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Quasi Static Axial Compression of Thin Walled Aluminum Tubes: Analysis of Flow Stress in the Analytical Models

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Abstract

This paper presents experimental work on quasi static compression tests on aluminum AA 6063 circular and square tubes. Specimen tubes with ratio of R/t = 12 and b/t = 24 for circular and square tubes respectively were prepared and validated with several analytical model developed by previous researchers. Two definitions of flow stress were used for validate the test result, first were proposed by Abramowicz and Jones (Wierzbicki & Abramowicz, 1983), defined as mean stress between yield stress and ultimate tensile stress. Secondly, using ultimate tensile stress as flow stress. For circular tubes test result is in better agreement with the analytical model by Guillow et al. (Guillow, Lu, & Grzebieta, 2001) using ultimate tensile stress as flow stress. Meanwhile for square tubes, test result agrees better with the analytical model by Abramowicz and Jones (Abramowicz & Jones, 1984a) using flow stress as the mean stress between yield stress and ultimate tensile stress.

Keywords: energy absorption, compression, thin walled tubes, flow stress, analytical model

1. Introduction

Safe design of components and systems for vehicle is of interest to general public. The impact of transport vehicle as an example is an unfortunate but common daily occurrence. It is becoming apparent that, in the future, transport structures will have to be designed to minimize effects from impacts and crashes. The current trend in producing lighter structures puts greater demands on the designer since more aspects of design become critical as the weight is reduced, and working stresses become closer to the ultimate strengths of the material (Abramowicz, 2003; Sun, Xu, Li, & Li, 2014).

Aluminum is quite a new material in modern car body design. A 25% weight reduction can be achieved by using aluminum compared to conventional steel structures. The lower weight reduces the fuel consumptions and the emission carbon dioxide (Kim & Wierzbicki, 2001; Kim, 2002).

An energy absorber is a system that converts, totally or partially, kinetic energy into another form of energy. Energy converted is either reversible, like strain energy in solids, or irreversible like plastic deformation energy. When designing a collapsible energy absorber, one aims at absorbing the majority of the kinetic energy of impact within the device itself in an irreversible manner, thus ensuring that human injuries and component damages are minimal (Abdewi, Sulaiman, Hamouda, & Mahdi, 2008; Abramowicz, 2003; Alghamdi, 2001; Jones, 2010; Reid & Reddy, 1986; Reid, 1993; Salehghaffari, Tajdari, Panahi, & Mokhtarnezhad, 2010; Santosa & Wierzbicki, 1998; Seitzberger et al., 2000; X. W. Zhang, Su, & Yu, 2009).

1.1 Thin Wall Tube

The earliest theoretical analysis of thin walled tube was pioneered by Alexander(Alexander, 1960). He proposed a rigid, perfectly plastic and simplified deformation pattern of progressive crushing of a circular thin wall tube. The model consisted of two limbs with a plastic hinge between the limbs. The plastic hinges are developed from the total of inside or outside folding and the overall length is potentially to be crushed during deformation. This model illustrates the general approach which appears to be basic the basic platform of many later studies. For example, Abramowicz and Jones (Abramowicz & Jones, 1984a) improved Alexander (Alexander, 1960) model, and introduced analytical model for square tubes. Wierzbicki and Bhat (Wierzbicki & Bhat, 1986) enhanced the analytical model by stiffening phase of the tube resistance. Guillow et al. (Guillow et al., 2001) improved the model by expand the ratio D/t to 10-450.

The potential of thin wall tubes as an excellent energy absorber is further explored by investigating the influence of geometrical structure on its energy absorption capacity. The effect of length, wall thickness and diameter are studied by varying these parameters. On the contrary, it has been observed that the deformation may occur in overall Euler buckling mode which is undesirable in term of energy absorption if the length of tube is greater than its critical length. Wall thickness and diameter appear to affect the impact response of thin wall tubes in the form of wall thickness to radius ratio. It showed that the initial peak stress remains at constant value with the same value of the ratio. In another study, the energy absorption is noted to be greater with increasing thickness for smaller section tubes.

Then new of the shape of tubes were introduced to study behavior and develop analytical models such as square tubes, (Abramowicz & Jones, 1984b, 1986, 1997; Alghamdi, 2001; Bodlani, Yuen, & Nurick, 2009; DiPaolo, Monteiro, & Gronsky, 2004; Feraboli, Wade, Deleo, & Rassaian, 2009; Fyllingen, Hopperstad, & Langseth, 2007; Hanssen, Langseth, & Hopperstad, 2000; Jensen, Langseth, & Hopperstad, 2004; Jones & Abramowicz, 1985; Jones, 2003, 2010; Kim, 2002; Langseth, Hopperstad, & Berstad, 1999; Langseth & Hopperstad, 1996; Ma & You, 2013; Mamalis & Johnson, 1983; Reid, 1993; Santosa & Wierzbicki, 1998; Tarigopula, Langseth, Hopperstad, & Clausen, 2006; Wierzbicki, Recke, Abramowicz, Gholami, & Huang, 1994; Yin, Wen, Liu, & Qing, 2014; Yuen & Nurick, 2009; X. Zhang, Cheng, You, & Zhang, 2007; X. Zhang, Wen & Zhang, 2014; X. W. Zhang et al., 2009), corrugated tubes,(Abdewi, Sulaiman, Hamouda, & Mahdi, 2006; Abdewi et al., 2008; Chen & Ozaki, 2009; Elgalai, Mahdi, Hamouda & Sahari, 2004; Mahdi, Mokhtar, Asari, Elfaki & Abdullah, 2006; Singace & El-Sobky, 1997) multicorner columns, frusta,(Alghamdi, 2001; Mamalis & Johnson, 1983) struts,(Alghamdi, 2001) honeycomb cells,(Alghamdi, 2001; Santosa & Wierzbicki, 1998) sandwich plates(Alghamdi, 2001; Mohr & Wierzbicki, 2003) and some other special shapes such as stepped circular thin walled tubes and top hat thin walled sections (Jones, 2010; Tarigopula et al., 2006).

The performance of the energy absorbing is evaluated by plotting a force-displacement curve. The total absorbed energy signified by the area under force-displacement curve. The performance of thin walled tubes can also be characterized quantitatively by applying several parameters namely specific energy absorber (SEA), crush force efficiency (CFE), mean force (P mean) and peak force (P max).

The aims of this paper are, (a) to perform compression quasi static test of circular and square aluminum alloy AA6063 temper T5; and (b) to validate an existing analytical model with test results.

1.2 Analytical Model

Alexander (Alexander, 1960) presented a rigid plastic analysis for the concertina mode of deformation. His model is based on the plastic work required for bending and stretching of an extensible thin cylinder. He gave the following expression for the mean crushing load P_{mean};

$$P_{mean} = \left(20.75\sqrt{\frac{2R}{t}} + 6.283\right) M_0 \quad (1)$$

where R is radius tube, t is thickness of tube and M0 is full plastic bending moment.

Abramowicz and Jones (Abramowicz & Jones, 1986) conducted axial compression tests on a range of thin walled circular and square steel tubes. They analytically considered both axi-symmetric and non-symmetric mode. For the square tubes, they also developed analytical models for axis-symmetric mixed collapse type A, type B and extensional collapse mode.

Wierzbicki and Bhat (Wierzbicki & Bhat, 1986) studied a moving hinge solution for axi-symmetric crushing tubes, where they developed new analytical model for circular tube. The solution features a stiffening phase of the tube resistance which follows the softening phase during the formation each buckle.

Wierzbicki et al. (Wierzbicki, Bhat, Abramowicz, & Brodkin, 1992) carried out the analysis of an axially compressed circular tube deforming in progressive axi-symmetric and they assumed an eccentricity factor relate the inward and outward parts of the folds. Singace et al. (Elsobky & Singace, 1996; Singace, Elsobky, & Reddy, 1995) reexamines the problem and produces a value eccentricity factor which conforms the experimental works.

Values of the critical angles required for the formation of the inward and outward fold s obtained from the analysis were substantiated by those obtain from experiments.

Guillow et al. (Guillow et al., 2001) conducted almost 70 experimental works to expand D/t ratio over previous study to D/t =10-450. A chart mode classification was developed and collapse mode were observed for L/D ≤ 10. P mean was developed and it was found that test results for both axi-symmetric and non-symmetric modes lie on single curve.

Another researchers followed another approach, instead purely analytical, they used experimental data of crushed tubes to develop empirical relations. Magee and Thornton (Magee, 1978) used crush test data of different columns of several different section geometries and developed a relationship for P mean for tubes;

$$P_{mean} = \eta \sigma_u \phi A_0 \quad (2)$$

where η is structural effectiveness, σ_u is ultimate tensile strength, ϕ is relative density, and A_0 is overall section area defined by the outer circumference.

For a circular tubes $\eta = 2\phi^{0.7}$, $\phi = 4t/2R$, P_{mean} becomes;

$$P_{mean} = 2\left(\frac{4t}{2R}\right)^{0.7} \sigma_u \left(\frac{4t}{2R}\right) A_0 \quad (3)$$

Meanwhile, for square tube, $\eta = 1.4\phi^{0.8}$, $\phi = 4t/b$, P_{mean} becomes; where b is side length of side.

$$P_{mean} = 17t^{1.8}\sigma_u b^{0.2} \quad (4)$$

2. Experimental Details

Compression tests were conducted using Shimadzu universal testing machine with loading capacity up to 250 kN. Loading rate and total crushing displacement were set at 5 mm/min and total crushing of 100 mm respectively. This loading rate considered quasi static since the strain rate is in the range of 10^{-4} s^{-1}.

Aluminum Alloy 6063 temper with T5 was used as tested material, which was cooled from elevated temperature shaping process then artificially aged. Two type of tube were used namely square(S) and circular(C) tubes. Table 1 gives dimension for both tubes. Ratio R/t for circular tube and ratio b/t for square tube was 11.99 and 23.99. Meanwhile for the both tubes ratio L/D was 5.33, and based on chart developed by Guillow et al. (Guillow et al., 2001), circular tube was expected having collapsible mixed mode.

Table 1. Dimension of Tubes

Sample	Length (mm)	Thickness (mm)	Outer diameter (mm)	Side length (mm)	Weight (kg)
C1	177.8	1.59	38.1		0.07
C2	177.8	1.59	38.1		0.07
C3	203.2	1.59	38.1		0.08
C4	203.2	1.59	38.1		0.08
S1	203.2	1.59		38.1	0.11
S2	203.2	1.59		38.1	0.11
S3	203.2	1.59		38.1	0.11
S4	203.2	1.59		38.1	0.11
S5	203.2	1.59		38.1	0.11

The chemical composition and material properties for this tube are given as per Tables 2 and 3 respectively.

Table 2. Chemical Composition of Tubes

%	Cu	Fe	Mg	Mn	Si	Ti	Zn	Cr	Other	Al
Min*	0.01	0.17	0.48	0.03	0.44	0.01	0.01	0.01	0.10	The rest
Max*	0.10	0.17	0.48	0.03	0.44	0.01	0.10	0.10	0.10	

*All in weight %

Table 3. Material Properties of Tubes

Yield Stress	Ultimate Tensile Stress	Density	Elastic Modulus
$\sigma_{0.2}$(MPa)	σ_u(MPa)	ρ(kg/m^3)	E(GPa)
118	152	2.71 x 10^3	69

3. Test Results

The test results are summarized in Table 4. The force-displacement curve for both tubes are presented in Figures 1 and 2. Initially tubes behave elastically until force rises to maximum load (peak), but as instability develops it falls off rapidly until a first fold is developed. A series of fluctuations about mean force develops, the peaks and troughs being directly related to the form and folding at the various buckling levels. This deformation pattern is referred to as progressive crushing.

Figure 3 shows comparison between square and circular tubes based on average data from samples. From Figure 3a, average P_{max} for circular tube is 73% more than square tube. Meanwhile average P_{mean} for the circular tubes is 12% more than square tube. Energy absorbed by circular tube almost twice than the square tube (Figure 3b). Even specimen C1 and C2 of different length than others, result in the similiar value for P_{max} and P_{mean}. Thus, length is insignificant to the response of tubes in compression. C1 has lower energy absorbed due to global buckling towards final displacement.

Table 4. Summary of Test Result

Sample	**P_{max}(kN)**	**Energy(kNmm)**	**P_{mean}(kN)**	**CFE**	**SEA(kJ/kg)**
C1	37.07	1859.43	18.59	0.50	26.56
C2	37.69	2087.02	20.87	0.55	29.82
C3	37.41	2124.37	21.22	0.57	26.56
C4	37.90	2131.81	21.32	0.56	26.65
C5	37.62	2125.40	21.25	0.57	26.57
Average Circular	37.54	2065.61	20.65	0.55	27.23
S1	22.94	966.76	9.67	0.42	8.79
S2	21.15	897.40	8.97	0.42	8.16
S3	22.61	1048.77	10.49	0.46	9.53
S4	21.56	995.30	9.95	0.46	9.05
S5	20.09	948.25	9.48	0.47	8.62
Average Square	21.67	971.30	9.71	0.45	8.83

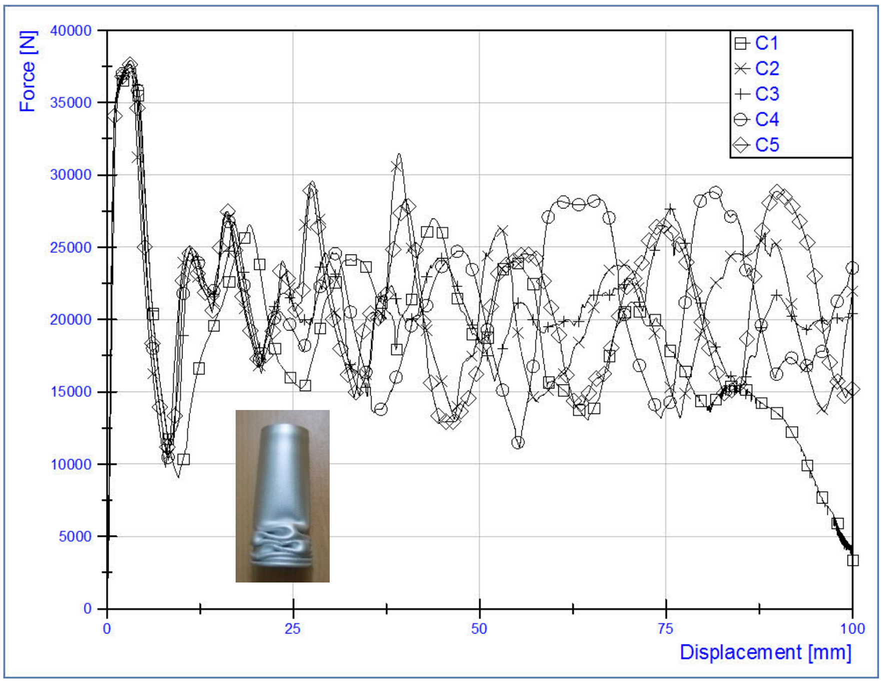


Figure 1. Force –displacement characteristic for circular tubes. An inset shows deformation pattern of the circular tube (C5 sample)

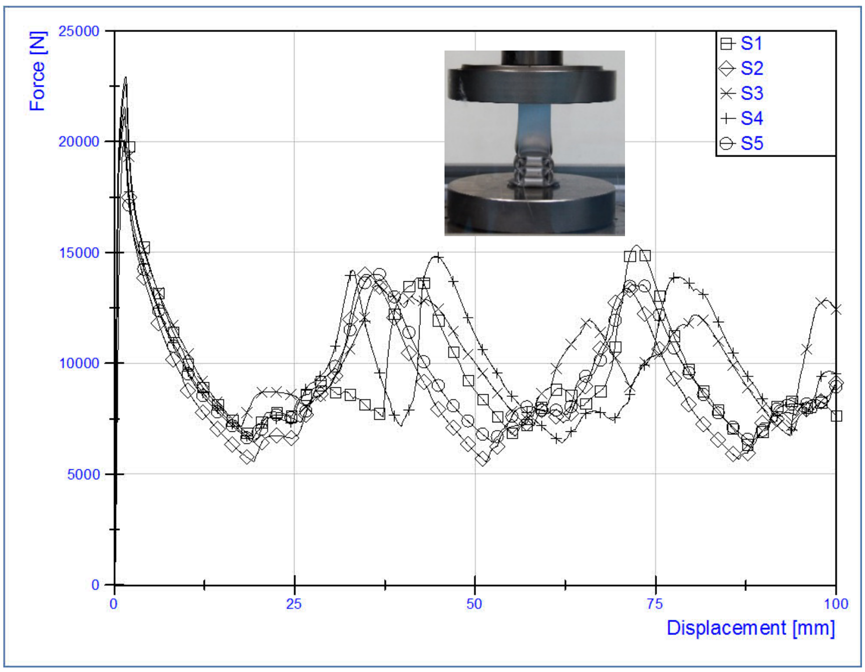
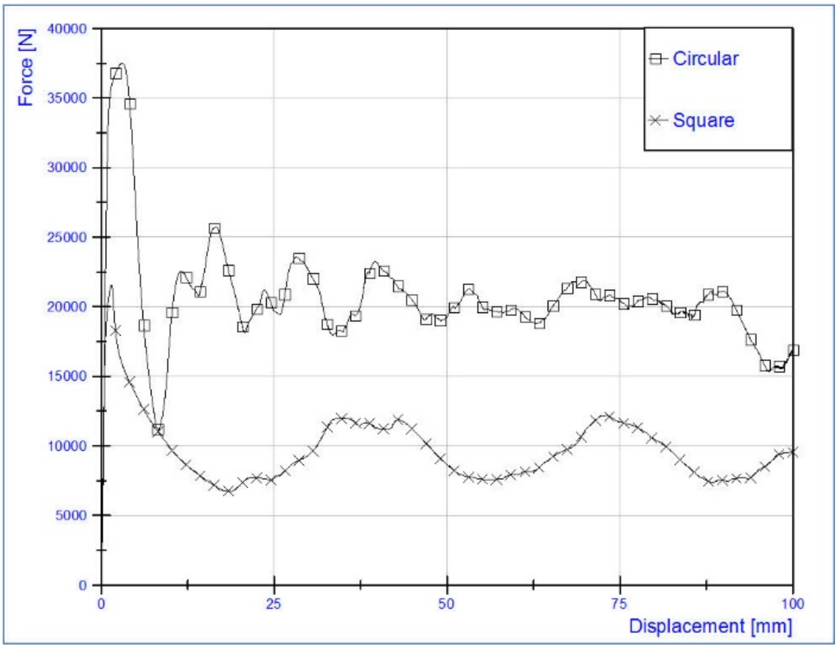
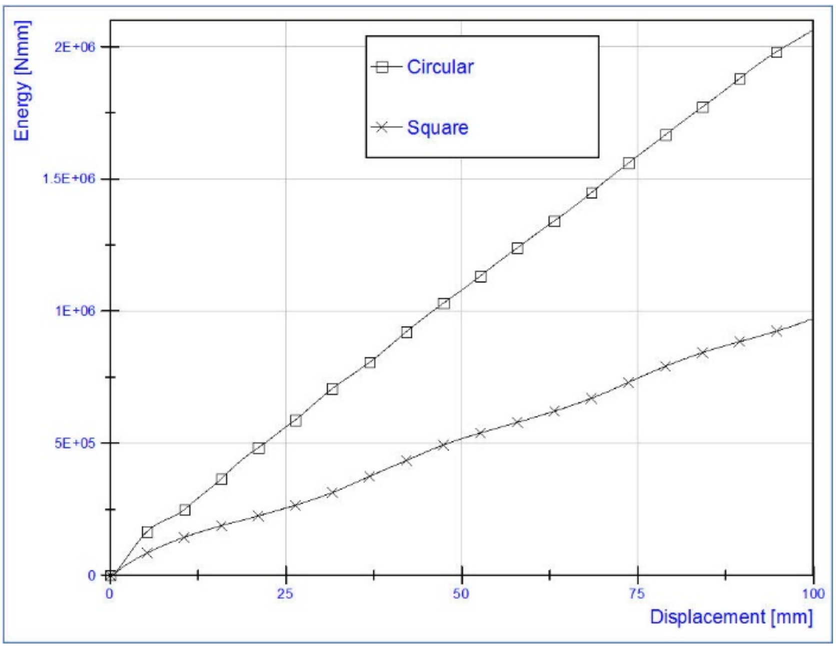


Figure 2. Force-displacement curve for square tubes. An inset shows deformation pattern of the square tube (S1 sample)



(a)



(b)

Figure 3. Comparison between square and circular tubes based on average data for (a) Force –displacement and (b) Energy –displacement curve

Table 5. Comparison between P_{mean} analytical models for circular tubes

Analytical Model (Circular Tubes)	Equation	P_m Analytical * (kN) $\left(\text{using } \sigma_0 = \frac{\sigma_u + \sigma_{0.2}}{2}\right)$	P_m Analytical + (kN) $(\text{using } \sigma_0 = \sigma_u)$
Alexander Model (Alexander, 1960)	$\frac{P_m}{M_0} = 20.75\sqrt{\frac{2R}{t}} + 6.283$	9.19	10.35
Abramovicz & Jones Model (Abramowicz & Jones, 1984a)	$\frac{P_m}{M_0} = 22.366\sqrt{\frac{2R}{t}} + 11.766$	10.35	11.65
Abramovicz & Jones Model (Abramowicz & Jones, 1986)	$\frac{P_m}{M_0} = 25.230\sqrt{\frac{2R}{t}} + 15.09$	11.83	13.31
Wierzbicki & Bhat Model (Wierzbicki & Bhat, 1986)	$\frac{P_m}{M_0} = 35.22\sqrt{\frac{2R}{t}}$	14.71	16.56
Wierzbicki et al Model (Wierzbicki et al., 1992)	$\frac{P_m}{M_0} = 31.74\sqrt{\frac{2R}{t}}$	13.26	14.93
Wierzbicki Model (Wierzbicki & Abramowicz, 1983)	$\frac{P_m}{M_0} = 62.88\sqrt[3]{\frac{2R}{t}}$	15.47	17.42
Singace et al Model (Singace et al., 1995)	$\frac{P_m}{M_0} = 22.27\sqrt{\frac{2R}{t}} + 5.632$	9.78	11.01
Guillow et al Model (Guillow et al., 2001)	$\frac{P_m}{M_0} = 72.3\left(\frac{2R}{t}\right)^{0.32}$	17.04	19.19
C. L Magee & P.H Thornton Model (Magee & P. H., 1978)	$P_m = 2\left(\frac{4t}{2R}\right)^{0.7}\sigma_u\left(\frac{4t}{2R}\right)A_0$	-	16.52

Table 6. Comparison between P mean analytical models for square tubes

Analytical Model (Square Tubes)	Equation	P_m Analytical * (kN) $\left(\text{using } \sigma_0 = \frac{\sigma_u + \sigma_{0.2}}{2}\right)$	P_m Analytical + (kN) $(\text{using } \sigma_0 = \sigma_u)$
Abramovicz & Jones Model (Abramowicz & Jones, 1984b)	$\frac{P_m}{M_0} = 38.12\sqrt[3]{\frac{b}{t}}$	9.38	10.56

Abramovicz & Jones Model (Abramowicz & Jones, 1986)	$\frac{P_m}{M_0} = 52.22\sqrt[3]{\frac{b}{t}}$	12.85	14.46
Abramovicz & Jones Model (Type A)(Abramowicz & Jones, 1986)	$\frac{P_m}{M_0} = 42.92\sqrt[3]{\frac{b}{t}} + 3.17\left(\frac{b}{t}\right)^{\frac{2}{3}} + 2.04$	12.98	14.61
Abramovicz & Jones Model (Type B)(Abramowicz & Jones, 1986)	$\frac{P_m}{M_0} = 45.90\sqrt[3]{\frac{b}{t}} + 1.75\left(\frac{b}{t}\right)^{\frac{2}{3}} + 1.02$	12.63	14.22
Abramovicz & Jones Model (Extension mode)(Abramowicz & Jones, 1986)	$\frac{P_m}{M_0} = 32.64\sqrt{\frac{b}{t}} + 8.16$	14.33	16.13
C. L Magee & P.H Thornton Model(Magee P. H., 1978)	$P_m = 17t^{1.8}\sigma_u b^{0.2}$	-	12.30

Tables 5 and 6 shows the comparison between various calculated P_{mean} using several analytical models for both tubes. Full plastic bending moment, M_0 is derived from equation,

$$M_0 = \frac{1}{4}\sigma_o t^2 \tag{5}$$

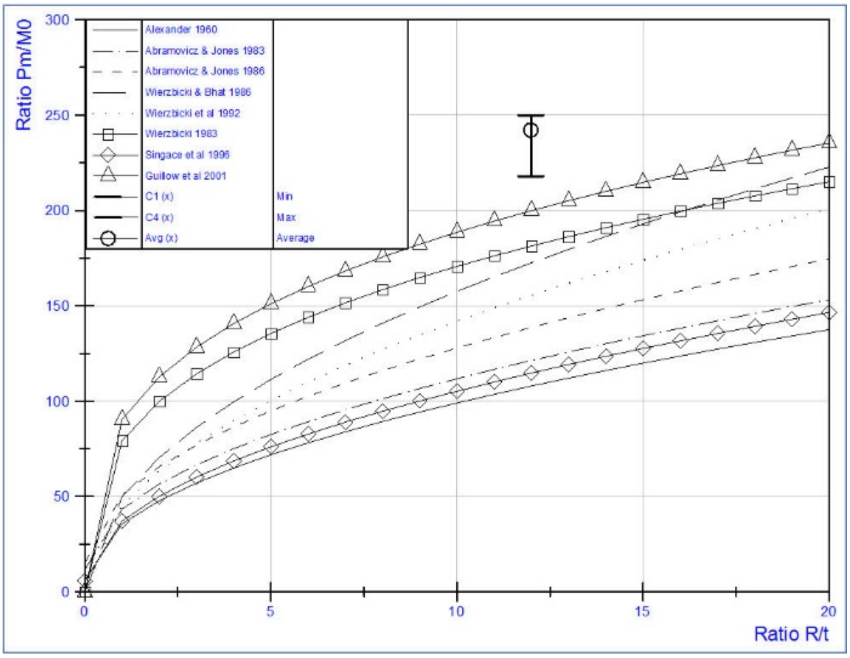
where σ_o and t refer to the flow stress and thickness of tubes respectively. Different researchers have used various different measures for the flow stress, σ_o. The author would like to use mean stress between yield stress and ultimate tensile strength (Equation 6), as it was proposed by Abramowicz and Jones(Abramowicz & Jones, 1986) and to propose ultimate tensile stress as flow stress, σ_o(Equation 7);

$$\sigma_o = \frac{\sigma_{0.2}+\sigma_u}{2} \tag{6}$$

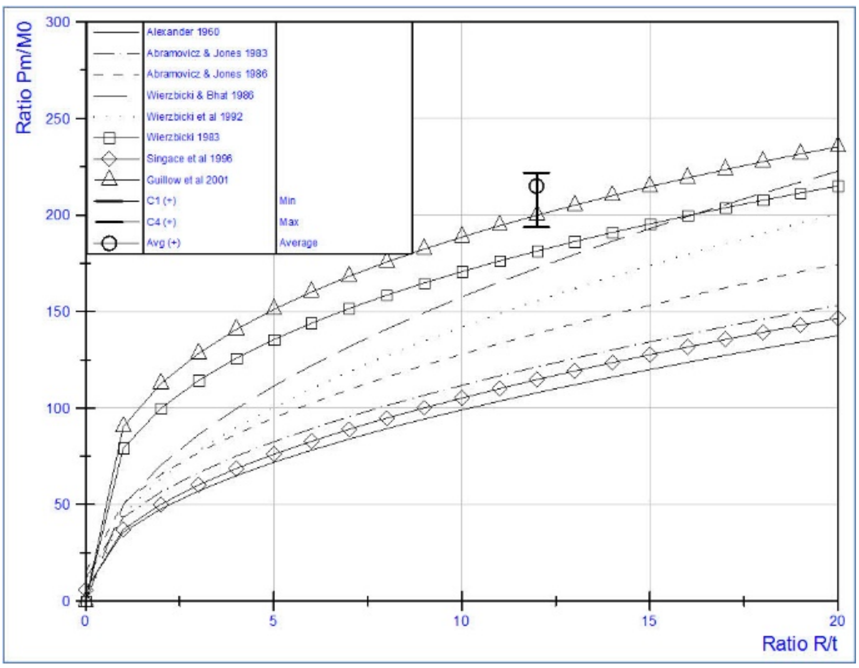
$$\sigma_o = \sigma_u \tag{7}$$

where $\sigma_{0.2}$ is the yield stress based on 0.2% strain and σ_u is ultimate tensile stress to compare the both equation to calculate P mean using analytical model. In Table 5 and 6, calculated P mean using flow stress Equation 6 and Equation 7 are referred as P_m Analytical * and P_m Analytical $^+$.

Figures 4 and 5 show comparison between test results and several analytical model for circular and square tubes respectively. P_{mean} for circular tubes shows a much better agreement between the test results and the model proposed by Guillow et al. (Guillow et al., 2001) using flow stress σ_o defined by Equation 7(Figure 4(b)). Meanwhile for square tubes, P_{mean} from test results is more accurately represented by the analytical model of Abramowicz and Jones (Abramowicz & Jones, 1984a) using flow stress σ_o defined by Equation 6 (Figure 5(a)).

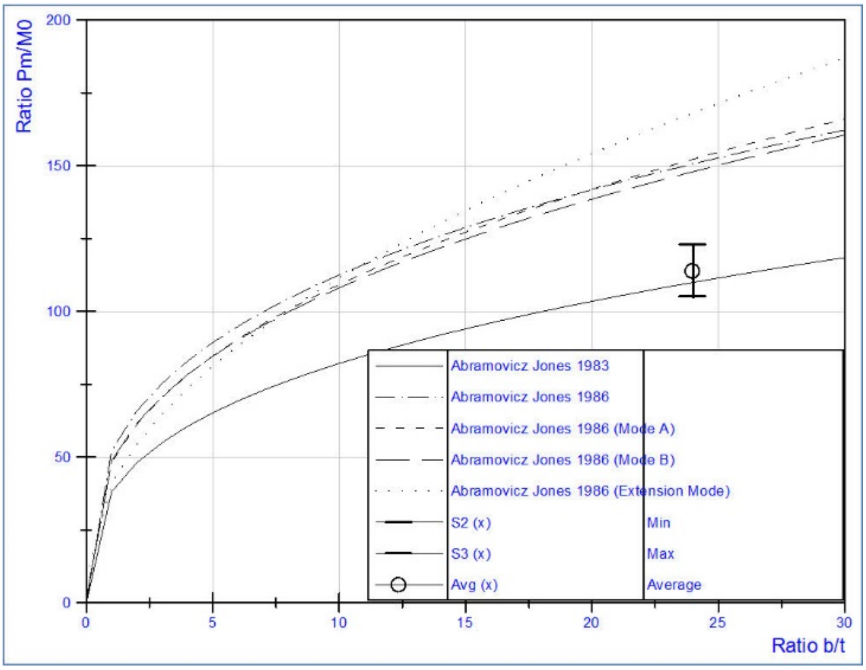


(a)

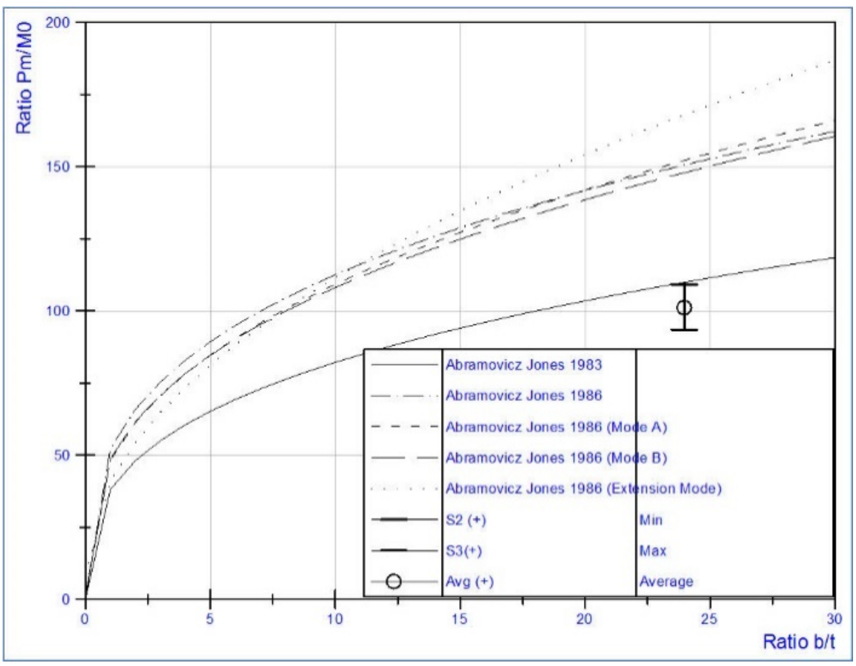


(b)

Figure 4. Comparison data between test result and several analytical model for circular tubes, (a) Using Flow Stress Equation 6 and (b) Using Flow Stress Equation 7



(a)



(b)

Figure 5. Comparison data between test result and several analytical model for square tubes, (a) Using Flow Stress Equation 6, and (b) Using Flow Stress Equation 7

4. Conclusions

Experimental quasi static test results of circular and square tubes are presented in this paper. Then, the test results are compared in order to verify and validate several analytical models proposed by previous researchers. The main conclusions from this study can be summarized as follows:

a) Circular tube gives the best crushing performance due to P_{max} for circular tube is 57% higher than the square tube. P_{mean} for circular tube is 46% higher than the square tube. Energy absorbed by the circular tube is twice more than the square tube.

b) Comparison between analytical model and test results, shows that P $_{mean}$ for circular tube is closer to model proposed by Guillow et al. (Guillow et al., 2001) and flow stress σ_o by using in Equation 7, which is $\sigma_o = \sigma_u$.

Meanwhile, P $_{mean}$ for square tube are closer to Abramowicz and Jones(Abramowicz & Jones, 1984a) model, with the flow stress is defined using in Equation 6, ($\sigma_o = \frac{\sigma_{0.2}+\sigma_u}{2}$).

Further works can be considered for testing specimens at various ratio of R/t and b/t for circular and square tube respectively.

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