Modern Applied Science; Vol. 9, No. 6; 2015
ISSN 1913-1844 E-ISSN 1913-1852
Published by Canadian Center of Science and Education

Research on Satellite Orbit Simulation Based on Recursive Algorithm

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Received: December 4, 2014 Accepted: December 7, 2014 Online Published: April 26, 2015

doi:10.5539/mas.v9n6p128 URL: http://dx.doi.org/10.5539/mas.v9n6p128

Abstract

In this paper, recursive algorithm is presented for the simulation of the soft landing orbit of Chang'e-3, for making sure the location of the landing orbit and the perilune point. Recursive algorithm is a method of derivation from the state in a moment to the next moment through their relationship. Frist, the coordinate model of soft landing and the relevant parameters are set up. Second, the soft landing orbit of Chang'e-3 derivate by recursive algorithm. Third, the soft landing orbit and the perilune point are carried out after analyzing the result from the simulation. The simulation of the soft landing orbit shows that recursive algorithm is innovative in the simulation of orbit in high probability.

Keywords: perilune point, satellite orbit simulation, coordinate model of soft landing, recursive algorithm

1. Introduction

Chang'e-3 successfully landed on the moon, accelerating the development of our nation's lunar exploration industry. As one of the outer space exploration development goals, the moon is rich in resources and worth exploring. However, human cognition of the moon is still insufficient. Especially in the flight orbit, different kinds of uncertainty and risk bring up difficulty to lunar exploration and high accuracy of the orbit is very indispensable. Thus, the study of the landing orbit is of great significance.

The orbit landing on the moon is divided into three kinds. The first one is direct landfall orbit, which is an ellipses orbit linking the earth and the moon. The spacecraft can hit on the moon making a hard landing when it reaches this orbit. The second one is earth orbit-lunar landing orbit, which also is hard landing. The spacecraft enters into the earth parking orbit first, choses the best moment and runs toward the moon directly. The third one is lunar circular orbit, which includes earth parking orbit segment, earth-moon transfer orbit segment, lunar satellite orbit segment and moon-landing orbit segment. The orbit of the Apollo manned spacecraft and the unmanned lunar probe are the third one. This research will mainly discuss the third one.

As for the study on the simulation of the landing satellite orbit, Fan et al. (2012) put forward a satellite orbit simulation model based on the spline interpolation. Compared with Lagrange's interpolation and Chebyshev interpolation method, cubic-spline interpolation in fitting the satellite orbit improved the accuracy and degraded the computational complexity. Fang & Lai (2010) proposed the idea that satellite orbit could be forecast by STK, which was a toolkit software including the satellite database. But the STK was expensive and non-open source, which imposed restrictions on the simulation. Liu et al. (2009) tried to simulate the low orbit based on the lunar gravitational field, analyzed the gravitational field GLGM-2 and LP165P using Kaula criterion and proposed a reliable control scheme on satellite orbit.

As for the application of the recursive algorithm, Shan (2011) and Tian (2007) applied the recursive algorithm to the design of algorithmic program. Compared with the traditional algorithm, the recursive algorithm in designing algorithmic program not only improved the execution efficiency but also highly increased the correctness and reliability. Hou (2006), Xiao (1994), Qiao (1999) and Luo (1996) applied the recursive algorithm to the mathematics. Since all the recursive relations were derived from the mathematical formula, the correctness of recursive algorithm was guaranteed by the preciseness of mathematics. Ma et al. (2013), Tian et al. (2008) and Tu (1996) applied the recursive algorithm to the engineering. The recursive algorithm applied in the traffic Prediction, Time-Frequency Analysis and video conference coding showed that recursive algorithm was not limited to mathematics and program design but with high applicability in many field.

Referring to Annex1 and Annex 2 of the Mathematical Contest (2014), the landing site of Chang'e-3 is designated on the sinus iridum area which is rather flat. Due to the complexity of the landform, it is difficult to find a preferable landing site. Chang'e-3 started to descent in the shape of a parabola from the perilune point with 15km away from the surface in the relative velocity of 1.7 kilometers per second. Since there is no atmosphere around the moon, falling by parachute will not work. Thus, Chang'e-3 landed on the moon successfully by a variable thrust engine which slow the lander down during the landing.

The process of landing is divided into six phases. Landing preparation phase means that the satellite flies around the moon in an elliptical orbit and main deceleration phase starts from the perilune point to the layer with 3km away from the surface. Rapid adjustment phase is responsible for adjusting the attitude of the spacecraft. Rough obstacle avoidance phase and meticulous obstacle avoidance phase are both for identifying the lunar pits. Slow falling phase is the last phase in which the satellite falls vertically. With a long braking distance and high variation of velocity, the main deceleration phase plays a decisive role in the process of landing.

With an altitude of -2640m, the designated landing location in the sinus iridium area has the latitude and longitude of 19.51°W, 44.12°N. Chang'e-3 entered to the landing preparation orbit after circling in a polar orbit, which meant Chang'e-3 would fly across both poles of the moon. And the sinus iridium area presented in Figure 1 is distributed like a rectangular along the longitude direction. Due to the moon's rotation, the selecting of perilune point in the main deceleration phase would influence whether Chang'e-3 reached the designated landing location successfully in the sinus iridium area. Thus, it is important to start slowing Chang'e-3 down exactly in the perilune point in order to reach on the designated location accurately.

Therefore, this research will put forward a method of simulation of orbit based on the recursive algorithm and discuss the orbit of Chang'e-3 in the main deceleration phase. By analyzing the simulation of orbit and the designated landing location, the position of the perilune point of Chang'e-3 can be derived.

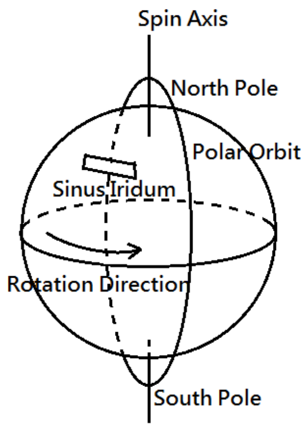

Figure 1. The Polar Orbiting of Chang'e-3

2. Satellite Orbit Simulation Based on Recursive Algorithm

2.1 Coordinate Model of Soft Landing

In the main deceleration phase, since the satellite is far away from the moon with long flight distance, it will bring much deviation in modeling if the moon is considered as a plane. Thus, it is necessary to consider the moon as a globe while setting up model. Under the assumption of the uniform gravitational field and the ignoration of the moon's rotation, the coordinate model of soft landing presented in Figure 2 is presented based on the axes coordinate system with the elliptical lunar orbit (100km×15km).

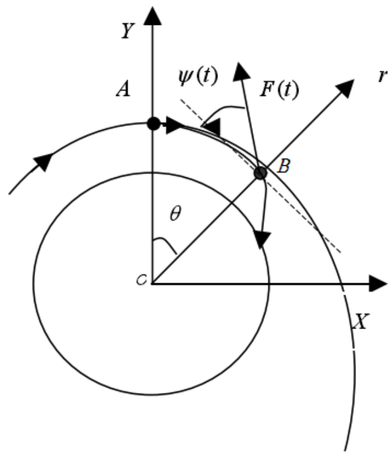

Figure 2. Soft Landing Model Based on Axes Coordinate System

Where, the movement of soft landing can be described in a plane. With the origin O at the center of the moon, soft landing model is presented based on the axes coordinate system Oxy . In this axes coordinate system, axis Oy points to the perilune point A , variable r denotes the distance between the satellite B and the center O , symbol θ denotes the angle between the axis Oy and axis Or , symbol F denotes braking thrust vector, R denotes the semi diameter of the moon and symbol ψ denotes the angle between the braking thrust vector F and the axis Or .

According to Newton's second law, the acceleration formula of the satellite can be defined as:

$$a_r = \frac{\bar{F}}{m} - \frac{GM}{r^2}\left(\frac{\bar{r}}{r}\right) \tag{1}$$

Where, symbol a_r denotes the acceleration vector of the satellite, symbol $\bar{F}$ is a thrust vector, symbol M is the mass of the moon, symbol m is the mass of the satellite, symbol G is the constant of universal gravitation.

Set

$$\mu = GM$$

Then the formula (1) can be rewritten in the orthogonal decomposition form, that is,

$$\begin{cases} a_x = \dfrac{F\sin\psi\sin\theta}{m} - \dfrac{F\cos\psi\cos\theta}{m} - \dfrac{\mu x}{r^3} \\ a_y = \dfrac{F\sin\psi\cos\theta}{m} + \dfrac{F\cos\psi\sin\theta}{m} - \dfrac{\mu y}{r^3} \end{cases} \tag{2}$$

Here,

$$\begin{cases} x = r\sin\theta \\ y = r\cos\theta \end{cases} \tag{3}$$

and symbol a_x denotes the acceleration vector along x-coordinate axis of the satellite, symbol a_y denotes the acceleration vector along y-coordinate axis of the satellite, symbol F is a thrust scalar of the satellite, symbol m is the mass of the satellite, symbol μ is a constant, symbol r denotes the distance between the satellite and the center of the moon.

By the above discussion, we can dreive the expression (2) about the movement from the perilune point with 15km away from the surface to the landing location with 3km away from the surface and the equation of acceleration along both x-coordinate axis and y-coordinate.

In the coordinate system of soft landing model, point A denotes the position of the perilune point with the coordinate of $(0, R+15\text{km})$ and point B denotes the landing location with 3km away from the surface. By the above discussion, the satellite rotates around the moon in the polar orbit. That is to say the longitude of the perilune point and the longitude of the landing location being almost the same. Besides, the latitude position of point A can be derived from the angle θ and the position of the point B by the rules of geometry, presented in Figure 3.

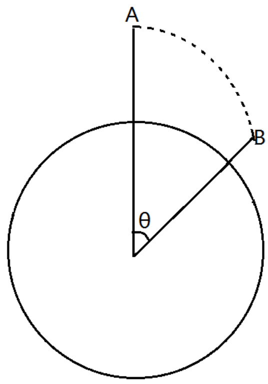

Figure 3. Angle between perilune point and landing point

Then, the acceleration vector a_x and a_y of every position in the axes coordinate system presented in Figure 2 can be calculated according to the formula (2). Based on the rules of geometry, the geometric state of every points in the x-coordinate axis can be confirmed, such as the angle θ, the angle ψ, the distance r. Based on the kinematical state of every points in the x-coordinate axis can be confirmed, such as the displacement on the x-coordinate axis and the y-coordinate axis Sx, Sy, the velocity on the x-coordinate axis and the y-coordinate axis v_x, v_y, the flight time t.

2.2 Recursive Algorithm and Simulation

Recursive algorithm is a method to reduce a problem into a set of sub-problems and solve them based on their recursive relationship. Analyzing the relationship between the sub-problems, starting from the original problem, deriving to the next sub-problem according to the recursive algorithm until the original problem has been solved. Recursive algorithm is applicable to these large-scale and complex problem which can be reduced to small-scale sub-problems.

Since the geometric state and the kinematical state in the axes coordinate system can be calculated according to the formula (2), the geometric state and the kinematical state in this moment can derived to the state of next moment based on the recursive algorithm. Setting the geometric and kinematical state of point A as the initial state, it can be derived a next state from the original state while moving a unit step ΔS in the x-coordinate axis. Thus, the following state can be worked out from its previous state until the satellite reached point B .The process of the recursive algorithm can be described as followed:

Step 1: Determination of the initial conditions

By the above discussion, the scope of landing trajectory is equivalent in x-coordinate axis with the interval of $[0, R]$, which can be divided into a series of unit interval with the increments ΔS . Similarly, the scope of landing trajectory is equivalent in y-coordinate axis with the interval $[0, R+15]$, which also can be divided into a series of unit interval with the increments ΔS , with perilune point in the height of $R+15\text{km}$.

Then, the original kinematical state of point A can be set as:

$$Sx(0)=0 \quad \text{and} \quad Sy(0)=R+15\text{km}$$

and the original geometric state of point A can be set as:

$$\theta(0)=0,\ \ \psi(0)=0 \quad \text{and} \quad r(0)=R+15\text{km}$$

and the original acceleration vector along x-coordinate axis and y-coordinate axis can be set as:

$$a_x(0)=\frac{F\sin\psi(0)\sin\theta(0)}{m}-\frac{F\cos\psi(0)\cos\theta(0)}{m}-\frac{\mu Sx(0)}{r(0)^3} \tag{4}$$

and

$$a_y(0)=\frac{F\sin\psi(0)\sin\theta(0)}{m}-\frac{F\cos\psi(0)\cos\theta(0)}{m}-\frac{\mu Sy(0)}{r(0)^3} \tag{5}$$

Where, the symbol Sx denotes the out-of-origin displacement in x-coordinate axis. Similarly, the symbol Sy denotes in y-coordinate axis.

Step 2: Determination of recursive relationship

Here, symbol ΔS denotes the unit step in the x-coordinate axis, which is represented in the form:

$$Sx(i) = Sx(i-1) + \Delta S \text{ , } t = 1, 2, 3, ...$$

According to the rules of geometry and kinematics, the state of the satellite will refresh after it moves a unit step ΔS in the x-coordinate axis, and the new state is expressed by the equation:

$$\begin{cases} v_x(i)^2 - v_x(i-1)^2 = 2a_x(i-1)\Delta S \\ t(i) = \dfrac{v_x(i) - v_x(i-1)}{a_x(i-1)} \\ v_y(i) = v_y(i-1) + a_y(i-1)t(i) \\ Sy(i) = Sy(i-1) - \dfrac{v_y(i)^2 - v_y(i-1)^2}{2a_y(i-1)} \end{cases} \tag{6}$$

Where, symbol $t(i)$ denotes the time interval from the state $(i-1)$ to the state (i),

and

$$\begin{cases} a_F \cos\psi(i) = a_x(i)\cos\theta(i) + a_y(i)\sin\theta(i) \\ \theta(i) = \arctan\dfrac{Sx(i)}{Sy(i)} \\ r(i)^2 = Sx(i)^2 + Sy(i)^2 \end{cases} \tag{7}$$

Where, symbol a_F denotes thrust acceleration.

Step 3: Ending condition of the algorithm

Every time the satellite moves a unit step ΔS in the x-coordinate axis, it is necessary to judge whether the main deceleration phase is over and the satellite enters to the next phase with the height of less than 3km. The judgment expression can be written in the form:

$$r(i) > R + H + 3\text{km}$$

Where, symbol H is the altitude of the sinus iridium area.

If the inequality is right, it means that the satellite is not in the main deceleration phase and enters to the next phase with the height of less than 3km. And then stop calculating the geometry and kinematics state. The current state marks the end of the process of recursive algorithm. It can be proved that the landing position B is

$$(Sx(i), Sy(i))$$

Where, the symbol $Sx(i)$ denotes the out-of-origin displacement in x-coordinate axis of the current state. Similarly, the symbol $Sy(i)$ denotes in y-coordinate axis of the current state. Besides, the total time from the perilune point to the landing location can be expressed by the formula:

$$T = \sum_{k=1}^{i} t(k)$$

and the current angle $\theta(i)$ can be calculated by formula (7).

Otherwise go to Step2. And then, keep on moving a unit step ΔS in the x-coordinate axis and calculating the next geometry and kinematics state with the formula

$$Sx(i+1) = Sx(i) + \Delta S$$

3. Example Analysis

The initial parameter of the simulation can be set up referring to Annex1 and Annex 2 of the Mathematical Contest [14]. It is known that the mass m of the Chang'e-3 satellite in the flight orbit is 2.4×10^3 kg , and the variable thrust engine mounted on the satellite can supply a thrust from 1500N to 7500N. Besides, the average radius R of the moon is 1737.013km, the mass M is 7.3477×10^{22} kg , the rotation angular velocity ω is $0.00015°/s$ and the altitude H of the sinus iridum area is -2640m.

During the main deceleration phase, the fuel consumption of Chang'e-3 is huge due to the rapid deceleration from the speed of 1.7km/s to 57m/s. In order to reduce the fuel consumption during the landing process, the fuel consumption should be considered as the optimization goals. And the typical issue of fuel expense is minimising the time on transformation. In order words, the less time it takes to the deceleration, the less fuel is lost. To shorten the time in the main deceleration phase, the variable thrust engine should work at maximum thrust of 7500N.

By the above discussion, Chang'e-3 circles in a polar orbit and fly across both poles of the moon before entering to the landing preparation orbit. And the landing site B is designated on the sinus iridium area with the longitude and latitude of 19.51°W, 44.12°N.

In the process of simulation, the original state of point A can be set as followed:

The original displacement in x-coordinate axis and y-coordinate axis

$$Sx(0)=0 \quad \text{And} \quad Sy(0)=R+15\text{km}$$

The original angle

$$\theta(0)=0 \quad \text{And} \quad \psi(0)=0$$

The original distance to the center of the moon

$$r(0)=R+15\text{km}$$

The original velocity in x-coordinate axis and y-coordinate axis

$$v_x=1.7\text{km/s} \quad \text{And} \quad v_y=0$$

Simulate the soft landing process from the above original state, until the Chang'e-3 enters to the next phase. In other word, if the inequality

$$r(i)<R+H+3\text{km}=1737.37\text{km}$$

is right, stop the simulation.

4. Experiment Result

MATLAB is adopted to carry out simulation on the thrust both 7500N and 6000N and the preferable result is presented in Figure 4. The result shows that with the thrust of 7500N the coordinate of the landing site B is (24.4km,1720km), and the angle θ between the perilune point A and the landing site B relative to the center of the moon is 8.073°. Similarly, with the thrust of 6000N, the coordinate of point B and the angle θ are $(27.4\text{km},1715.6\text{km})$ and 9.076°, respectively.

Besides, the result of landing time interval in every steps in the simulation is presented in Figure 5. And the result shows that with the thrust of 7500N the total time T from perilune point A to the landing site B is 706.4s. Similarly, with the thrust of 6000N, the total time T is 774.5s.

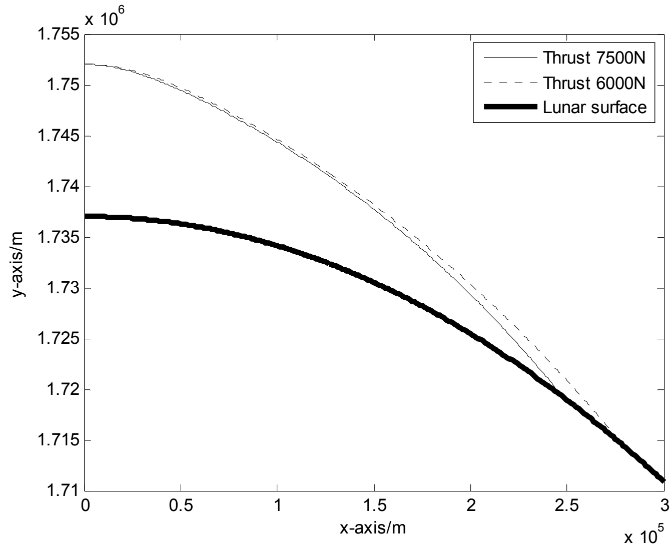

Figure 4. Orbit Simulation

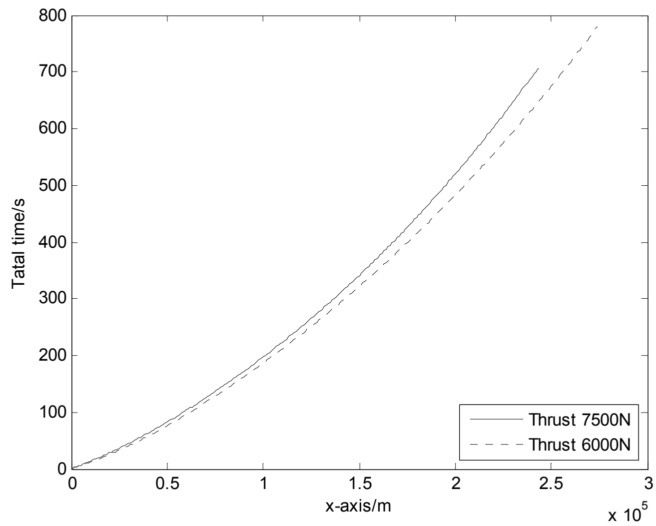

Figure 5. Total Time of the Landing

Take the thrust 7500N for discussion, by the above result, it is known that the longitude and latitude of the landing site B is 19.51°W, 44.12°N, and that the angle θ between the perilune point A and the landing site B relative to the center of the moon is 8.078°. Supposed that the moon's rotation has no influence on the simulation, the longitude of the perilune point A is mostly equal to the landing site B with the longitude of 19.51°W. Presented in Figure 6, the Chang'e-3 flied from perilune point A to the landing site B, and the latitude of perilune point A can be calculated with the angle θ and the latitude of landing site B. Thus, the latitude of perilune point A can be proved as 36.047°N with the angle θ of 8.078° and the latitude of 44.12°N.

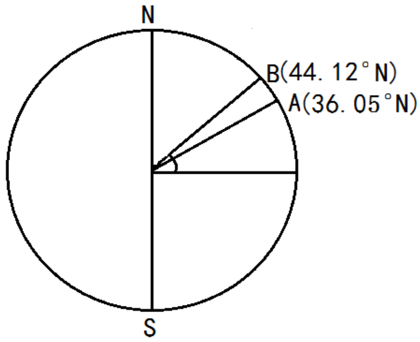

Figure 6. Latitude Location of the Perilune Point A

If ignoring the moon's rotation, the latitude of perilune point A and landing site B are almost the same. But actually, the moon's rotation has impact on the latitude of the location in the simulation and the position of the designated landing site B will be deviated due to the moon's rotation. Thus, the total time T and the rotation angular velocity ω should be considered into the simulation. And the product of total time T and rotation angular velocity ω can be set as the modification value in the latitude of perilune point A. By the above result, the modification value of the latitude can be proved as 0.106°, and the final latitude of perilune point A is 19.40°W, 36.06°N.

5. Conclusions

By the above analysis, recursive algorithm can be applied in the simulation of the satellite orbit. Based on the soft landing model, the orbit and landing process in the main deceleration phase can be calculated and forecasted by the recursive algorithm. Finally, conclusions come out as follows:

(1) The orbit simulation of Chang'e-3 is of great significance in the aerospace business. By setting the thrust and the designated landing site, the position of perilune point can be proved based on the recursive algorithm. Thus, the result of simulation provide references to the selection of the perilune point and enhances the accuracy rate of the orbit and the success rate of soft landing.

(2) The above soft landing model is a two-dimensional plane coordinate system model. It is known that Chang'e-3 flied across both poles which means that the orbit was completely mapped on a plane with two poles. Thus, setting up a two-dimensional model based on this plane can simplify the calculation procedure without considering other planes which have nothing to do with the orbit.

(3) Compared with other classical algorithm in simulation of the satellite orbit, the recursive algorithm applied in the simulation is understandable and reliable. By setting the initial state of the orbit and deriving the following state from the previous state, the whole state of the orbit can be proved completely. Thus, the application of recursive algorithm in orbit simulation is feasible and innovative.

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