & Spadaccini 2002). The heat will be generated in tube wall by the electric current and resistance of the tube as shown in figure 1. The convective heat transfer with nearly constant heat flux will take place when n-decane flows inside such a tube. As n-decane flows, the absorbed heat will drive it to higher temperatures T, lower densities ρ, and lower viscosities μ. If the mass flux, tube inner diameter d_i, and electric current inside the tube wall have appropriate constant values, steady flow will be established, and the Reynolds number ($Re=\rho U d_i/\mu$, here ρ, U and μ are mean values at a cross section) can increase from 886 at inlet to 15,000 at outlet. These can be seen in figure 2.

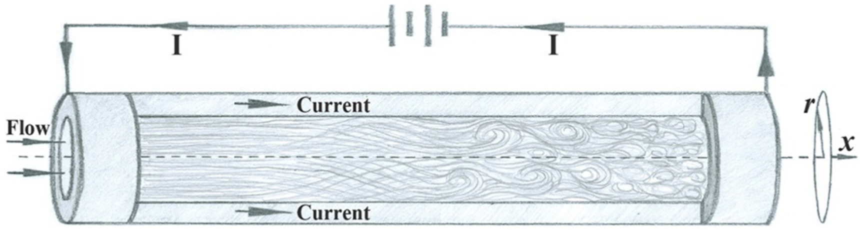

Figure 1. Flow inside an electrically heated tube

The heat flux of the tube can be controlled by selecting the electric current, which may drive the fluid temperature to increase from room temperature to as high as 800K. The thermophysical properties, such as density ρ, viscosity μ, and thermal conductivity k, decrease over a certain range from the inlet to outlet. In figure 2, ρ, μ and k of n-decane at different temperature and pressure are calculated using a program for thermophysical properties of high temperature hydrocarbon mixtures.

It is well known that the natural transition will begin if $Re\approx2300$ (Eckert & Drake 1972), so there can be a full process of transition from laminar to turbulent flow inside the tube. The laminar flow in tube is linearly stable and finite amplitude disturbance is needed to trigger the transition even if *Re* is big enough. When the transition starts and ends in tube are case-dependent problems and depend on the disturbance in tube, and the transition starting *Re* is very much different for the forced transition. No recognized theory has been established to interpret the process of laminar-to-turbulent transition since Reynolds' original experiments in 1883 (Mullin 2011; Durst & Ünsal 2006). The nature of transition of the pipe flow is still a puzzle in fluid dynamics. The same is true for the interpretation of convective heat transfer during the process.

Some laminar-to-turbulent transition was treated as a phase transition of nonequilibrium thermodynamic system (Reichl 1998). Order parameters are usually used to describe phase transitions (e.g. normal conductor to superconductor), and they fluctuate near the phase transition point. Large fluctuations have been met in the process of laminar-to-turbulent transition in pipe-flows (Durst & Ünsal 2006; McComb 1992), but very few has been discussed adopting the point view of nonequilibrium phase transition.

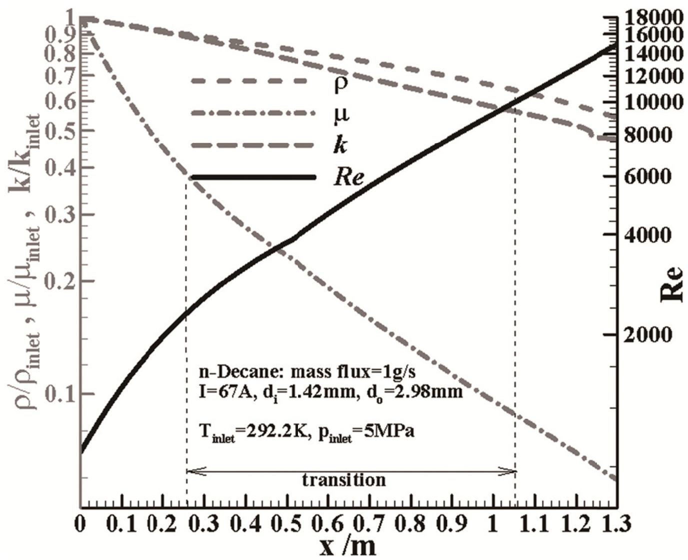

Figure 2. Density, viscosity, thermal conductivity and *Re* inside the tube

In axisymmetric transition flow, the random fluctuations of velocity have certain statistic characteristics (Durst & Ünsal 2006). The problem how the statistics of fluctuations evolve as *Re* increases in transition flow is very important, even if the starting *Re* and ending *Re* of transition flow in tube are known. This paper attempts to answer how the statistics of fluctuations evolve between the start and end of transition flow in the tube, and how the statistics of fluctuations affect the flow and convective heat transfer. The method includes three steps. In the first step, the equations for solving the laminar and turbulent flows in tube are given, then the transition flow is treated as a composition of laminar and turbulent flows by assuming the fluctuating velocity in radial direction has the same value as that of the fully turbulent flow. The composite ratios are used to define the composite flow. In the second step, the fluctuations of the composite ratios in transition flow are introduced, and then the minimum entropy production method is used to derive an equation which describes how the statistics of fluctuations of the composite ratios and transition evolve. In the last step, after some analogies are made between laminar-to-turbulent transition and phase transition, one fluctuation function is given and some comparisons with experimental phenomena are made.

2. Equations for Fully Developed Laminar Flow

One can see in figure 2 that the decrease of density ρ and thermal conductivity k in tube is about half of their original values. At outlet the viscosity μ decreases to about 6 percent of its original value, accordingly *Re* increases to about 17 times of its original value, since the mass flow rate and the tube inner diameter d_i are constant.

Under the flow conditions of figure 2, *Re* increases continuously from 886 at inlet to 15,000 at outlet. The natural transition starts at the position of $x \approx 0.26$m where $Re \approx 2300$, which is about 180 times of d_i (1.42 mm). For heat transfer (temperature), the transition ends at the position of $x \approx 1.05$m where $Re \approx 10{,}000$ (Bergman, Lavine, Incropera & DeWitt 2011; Rohsenow, Hartnett & Cho 1998), which is about 740 times of d_i. According the measurements of the fluctuations of flow velocity, the difference of transition starting and ending Reynolds numbers for flow (velocity) is much smaller (Durst & Ünsal 2006), and the transition length is around 100 times of d_i (from Re≈2300 to Re≈3000) in this case.Before the position of x=0, an additional segment (longer than 500d_i) which belongs to the same tube as the heated part shown in figure 1 can be placed. In followings the flow at each station from just before the transition to the outlet is assumed to be both hydrodynamically and thermally fully developed.

For the fully developed axisymmetric flow inside the tube, the parameters at each cross section (station) can be determined by the mean velocity and temperature and the boundary conditions (the heat flux and zero velocity at the tube wall surface). The length of tube is about 1000 times of d_i. When studying the flow velocity in x-direction which is u and fluid temperature T at a cross section, one can neglect the variations of some quantities along x-direction. The reason is that their variations along x-direction are small when compared with the variations of u and T along r-direction. In following, u and the fluid properties such as μ and k are assumed to be not varying with x when finding the solution at each cross section. Only the variations of T and the pressure p with x are considered. So at each station ρ and u are functions of r along the radius. The whole flow in the tube can be solved by determining the parameters at each station.

In fact the changes of μ and k are very large from the inlet to outlet. Two reasons can be said about treating them as not varying with x when finding the solution at each cross section. Firstly, in the following formulations the derivatives of μ and k with x are not used. Secondly, when finding the solution at each cross section, the values of local μ and k corresponding to the local T and p are used. In this way the influences of changes of μ and k on the solution at each cross section are considered.

For axisymmetric laminar flow in the tube, u satisfies (Eckert & Drake 1972; Landau & Lifshitz 1987)

$$\frac{dp}{dx}=\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right)\Rightarrow\frac{dp}{dx}=\frac{2}{r}\mu\frac{du}{dr} \tag{1}$$

The temperature increase is mainly due to heat transfer in radial direction. Following the analysis of Eckert and Drake (1972), the temperature variations due to viscous friction and axial heat transfer are neglected, so

$$\rho c_p u\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) \tag{2}$$

If the method of separation of variables is used, T can be expressed as a product of two parts. Of the two parts, one varies with x and the other varies with r. Let

$$T=\mathrm{X}(x)\Theta(r) \tag{3}$$

then Eq. (2) becomes

$$\frac{1}{\mathrm{X}}\frac{d\mathrm{X}}{dx}=\frac{1}{\rho c_p u}\frac{1}{r\Theta}\frac{d}{dr}\left(kr\frac{d\Theta}{dr}\right) \tag{4}$$

The velocity in r-direction is neglected here. According to the mass balance equation, the product of ρ and u is same at different cross section if r is equal, Eq. (4) can be divided into two linear equations. The physical properties such as μ, k and c_p is determined by the local temperature and pressure, so it is not difficult to understand that Eqs. (1) and (2) are coupled. For the cases of interest, Eqs. (1) and (2) are only weakly coupled through the dependence of physical properties on temperature and pressure.

Fluid is consisted of vast number of molecules. The molecules collide all the time. These collisions do not produce or annihilate energy or momentum of the fluid. So momentum and energy equations do not have source terms when conserved variables are used. But the entropy balance equation has source terms because of the dissipative processes accompanying the collisions. These processes change the nonequilibrium distribution, tend to drive the flowing fluid to approach equilibrium, and produce entropy. The entropy production due to temperature and velocity gradients is (Reichl 1998; Landau & Lifshitz 1987; Lifshitz & Pitaevskii 1980)

$$\sigma_{Lam}=\frac{k}{T^2}(\nabla T)\cdot(\nabla T)+\frac{2\mu}{T}(\nabla\boldsymbol{v})^s:(\nabla\boldsymbol{v})^s \tag{5}$$

in which, σ is the entropy production, subscript Lam means laminar, $\boldsymbol{v}$ is the velocity vector, and $(\nabla\boldsymbol{v})^s$ is the symmetric tensor with zero trace of $\nabla\boldsymbol{v}$. In cylindrical coordinate $(\nabla\boldsymbol{v})^s$ is

$$(\nabla \boldsymbol{v})^s = \frac{1}{2}\left[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right] - \frac{1}{3}Tr(\nabla \boldsymbol{v})\bar{\bar{U}} = \begin{pmatrix} \frac{2}{3}\frac{\partial v_r}{\partial r} - \frac{1}{3}\frac{v_r}{r} & 0 & \frac{1}{2}\frac{\partial v_x}{\partial r} \\ 0 & -\frac{1}{3}\frac{\partial v_r}{\partial r} + \frac{2}{3}\frac{v_r}{r} & 0 \\ \frac{1}{2}\frac{\partial v_x}{\partial r} & 0 & -\frac{1}{3}\frac{\partial v_r}{\partial r} - \frac{1}{3}\frac{v_r}{r} \end{pmatrix} \tag{6}$$

in which, $\bar{\bar{U}}$ is the unit tensor, superscript T means transpose, Tr means the trace of a tensor, v_r is the velocity along radius, and v_x is the velocity along x-direction. In present case, $v_x=u$ and $v_r=0$.

The terms of temperature variation due to viscous friction and the axial heat transfer are omitted in Eq. (2). So the second term and axial part in the first term of Eq. (5) can be neglected in order to be consistent with Eq. (2). Nevertheless, these terms are retained when the entropy production is discussed in following. One will see this retainment does not affect the conclusion.

In cylindrical coordinate, the first term in Eq. (5) is

$$\sigma_{Lam,T} = \frac{k}{T^2}(\nabla T)\cdot(\nabla T) = \sigma_{Lam,\mathrm{X}} + \sigma_{Lam,\Theta} = \frac{k}{\mathrm{X}^2}\left(\frac{d\mathrm{X}}{dx}\right)^2 + \frac{k}{\Theta^2}\left(\frac{d\Theta}{dr}\right)^2 \tag{7}$$

and the second term in Eq. (5) is

$$\sigma_{Lam,V} = \frac{2\mu}{T}(\nabla \boldsymbol{v})^s : (\nabla \boldsymbol{v})^s = \frac{\mu}{T}\left(\frac{du}{dr}\right)^2 \tag{8}$$

In Eqs. (7) and (8), the subscripts T, Χ, Θ and V denote partial entropy productions due to the corresponding gradients, and V means velocity. One can see that the entropy production includes square terms of temperature and velocity gradients.

The hydrodynamic equations for laminar flow are also called Navier-Stokes equations. In the equations considered here the relationships between the generalized currents (e.g. the stress tensor) and the generalized forces (e.g. the tensor $\nabla \boldsymbol{v}$) are linear (Reichl 1998). The fluid elements of laminar flow inside the tube are in nonequilibrium, and the dissipative transport processes inside them increase the entropy. For nonequilibrium system, a state of minimum entropy production is a stationary state in linear regime, and this was first established by Prigogine (Reichl 1998). This criterion corresponds to the minimum free energy which is applicable for equilibrium systems.

3. Equations for Fully Developed Turbulent Flow

For fully developed axisymmetric turbulent flow inside the tube, at each station $\bar{\rho}$ and $\bar{u}$ are also functions of r, and the momentum balance equation can be written as (Eckert & Drake 1972)

$$\frac{d\bar{p}}{dx} = \frac{1}{r}\frac{d}{dr}\left(\mu r\frac{d\bar{u}}{dr}\right) - \frac{1}{r}\frac{d}{dr}\left(r\bar{\rho}\overline{\tilde{u}\,\tilde{v}}\right) \tag{9}$$

in which, *overbar* above the physical quantities means time averaged values, while *tilde* means instantaneous values minus their time averaged values, which can be called fluctuating values. Since the mean velocity in r-direction is 0, the instantaneous velocity in r-direction is $\tilde{v}$.

For fully developed turbulent flow, the equation of temperature is (Eckert & Drake 1972)

$$\bar{\rho}\, c_p \bar{u}\frac{\partial \bar{T}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial \bar{T}}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\bar{\rho}c_p r\overline{\tilde{T}\,\tilde{v}}\right) \tag{10}$$

Integrating Eq. (9) once along radial direction yields Eq. (1) for laminar case. So the first terms in the right hand sides of Eqs. (9) and (10) are consistent with those laminar dissipative terms in Eqs. (1) and (2). The second terms in the right hand sides are called turbulent dissipative terms, which can be expressed by gradients of $\bar{u}$ and $\bar{T}$ if some assumptions are used. Using these assumptions and integrating Eq. (9), one has (Eckert & Drake 1972).

$$\frac{d\overline{p}}{dx}=\frac{1}{r}\frac{d}{dr}\left[\left(\mu+\overline{\rho}\varepsilon_m\right)r\frac{d\overline{u}}{dr}\right]\Rightarrow\frac{d\overline{p}}{dx}=\frac{2}{r}\left(\mu+\overline{\rho}\varepsilon_m\right)\frac{d\overline{u}}{dr}\tag{11}$$

$$\frac{\partial\overline{T}}{\partial x}=\frac{1}{\overline{\rho}\,c_p\overline{u}}\frac{1}{r}\frac{\partial}{\partial r}\left[\left(k+\overline{\rho}c_p\varepsilon_H\right)r\frac{\partial\overline{T}}{\partial r}\right]\tag{12}$$

in which, $\varepsilon_m=\varepsilon_m(r,Re)$ is called turbulent dissipative coefficient of momentum, and $\varepsilon_H=\varepsilon_H(r,Re)$ is called turbulent dissipative coefficient of energy. The variations of $\overline{\rho}\,\overline{u}$ with x are neglected (the product of $\overline{\rho}$ and $\bar{u}$ is same at different cross section if r is equal), so from Eqs. (11) and (12) one can see that the equations for solving velocity and temperature are also weakly coupled through the dependence of physical properties on temperature and pressure.

The method of separation of variables can be used to divide Eq. (12) into two equations, because $\bar{u}$, $\overline{\rho}$ and ε_H are all functions of r at a station. When doing so, let the fluctuating part contained in Θ, which means

$$T_{turb}=\mathrm{X}(x)\Theta_{turb}(r)=\mathrm{X}(x)\left[\overline{\Theta}(r)+\tilde{\Theta}(r)\right]\tag{13}$$

$$\tilde{T}=\mathrm{X}(x)\tilde{\Theta}(r)\tag{14}$$

The treatment of separation of variables here is similar to Eqs. (3) and (4). X and $d\mathrm{X}/dx$ have identical values if the mass flux and heat flux are same for both laminar and turbulent flows. If both fluxes reverse to their opposite values, according to Eqs. (1), (2) and (4) for laminar case, the gradients of u, X and Θ change signs. The same should be true for turbulent case, and it is required that the reversals of both fluxes do not affect $\tilde{v}$, ε_m and ε_H. So for turbulent flow, according to Eqs. (11), (12) and (13), the reversals of both fluxes result in that the gradients of $\overline{u}$, X and $\overline{\Theta}$ change signs. According to Eqs. (9), (10) and (14), $\tilde{u}$ and $\tilde{\Theta}$ will also change signs.

Turbulent flow obeys the same full hydrodynamic equations as laminar flow, so its entropy production should be the same as Eq. (5) except that instantaneous values are used. The mean entropy production is

$$\overline{\sigma_{Turb}}=\overline{\sigma_{Turb,T}}+\overline{\sigma_{Turb,V}}=\overline{\frac{k}{T^2}(\nabla T)\cdot(\nabla T)}+\overline{\frac{2\mu}{T}(\nabla\boldsymbol{v})^s:(\nabla\boldsymbol{v})^s}\tag{15}$$

in which, the subscript *Turb* means turbulent, the subscripts T and V have the same meanings as in Eqs. (7) and (8). If the effects of statistical dependence between the numerators and denominators are ignored, in cylindrical coordinate the first term in Eq. (15) is

$$\overline{\sigma_{Turb,T}}=\overline{\frac{k}{T^2}(\nabla T)\cdot(\nabla T)}=\overline{\sigma_{Turb,\mathrm{X}}}+\overline{\sigma_{Turb,\Theta}}\tag{16}$$

in which,

$$\overline{\sigma_{Turb,\mathrm{X}}}=\frac{k}{\mathrm{X}^2}\left(\frac{d\mathrm{X}}{dx}\right)^2\tag{17}$$

$$\overline{\sigma_{Turb,\Theta}}=\frac{k}{\overline{\Theta}^2}\left(\frac{d\overline{\Theta}}{dr}\right)^2+\frac{k}{\overline{\Theta}^2}\overline{\left(\frac{d\tilde{\Theta}}{dr}\right)^2}\tag{18}$$

The second term in Eq. (15) is

$$\overline{\sigma_{Turb,V}}=\overline{\frac{2\mu}{T}(\nabla\boldsymbol{v})^s:(\nabla\boldsymbol{v})^s}=\overline{\sigma_{Turb,u}}+\overline{\sigma_{Turb,v}}\tag{19}$$

in which,

$$\overline{\sigma_{Turb,u}}=\frac{\mu}{\overline{T}}\left[\left(\frac{d\overline{u}}{dr}\right)^2+\overline{\left(\frac{d\tilde{u}}{dr}\right)^2}\right]\tag{20}$$

$$\overline{\sigma_{Turb,v}} = \frac{4}{3}\frac{\mu}{\overline{T}}\left[\overline{\left(\frac{d\tilde{v}}{dr}\right)^2} + \overline{\left(\frac{\tilde{v}}{r}\right)^2} - \overline{\frac{d\tilde{v}}{dr}\frac{\tilde{v}}{r}}\right] \tag{21}$$

In Eqs. (16) through (18), the subscripts X and Θ have the same meanings as in Eqs. (7) and (8). The subscripts u and v in Eqs. (19) through (21) denote partial entropy productions produced by the corresponding velocity components.

4. Equations for Natural Transition Flow

Comparing the equations for solving velocity and temperatures, which are Eqs. (1) and (2) for laminar flow and Eqs. (11) and (12) for turbulent flow, one can see that the two flow modes obey the same equations if ε_m and ε_H are taken as 0 for laminar flow.

The equations for transition flow have the same forms as Eqs. (9) and (10). If one assumes that the fluctuating velocity $\tilde{v}$ in radial direction has the same value as that of the fully turbulent flow, at each point in natural transition region the flow can be treated as a composition of fully developed laminar and turbulent flows. Under this assumption u and Θ at each point are consisted of two parts, of which one is contributed by laminar flow and the other by turbulent flow, respectively. The composite ratios are denoted as 1-η and η for laminar and turbulent flows, respectively. The variation of u is

$$(du)_{Tran} = (1-\eta)(du)_{Lam} + \eta(du)_{Turb} = (1-\eta)du + \eta d(\overline{u} + \tilde{u}) \tag{22}$$

in which, the subscript *Tran* means transition.

The momentum equation is

$$(1-\eta)\frac{dp}{dx} + \eta\frac{d\overline{p}}{dx} = (1-\eta)\frac{2}{r}\mu\frac{du}{dr} + \eta\frac{2}{r}(\mu + \overline{\rho}\varepsilon_m)\frac{d\overline{u}}{dr} \tag{23a}$$

$$\frac{dp}{dx} - \frac{2}{r}\mu\frac{du}{dr} = \frac{d\overline{p}}{dx} - \frac{2}{r}(\mu + \overline{\rho}\varepsilon_m)\frac{d\overline{u}}{dr} = 0 \tag{23b}$$

Equation (23b) is the result of momentum conservation for laminar and turbulent flows. In Eq. (23) the variables without *bar* are values of laminar flow and those with *bar* are mean values of turbulent flow.

It should be noted that only one pressure gradient is permitted at one station and at same instant. So η should be same at one station, and u and Θ and their derivatives with respect to r are consisted of laminar and turbulent parts. Turbulent flow in the tube has a fluctuating velocity in r-direction which is $\tilde{v}$. The validity of Eqs. (9) and (23) requires that $\tilde{v}$ exists as if the flow is fully turbulent in transition flow region. So in transition flow region, the velocity vector is $\boldsymbol{v}=(v_r, v_\theta, v_x)=(\tilde{v}, 0, u_{Tran})$. One can substitute this velocity vector into the full momentum equation (9) to derive Eq. (23a) (will be further explained using Eq. (26)). The transition length for heat transfer (temperature) is several hundred times of the inner diameter d_i, and the transition length for flow (velocity) is around a hundred times of the inner diameter d_i. In Eq. (23a) η is treated as not varying with x when finding the solution at each cross section.

For temperature one can write similar equations to Eqs. (22) and (23). Like those in pure laminar or turbulent case, the variations of ρu and $\overline{\rho}\,\overline{u}$ with x are neglected (the product of ρ and u is same at different cross section if r is equal, and so is true for the product of $\overline{\rho}$ and $\bar{u}$), and the equations for velocity and temperature are also weakly coupled through the dependence of physical properties on temperature and pressure in transition flow. The laminar, transition, or turbulent flow and heat transfer have certain type of profiles of u and Θ, or $\overline{u}$, $\overline{\Theta}$, $\tilde{u}$ and $\tilde{\Theta}$, and the profiles of physical properties such as μ, k and c_p may be affected by them. Since it is the *Re* which affects the type of profiles of variables, the physical properties alone do not change the type of profiles of variables or the mode of flow and heat transfer. The weakly coupled equations for velocity and temperature can have such a characteristic that the type of profile of velocity and its fluctuation do not affect the type of profile of temperature and its fluctuation, and vice versa. According to Eqs. (5) and (15), the square terms of temperature and velocity gradients in the expression of entropy production are mutually separate. So a different η can be adopted for temperature. For simplicity the same η for temperature is considered below. The variation of Θ is

$$(d\Theta)_{Tran} = (1-\eta)(d\Theta)_{Lam} + \eta(d\Theta)_{Turb} = (1-\eta)d\Theta + \eta d(\overline{\Theta} + \tilde{\Theta}) \tag{24}$$

and the temperature equation is

$$\frac{1}{X}\frac{dX}{dx}=(1-\eta)\frac{k}{\bar{\rho}c_p\bar{u}}\frac{1}{r\Theta_{Tran}}\frac{d}{dr}\left(r\frac{d\Theta}{dr}\right)+\eta\frac{1}{\bar{\rho}c_p\bar{u}}\frac{1}{r\Theta_{Tran}}\frac{d}{dr}\left[\left(k+\bar{\rho}c_p\varepsilon_H\right)r\frac{\partial\bar{\Theta}}{\partial r}\right] \tag{25a}$$

$$\frac{d\Theta}{dr}=\left(1+\frac{\bar{\rho}c_p\varepsilon_H}{k}\right)\frac{\partial\bar{\Theta}}{\partial r} \tag{25b}$$

Equation (25b) is the result of same heat flux for laminar and turbulent flows. Here as that in Eq. (23a), η is also treated as not varying with x when finding the solution at each cross section.

The introduction of ε_m and ε_H in last and this sections is not indispensable for the derivations of composition of flows. For example, the momentum equation for transition flow has the same forms as Eq. (9). Since η is same at one station, at each point along a radius, the composite velocities $\bar{u}_{Tran}$ and $\tilde{u}_{Tran}$ can be found by integrating Eq. (22), of which the composite mean velocity in x-direction is $\bar{u}_{Tran}=(1-\eta)u+\eta\bar{u}$ and the composite fluctuating velocity in x-direction is $\tilde{u}_{Tran}=(1-\eta)0+\eta\tilde{u}=\eta\tilde{u}$. Substituting them into Eq. (9), under the assumption of $\tilde{v}$ in transition region having the same value as that of the fully turbulent flow, one has

$$(1-\eta)\frac{dp}{dx}+\eta\frac{d\bar{p}}{dx}=\frac{1}{r}\frac{d}{dr}\left\{\mu r\left[(1-\eta)\frac{du}{dr}+\eta\frac{d\bar{u}}{dr}\right]\right\}-\frac{1}{r}\frac{d}{dr}\left\{r\bar{\rho}\overline{\left[(1-\eta)0+\eta\tilde{u}\right]\left[(1-\eta)\tilde{v}+\eta\tilde{v}\right]}\right\} \tag{26a}$$

$$(1-\eta)\frac{dp}{dx}+\eta\frac{d\bar{p}}{dx}=(1-\eta)\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right)+\eta\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{d\bar{u}}{dr}\right)-\eta\frac{1}{r}\frac{d}{dr}\left\{r\bar{\rho}\overline{\tilde{u}\tilde{v}}\right\} \tag{26b}$$

in which, the fluctuating velocity in x-direction in laminar flow region is 0, and η is not fluctuating here (its fluctuation will be discussed in next section). Since η is same at one station and not fluctuating, one can easily find here that Eq. (26a) is identical to Eq.(26b), and they both are equivalent to Eq. (23a). From Eq. (26) one can see that the flow in transition region is a composite motion of fully developed laminar and turbulent flows, because $(1-\eta)u+\eta\bar{u}$ is **the composite mean velocity in *x*-direction**, $(1-\eta)0+\eta\tilde{u}=\eta\tilde{u}$ is **the composite fluctuating velocity in *x*-direction**, and $(1-\eta)\tilde{v}+\eta\tilde{v}=\tilde{v}$ is **the composite fluctuating velocity in *r*-direction**. For the temperature equation, Eq. (25a) can also be rewritten in a similar form to Eq. (26). The introduction of ε_m and ε_H in last and this sections helps to shorten the formulations and explain the weak coupling of velocity and temperature equations in turbulent case.

The assumption of $\tilde{v}$ in transition region having the same value as that of the fully turbulent flow can be further explained. It is well known that finite amplitude disturbance is needed to trigger the transition even if *Re* is big enough because the laminar flow in tube is linearly stable. A full process of natural transition can happen due to some finite amplitude disturbance and the increase of *Re* inside the tube which is concerned. For a given flow inside the tube, this assumption is equivalent to that $\tilde{v}$ is same no matter the flow is laminar, transitional or turbulent, and this $\tilde{v}$ originates from the finite amplitude disturbance existing in laminar flow region. The assumption of $\tilde{v}$ being same inside the tube does not violate the mass balance equation, because $\tilde{v}$ corresponds to a finite amplitude disturbance, and there are two other fluctuating velocities in the other two directions. The finite amplitude disturbance determines the magnitudes of the three-dimensional fluctuating velocities in the laminar flow region, and the fluctuating velocities except $\tilde{u}$ do not change in regions of different flow modes. The influence of the three-dimensional fluctuating velocities can be neglected in the laminar flow region, but whether the transition happens or not depends on the amplitude of disturbance. The transition starting and ending Reynolds numbers are very much different for the forced transitions with different amplitudes of disturbance.

In transition flow the values of u and Θ for laminar ingredient may be different from those for corresponding pure laminar flow. In the case of heated flow in tube, the different flow modes have different profiles of temperature at a station, so the physical properties such as c_p, μ and k have different profiles. It is not difficult to understand that different profiles of c_p, μ and k lead to different profiles of u and Θ at a station. In transition flow the profiles of u and Θ for laminar ingredient are related to the instant profiles of c_p, μ and k corresponding to the local instant temperature and pressure. If the differences of the physical properties caused by different flow modes are neglected, in transition flow the values of u and Θ for laminar ingredient are the same as those for corresponding pure laminar flow. The same is true for the values of $\bar{u}$ and $\bar{\Theta}$ for turbulent ingredient.

Obviously, arbitrary η can satisfy Eqs. (23), (25) and (26). So the fluctuating velocity in radial direction having the same value as that of fully turbulent flow, makes this composition of flows both mathematically and mechanically possible. From Eq. (26), one can see that this composition is still valid even if one considers the nonzero fluctuating velocity in x-direction in the laminar flow region. Since it is the composition of two types of flow motion for a fluid element which is discussed, it is permitted even for $\eta<0$ or $\eta>1$. A negative η or $1-\eta$ means negative contributions of the mass flux, pressure gradient, and heat flux, accordingly resulting in negative values of gradients of u, X and Θ, and negative values of $\tilde{u}$ and $\tilde{\Theta}$.

In this section the transition flow is decomposed into laminar and turbulent ingredients by assuming the fluctuating velocity $\tilde{v}$ exists as if the flow is always fully turbulent. The flow is laminar when $\eta=0$, while the flow is turbulent when $\eta=1$. There is one more variable (η) now, so Eqs. (23) and (25) alone cannot determine the transition behavior.

5. Fluctuations of Natural Transition Flow Inside the Tube

The dynamic equation of η must be found if the aim is to determine how natural transition evolves inside the tube. Let Re_L denote the transition starting Re and Re_R denote the ending Re. It has been found in heat transfer experiments that the natural transition starts at $Re_L\approx2300$ and ends at $Re_R\approx10000$ in circular tube (Bergman, Lavine, Incropera & DeWitt 2011; Rohsenow, Hartnett & Cho 1998). The difference of Re_L and Re_R for flow is much smaller according the measurements of the fluctuations of flow velocity (Durst & Ünsal 2006). The exactness of when transition starts and ends is case-dependent. Here the interests are how transition evolves between its start and end, and how the evolution affects the flow and convective heat transfer characteristics.

Before giving the expression of entropy production in transition flow region, the fluctuation of η is introduced. In natural transition region, η is assumed to be consisted of two parts, which are the mean value $\bar{\eta}$ and the fluctuating value $\tilde{\eta}$. So

$$\eta = \bar{\eta} + \tilde{\eta} \tag{27}$$

Substituting Eq. (27) into Eqs. (22) and (24), one has

$$\left(du\right)_{Tran} = \left(1-\bar{\eta}\right)du + \bar{\eta}\,d\bar{u} + \bar{\eta}\,d\tilde{u} + \tilde{\eta}d\left(\bar{u} - u + \tilde{u}\right) \tag{28}$$

$$\left(d\Theta\right)_{Tran} = \left(1-\bar{\eta}\right)d\Theta + \bar{\eta}\,d\bar{\Theta} + \bar{\eta}\,d\tilde{\Theta} + \tilde{\eta}d\left(\bar{\Theta} - \Theta + \tilde{\Theta}\right) \tag{29}$$

in which, $\bar{\eta}$ is between 0 and 1, and is a monotonic ascending function of Re between Re_L and Re_R. Equations (28) and (29) show that both $(du)_{Tran}$ and $(d\Theta)_{Tran}$ have two fluctuating components, which are turbulent fluctuating quantities and the quantities corresponding to the fluctuation of η, respectively.

The introduction of $\bar{\eta}$ and $\tilde{\eta}$, can give the instantaneous value of composite ratio to describe the state of the transition flow at each instant and discern the **disordered motions** which are neither laminar nor turbulent. It was reported by Mullin(2011) and Durst &Ünsal (2006) that during transition there are typical instantaneous **disordered motions** which are different than the generic turbulence (many other papers also reported such motions). The commonly used intermittency factor γ cannot discern such instantaneous states of the transition flow. One can see that the introduction of $\bar{\eta}$ and can better describe the transition flow.

Unlike $\tilde{u}$, $\tilde{v}$ and $\tilde{\Theta}$ which are statistically dependent, between the turbulent fluctuating quantities and the fluctuation of η, the following relationships of statistical independence are assumed to be valid

$$\overline{\left(\tilde{\eta}\right)^n\left(\tilde{\phi}\right)^m} = \overline{\left(\tilde{\eta}\right)^n}\;\overline{\left(\tilde{\phi}\right)^m} \quad , \quad \overline{\left(\tilde{\eta}\right)^n\left(\frac{d\tilde{\phi}}{dr}\right)^m} = \overline{\left(\tilde{\eta}\right)^n}\;\overline{\left(\frac{d\tilde{\phi}}{dr}\right)^m} \tag{30}$$

in which, n and m are both positive integers, and $\tilde{\phi}$ denotes $\tilde{u}$, $\tilde{v}$ or $\tilde{\Theta}$. The validity of Eq. (30) may lie in the two different mechanics by which the fluctuations are produced. The productions of $\tilde{u}$, $\tilde{v}$ or $\tilde{\Theta}$ are determined by the flow mode of turbulence, while $\tilde{\eta}$ is produced by the fluctuating behavior of the flow modes in transition region.

Only one pressure gradient is permitted at one station and at same instant, so $\tilde{\eta}$ as well as $\bar{\eta}$ has only one value at one station. One can show that the introduction of $\bar{\eta}$ and $\tilde{\eta}$ does not affect the validity of Eq. (26b).

At each point along a radius, the composite velocities $\bar{u}_{Tran}$ and $\tilde{u}_{Tran}$ can be found by integrating Eq. (28), of which the composite mean velocity in x-direction is $\bar{u}_{Tran}=(1-\bar{\eta})u+\bar{\eta}\bar{u}$ and the composite fluctuating velocity in x-direction is $\tilde{u}_{Tran}=\bar{\eta}\,\tilde{u}+\tilde{\eta}(\bar{u}-u+\tilde{u})$. The composite fluctuating velocity in r-direction is always $\tilde{v}$. Substituting them and $\bar{\eta}$ and $\tilde{\eta}$ into Eq. (9), using Eq. (30), then one has an equation identical to Eq. (26b) except that $\bar{\eta}$ takes the place of η. So the introduction of $\bar{\eta}$ and $\tilde{\eta}$ does not affect the validity of composition of flows described in last section.

The mean entropy production is the same as Eq. (15) because the flow obeys the same full hydrodynamic equations. Substituting Eqs. (28) and (29) into Eq. (15), using Eq. (30), then after some manipulations one has

$$\begin{aligned}\overline{\sigma_{Tran}}&=\overline{\sigma_{Turb,\mathrm{X}}}+\overline{\sigma_{Turb,v}}+(1-\bar{\eta})(\sigma_{Lam,\Theta}+\sigma_{Lam,V})+\bar{\eta}\left(\overline{\sigma_{Turb,\Theta}}+\overline{\sigma_{Turb,u}}\right)\\&+\left[\overline{\tilde{\eta}^2}-(1-\bar{\eta})\bar{\eta}\right]\left\{\frac{k}{\overline{\Theta^2_{Tran}}}\left[\left(\frac{d\bar{\Theta}}{dr}-\frac{d\Theta}{dr}\right)^2+\overline{\left(\frac{d\tilde{\Theta}}{dr}\right)^2}\right]+\frac{\mu}{\bar{T}_{Tran}}\left[\left(\frac{d\bar{u}}{dr}-\frac{du}{dr}\right)^2+\overline{\left(\frac{d\tilde{u}}{dr}\right)^2}\right]\right\}\end{aligned}\tag{31}$$

in which, the subscript *Tran* means transition, $\sigma_{Lam,\Theta}$ is the same as in Eq. (7), $\sigma_{Lam,V}$ is Eq. (8), $\overline{\sigma_{Turb,\mathrm{X}}}$ is Eq. (17), $\overline{\sigma_{Turb,\Theta}}$ is Eq. (18), $\overline{\sigma_{Turb,u}}$ is Eq. (20), and $\overline{\sigma_{Turb,v}}$ is Eq. (21), if T and Θ^2 there in denominators are replaced with $\bar{T}_{Tran}$ and $\overline{\Theta^2_{Tran}}$. The effects of statistical dependence between numerators and denominators are ignored for the convenience of elucidation. Both $\bar{\eta}$ and $\overline{\tilde{\eta}^2}$ are functions of Re, and they do not vary with r.

The terms excluding $\overline{\sigma_{Turb,\mathrm{X}}}$ and $\overline{\sigma_{Turb,v}}$ (both same for different flow modes) in the right hand side of Eq. (31) remind us of the free energy of the binary mixture in equilibrium, for which the treatment is given by Reichl (1998) and Cowan (2005). Following the definition of *the free energy of mixing* for the binary mixture in equilibrium by Cowan (2005), the four terms ahead in Eq. (31) are dropped, and *the mean entropy production of composition* for a fluid element in the nonequilibrium natural transition flow inside the tube is defined as

$$\overline{\sigma_{Tran,c}}=\left[\overline{\tilde{\eta}^2}-(1-\bar{\eta})\bar{\eta}\right]\left\{\frac{k}{\overline{\Theta^2_{Tran}}}\left[\left(\frac{d\bar{\Theta}}{dr}-\frac{d\Theta}{dr}\right)^2+\overline{\left(\frac{d\tilde{\Theta}}{dr}\right)^2}\right]+\frac{\mu}{\bar{T}_{Tran}}\left[\left(\frac{d\bar{u}}{dr}-\frac{du}{dr}\right)^2+\overline{\left(\frac{d\tilde{u}}{dr}\right)^2}\right]\right\}\tag{32}$$

in which, the subscript c means *composition*. Following the criterion of minimum entropy production first established by Prigogine (Reichl 1998), the choice of $\bar{\eta}$ should give the minimum value of $\overline{\sigma_{Tran,c}}$. This requires the derivative of Eq. (32) with respect to $\bar{\eta}$ is 0. When $Re=Re_L$, $\bar{\eta}=0$ and $\tilde{\eta}=0$, and when $Re=Re_R$, $\bar{\eta}=1$ and $\tilde{\eta}=0$. So the η-term, which is the sum of terms in the first square brackets of the right hand side of Eq. (32), is 0 when $\bar{\eta}=0$ or $\bar{\eta}=1$. The sum of other terms in the parentheses of the right hand side of Eq. (32) varies with r, but $\bar{\eta}$ should not vary with r. Let the derivative of the η-term with respect to $\bar{\eta}$ is 0, which is

$$\frac{d\left(\overline{\tilde{\eta}^2}\right)}{d\bar{\eta}}-1+2\bar{\eta}=0\tag{33}$$

then the η-term is always 0 for $0 \le \bar{\eta} \le 1$. So the derivative of $\overline{\sigma_{Tran,c}}$ with respect to $\bar{\eta}$ is 0 in transition region.

This choice of $\bar{\eta}$ corresponds to the minimum entropy production which is required to maintain the movement of a fluid element in transition flow region. $\bar{\eta}$ is independent of r and is only a function of Re. So Eq. (33) describes the evolution of fluctuations and natural laminar-to-turbulent transition flow inside the tube. The retainment or discard of terms in the entropy production equation corresponding to terms of viscous friction and axial heat transfer which are neglected in the temperature equation does not affect Eq. (33).

6. One fluctuation function and Comparisons with Experimental Phenomena

The transition from laminar to turbulent flow was treated as a phase transition of nonequilibrium thermodynamic system by Reichl (1998). Order parameters are used to describe phase transitions. If an order parameter is adopted to describe the transition flow inside the tube, it should be a linear function of η.

Fluctuations of the order parameter are discussed by Landau and Lifshitz (1980) when dealing with phase transitions of the second kind in thermodynamic equilibrium system. Near the phase transition point, there exists a narrow range of temperature where the physical nature of the thermodynamic function consists in an anomalous increase in the fluctuations of the order parameter. This range is called the fluctuation range where the fluctuations of order parameter play the dominant role. It is stated by Henkel, Hinrichsen & Lübeck (2008) that much of what is known about equilibrium phase-transitions can be extended to the non-equilibrium cases.

The Landau theory of phase transition, which does not consider the fluctuations of order parameter, is inapplicable in the fluctuation range (Cowan 2005; Landau & Lifshitz 1980). In this range the thermodynamic potential cannot be represented as a function of only the order parameter (and its derivatives with respect to coordinates) and other thermodynamic variables. The fluctuation of η has been introduced in Eq. (31). The derivation of Eq. (33) is in accordance with the spirit of Landau theory in the treatment regarding the order parameter, which is treating the order parameter as an independent variable whose value is determined by minimizing the thermodynamic potential (Landau & Lifshitz 1980).

Equation (33) shows the evolutions of statistics of fluctuations and laminar-to-turbulent transition are not dependent on either laminar or time-averaged turbulent profiles. This is in accordance with the critical phenomenon of phase transition. Near the critical point microscopic details do not determine the behavior of thermodynamic system, and dissimilar systems share many same properties (Cowan 2005).

For the studied natural transition flow, $\bar{\eta}$ is a function of Re. It is assumed here that $\overline{\tilde{\eta}^2}$ includes some powers of derivatives of $\bar{\eta}$ with respect to Re. If the fluctuation function of η is taken as

$$\tilde{\eta} = C\frac{d\bar{\eta}}{dRe}\tilde{g}(t) \quad , \quad \overline{\left[\tilde{g}(t)\right]^2} = 1 \tag{34}$$

$\overline{\tilde{\eta}^2}$ will contain the square of the derivative of lowest order and a positive coefficient. So Eq. (33) becomes

$$2C^2\frac{d^2\bar{\eta}}{dRe^2} - 1 + 2\bar{\eta} = 0 \tag{35}$$

The transition starts at Re_L and ends at Re_R, so the restrictions on $\bar{\eta}$ are

$$\bar{\eta}\left(Re_L\right) = 0 \quad , \quad \bar{\eta}\left(Re_R\right) = 1 \tag{36}$$

The fluctuations of η are both zero in laminar and turbulent flows, and $\bar{\eta}$ is a monotonic ascending function of Re, so the restrictions on the derivative of $\bar{\eta}$ are

$$\frac{d\bar{\eta}}{dRe} \ge 0 \quad , \quad \left.\frac{d\bar{\eta}}{dRe}\right|_{Re_L} = \left.\frac{d\bar{\eta}}{dRe}\right|_{Re_R} = 0 \tag{37}$$

The solution which satisfies all these conditions is

$$\overline{\eta} = \frac{1+\sin(\theta)}{2} \quad , \quad \theta = \frac{2Re - Re_R - Re_L}{Re_R - Re_L}\frac{\pi}{2} \tag{38}$$

and $C = (Re_R - Re_L)/\pi$. Substituting Eq. (38) into Eq. (34) yields $\tilde{\eta} = \frac{\cos\theta}{2}\tilde{g}(t)$.

This solution agrees with the experimental results of Zhang, Zhang, Xiao, Jiang & Le (2013), in which the experimental Nusselt number *Nu* in natural transition region is expressed as a weighted superposition of laminar *Nu* and turbulent *Nu* at same *Re*, and the weighting factor is the same as Eq. (38). Apparently, same rule of weighted superposition can be applied to the friction factor.

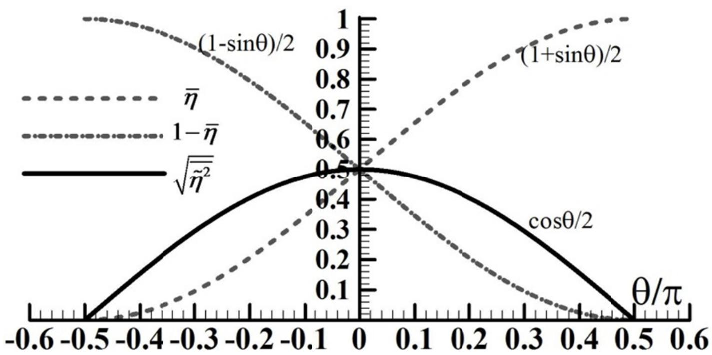

Figure 3. $\overline{\eta}$, 1-$\overline{\eta}$ and $\sqrt{\overline{\tilde{\eta}^2}}$ as functions of θ

It is plotted in figure 3 for $\overline{\eta}$, 1-$\overline{\eta}$ and $\sqrt{\overline{\tilde{\eta}^2}}$ as functions of *θ*. One can see the fluctuations of *η* are large in transition region. The root mean square values of $\tilde{\eta}$ ($\sqrt{\overline{\tilde{\eta}^2}}$) are greater than $\overline{\eta}$ for *θ*<0, and greater than 1-$\overline{\eta}$ for *θ*>0. So at some instants, *η* may be negative when *θ*<0, and 1-*η* may be negative when *θ*>0. It is possible because it is the composition of motions of a fluid element which is concerned, not the mixing of two matters.

The above stated large fluctuations can explain the strange effect mentioned by McComb (1992). When a speed is reached in pipe--flow experiment, a critical *Re* is also reached, and the manometer reading begins to oscillate wildly. This behavior continues over a range of speeds, until a speed is reached where the manometer reading steadies again and thereafter remains steady.

Since *η* is same at a cross section, u_{Tran} is a weighted superposition of laminar and turbulent values at same *Re* in transition region. One can have $\overline{u}_{Tran}$ and $\tilde{u}_{Tran}$ by integrating Eq. (28) along a radius, which are

$$\overline{u}_{Tran} = (1-\overline{\eta})u + \overline{\eta}\,\overline{u} \tag{39a}$$

$$\tilde{u}_{Tran} = \overline{\eta}\,\tilde{u} + \tilde{\eta}(\overline{u} - u + \tilde{u}) \tag{39b}$$

These are the mean longitudinal velocity and the instantaneous fluctuation velocity. They can be measured at the center line in flow experiment, and the results can be compared with those predicted by Eq. (39).

Water at room temperature was used in pipe-flow experiments by Durst & Ünsal (2006), Nishi, Ünsal, Durst & Biswas (2008) and Nishi (2009), which includes many forced transitions and a small quantity of natural transitions without heating. According to $Re=\rho\overline{U}\,d_i/\mu$, experiments with different *Re* can be conducted by changing the mass flux of water, then the statistics of the longitudinal velocity and its fluctuations can be measured between the start and end of transition. The present derivations can be used to explain such transition processes in which the physical properties are constant.

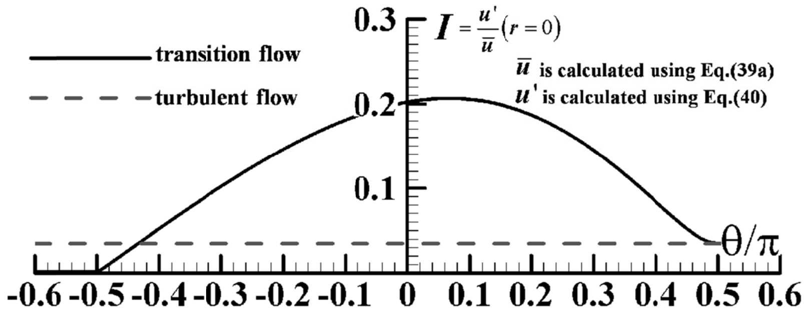

Figure 4. The large overshoot of I at the center line in transition region

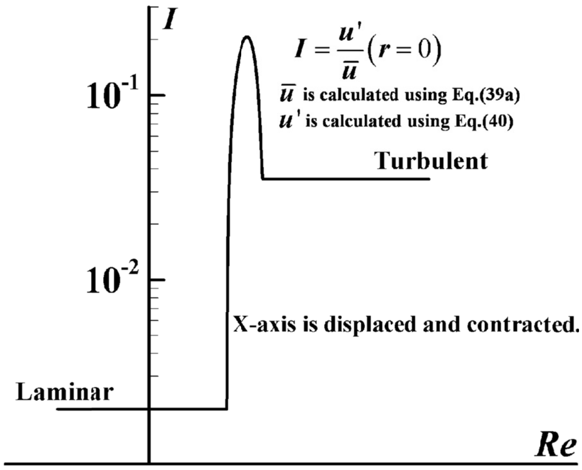

Figure 5. The large overshoot of I at the center line in transition region, both the shape and values can be compared with the measurements by Durst & Ünsal (2006)

The difference between laminar and turbulent mean velocities at same *Re* is large compared with $\tilde{u}$, so from Eq. (39) one can see there should be an *overshoot* of the longitudinal velocity fluctuations in the process of laminar-to-turbulent transition of pipe-flow. This was clearly measured by Durst &Ünsal (2006). For comparison one can calculate the fluctuation intensity I ($= u'/\bar{u}_{Tran}$) at the center line. If the change of $u'_{r=0}/\bar{u}_{r=0}$ for fully turbulent flow with *Re* is ignored, and the data in figure 3 of Durst &Ünsal (2006) are chosen for estimation, which are $\bar{u}_{r=0} = 1.4U_{mean}$ and $u'_{r=0} = 0.035\bar{u}_{r=0}$ for turbulent flow, and $u_{r=0} = 2.0U_{mean}$ for laminar flow (in which the fluctuation intensity is about 0.002), Eq. (39) gives a peak value of 20.7% at $\theta/\pi\approx 0.07$ for I at the center line, which is shown in figure 4. The value of measurement of Durst &Ünsal (2006) is around 20% when Re_L>4000. In figure 4, the value of u' is defined from Eq. (39b) as following

$$u' = \sqrt{\overline{\tilde{u}^2_{Tran}}} = \sqrt{\overline{\eta}^2\,\overline{\tilde{u}^2} + \overline{\tilde{\eta}^2}\left[\left(\bar{u}-u\right)^2 + \overline{\tilde{u}^2}\right] + \overline{\tilde{u}^2_{Lam}}} \tag{40}$$

In Eq. (40) the fluctuation intensity (about 0.002 in figure 3 of Durst & Ünsal (2006)) is considered for laminar flow.

Figure 5 is a different form of figure 4. The *y*-axis is in logarithm scale and the *x*-axis is displaced and contracted in figure 5, so one can easily make a comparison with the measurements by Durst & Ünsal (2006). Both the shape and values in figure 5 are very close to those in figures 3, 5 and 10 (measurements) of Durst & Ünsal (2006) when Re_L>4000 (the experimental data are inappropriate to be extracted so that one can make a detail comparison). The similar measurement of such overshoot was given by Nishi, Ünsal, Durst & Biswas (2008) and Nishi (2009). It was also reported by Durst & Ünsal (2006) that the overshoot of I did not result in a corresponding overshoot of the friction factor (should be the averaged value). This can also be explained by the present formulations.

The measured mean values of longitudinal velocity and its fluctuations at the center line during the forced transition were given by Nishi (2009). They are compared with present predictions in figure 6 and figure 7. The change of $u'_{r=0}/\overline{u}_{r=0}$ for fully turbulent flow with *Re* is ignored in predictions. From figure 6 one can see that present predictions using Eq. (39a) agree with the measurements for the longitudinal velocity at the center line during the transition. From figure 7 one can see that present predictions using Eq. (40) approximately give the trend of the mean values of fluctuation of longitudinal velocity at the center line during the transition when $Re_L \approx 2120$. But the predictions have a larger peak value, and the values or even the trends are different from those of the measurements just before or after the transition. From figure 10 of Durst & Ünsal (2006) one can see that the fluctuation intensities (*I*, see figure 5) of longitudinal velocity at the center line when $Re_L<3000$ have smaller peak values than those when $Re_L>4000$. The reason needs further studies. When $Re_L>4000$, all the experimental data by Durst & Ünsal (2006), Nishi, Ünsal, Durst & Biswas (2008) and Nishi (2009) are inappropriate to be extracted for detail comparisons. For the data further before or after the transition, the calculated values and trends in figure 7 agree with those of the measurements by Durst & Ünsal (2006), Nishi, Ünsal, Durst & Biswas (2008) and Nishi (2009).

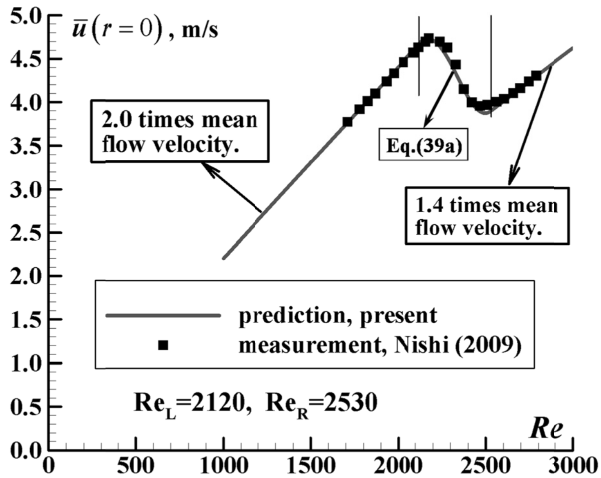

Figure 6. The calculated and measured mean values of longitudinal velocity at the center line

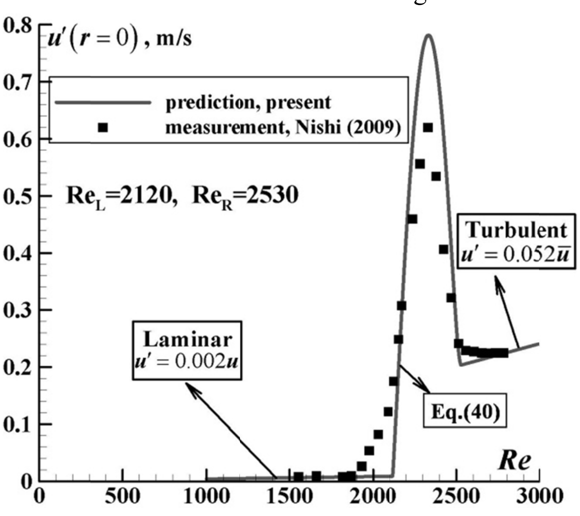

Figure 7. The calculated and measured mean values of the fluctuations of longitudinal velocity at the center line

$\overline{\sigma_{Tran,c}}$ in Eq. (32) is 0 when Eq. (33) are valid. So Eq. (31) can be rewritten as

$$\overline{\sigma_{Tran}} = \overline{\sigma_{Turb,\mathrm{X}}} + \overline{\sigma_{Turb,v}} + \left(1-\overline{\eta}\right)\left(\sigma_{Lam,\Theta} + \sigma_{Lam,V}\right) + \overline{\eta}\left(\overline{\sigma_{Turb,\Theta}} + \overline{\sigma_{Turb,u}}\right) \quad (41)$$

Comparing Eq. (41) with Eq. (31), one can see that the evolution of transition takes place in such a way that, the negative entropy production due to the composition of two flow modes balances out the positive entropy production due to the fluctuation of η. In this way $\overline{\eta}$ progresses continuously from 0 to 1, and the entropy production in transition flow is formally a weighted superposition of those in laminar and turbulent flows at same *Re*.

In laminar-to-turbulent transition η is adopted to describe the composite flow. The evolution of transition flow is given by Eq. (38) which is derived from Eq. (34). Equation (34) gives the fluctuation of η. Different form of Eq. (34) leads to different form of Eq. (38). From the comparisons with the experimental phenomena given above, one can conclude that the statistics of fluctuations and the flow and convective heat transfer behaviors of natural transition flow inside the tube can be explained using the present derivations, and the fluctuation of η governs the evolution of natural transition flow inside the tube.

Lastly, if a different η is adopted for temperature, the corresponding entropy production can be obtained in a similar form to Eq. (31). Because of the weak coupling of the equations for solving velocity and temperature, and the separateness of the square terms of temperature and velocity gradients in the expression of entropy production, this adoption does not change much the discussions above.

7. Conclusions

The statistics of fluctuations and the flow and convective heat transfer behaviors of natural transition flow inside an electrically heated circular tube can be explained using composition of motions, the fluctuations of composite ratios, and the minimum entropy production criterion.

(1)The natural transition flow in the tube can be decomposed into fully developed laminar and turbulent ingredients. The composite flow in transition region are defined by the composite ratios which are the proportions of the two flow modes.

(2)The composite ratios fluctuate in transition flow, and the fluctuations of the composite ratios govern the natural transition behavior.

(3)The process of laminar-to-turbulent transition inside the tube, can be compared with phase transitions of the second kind in thermodynamic equilibrium system. The conceptions for the description of the latter, such as order parameter and the fluctuation range, can be adopted in the study of the laminar-to-turbulent transition.

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