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# Stress Intensity Factors of External Surface Cracks in a Vertical Cylindrical Steel Tank

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**Abstract**

The present research deals with external surface cracks in vertical steel tanks. Such defects occur very often. The analytical expression of stress intensity factor (SIF) is need, for prediction residual life of tanks with cracks. There are solutions for the SIF of surface cracks in hollow cylinders. However, all of them can not be used for steel tank. The main purpose of this research is to present analytical expression SIF for longitudinal external surface cracks in vertical cylindrical steel tanks. The finite element method is used for the SIF calculation of the crack in the tank. All SIF values are determined based on the submodel technique. Firstly, the whole shell model of tank without defects is generated. Real service load and tank size take into account. After that the solid submodel with crack is generated. A wide range of cracks geometries and filling level of oil are researched. The finite element models have a good agreement with wellknown analytical decisions. All SIF results are expressed with polynomial functions, that the fracture criteria can be used easily in the estimation of the residual life of tanks. SIF polynomial function should used in Paris-Erdogan equation. Results of this work could be useful for engineers in oil storage field.

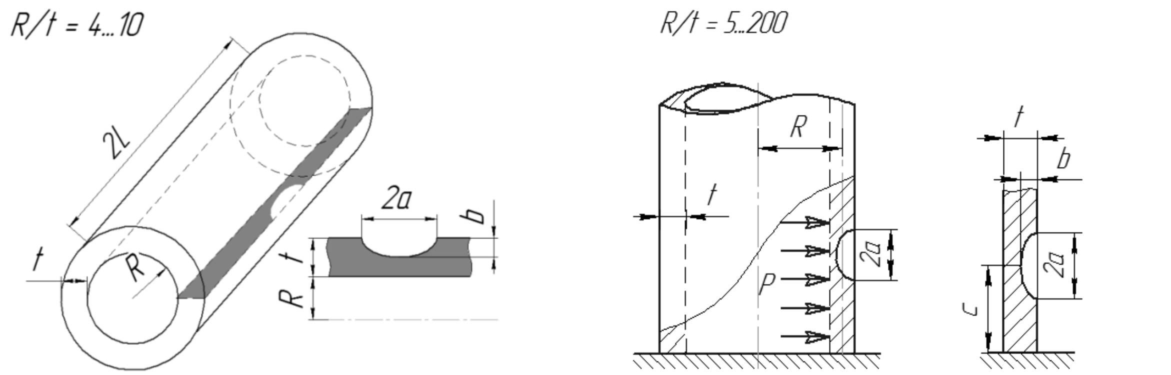
**Keywords:** stress intensity factor, cylindrical steel tank, longitudinal external surface crack, submodel technique

## 1. Introduction

Practical observations prove that failures and accident destruction of vertical cylindrical oil steel tanks happen either in the initial operating period in the first of three years (assembly defects) or after 15-20 years because of damage accumulation on stream. The very common defects are longitudinal surface cracks especially in walls, zones near nozzle, bottom-to-shell junctions, weldings and etc.

The fracture mechanical loading of the tank elements are studied using the methods of the linear fracture mechanics by determination a stress intensity factor (SIF) along the crack front. The SIF expresses the intensity of the stress field singularity near the crack tip. It depends on size, shape and location of the defect; the applied load and geometry of the structure according to Anderson and Sullivan (1966) and Kuna (2013). Also, it can be used in design strategies for brittle fracture and fatigue control of tanks (Alturi & Kathiresan, 1980). The linear fracture mechanics shows, that the crack will propagate only, when the SIF reaches the threshold value. Fracture occurs, when the SIF is equal to the critical stress intensity factor. So, it is necessary to have an expression for the SIF, which describes the special geometrical and loading conditions for the estimation of the safe operating life time of tanks.

There are analytical expressions of the SIF for the surface crack in pressurized cylinders in handbook of Murakami (1987). In Russian guidelines given by Russian Federation Direct Document 153-112-017-97 (1997) and Organization standard OS-03-001-06 (2006) rectangular plates with the internal crack are used. But, there is evidence, that the strength of through-cracked pressurized cylinders is less than that of a flat sheet having the same crack size (Anderson & Sullivan, 1966). The paper which was published by Newman J.C. et al. (1976, 1979) concerning an empirical equation of the SIF for the semi-elliptical surface cracks in pressurized cylinders, see Figure 1a. Atluri, et al. (1977, 1979) have also obtained solutions for inner or outer surface flaws. More recently, the geometry of large oil pressure tanks with a crack was considered by Yahsi O. S. and Erdogan F A. (1985). The authors have produced solutions for an axial part-through or through cracks in the pressurized cylindrical shell with a fixed end, see Figure 1b.




a) surface crack in pressurized cylinder under pressure

b) surface crack in pressurized cylindrical shell with a fixed end

Figure 1. Geometry of cylinder and loading system

However, cracks in the wall of vertical tanks are under biaxial load from tensile hoop (hydrostatic pressure) and compressive axial (weight of roof and equipment, self-weight of still and others and etc.) stresses. And, nozzles in the tank influence the stress distribution. Also, the ratio of radius to wall thickness is 500 to 2500 (Hoefakker, 2010). So, this solutions of the SIF for cracks are not useable for vertical steel tanks with flaws.

The purpose of this paper is, to present values for the mode I SIF for longitudinal external surface cracks in vertical cylindrical steel tanks with the capacity of 10 000 $m^3$ with nozzles (Table 1) under hydrostatic pressure and axial compression force. The stress intensity factors are calculated by a three-dimensional finite-element method (FEM) in Abaqus.

Table 1. Oil vertical cylindrical steel tanks with capacity 10000 $m^3$ specification

| Midsurface radius of the tank wall | 17.106 m |
|---|---|
| Height of the tank | 11.94 m |
| Thickness of the first shell course | 10.7 mm |
| Thickness of the second – sixth shell courses | 8.7 mm |
| Height of one course | 1.99 m |
| Maximum level of oil | 11.5 m |
| Weight of the roof | 879498.9 N |
| Weigh of the stationary equipment on the roof | 77618.98 N |
| Additional pressure of the evaporation of oil | 2 kPa |
| Oil density | 820 kg/$m^3$ |
| Steel | S235J2G3 |
| Nominal diameter of the nozzles | 500 |

## 2. Models and Technique

The calculation of the SIF is carried out in two steps. Firstly, a global finite element (FE) model of the whole tank with nozzles without crack is generated. The result of this step of analysis is the stress-strain state at the positions where a crack initiation from a defect would be likely. Then, the SIF is calculated in a submodel, in order to obtain more accurate results. Generally, this method uses two FE models: the global shell model of the whole tank with a coarse mesh and the solid submodel with a higher mesh density. The displacement results of the global model are prescribed on the submodel boundary (Simula Abaqus, 2008).

The numerical investigation of the SIF for the tank includes a number of assumed simplifications which are the following:

1. single longitudinal external surface cracks are researched;
2. the filling level of tank $H$ is 15%, 35%, 55%, 75%, 95% of tank height $h$;
3. the crack is located on height $c$ from the tank bottom as equal 20%, 40%, 60%, 80% from first course length $l$;

4. the crack is inserted between the nozzles and position far from of them;
5. the ratio of the crack depth $b$ to the wall thickness $t$ is equal to $b/t$ = 0.4, 0.6, 0.8;
6. the ratio of the crack depth $b$ to the crack length $2a$ is $b/a$ = 1/3;
7. elastic material is used.

*2.1 Global Model*

A static strength analysis is used to determine the stress and the strain state of the tank. A tank (material properties: elastic modulus $E$=$2\cdot10^{11}$ Pa, Poison ratio $\nu$=0.3; steel density $\gamma$=7850 kg/m$^3$) is designed according to Table 1. Due to the symmetry of the tank, a quarter model is sufficient for the analysis. Finite shell elements are used. The FE model is shown in Figure 2.

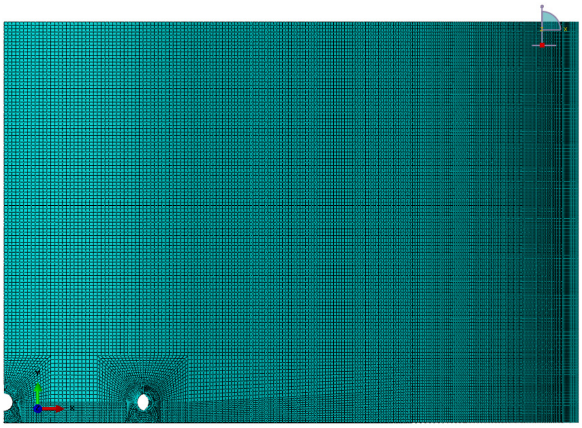


Figure 2. Model of the tank

This loads and boundary conditions are applied to the model: fixed support of the bottom edge, symmetry conditions at the end edges of the tank, dead load (the weight of the steel sheet), superimposed loads (live load of roof, equipment, snow), hydrostatic pressure and evaporation pressure. Wind load has not taken into account. The temperature distribution over the cross section of the tank wall is uniform.

*2.2 Submodel*

The accuracy of SIF values depends on the mesh quality, especially near singularity stresses. So, the size of the finite elements must be very smaller near to the crack tip. However, this increases the calculation time. In order to reduce the effort for the discretizing and computing, the submodel technique was used for the SIF calculation.

Figure 3 shows a crack template with regular and hexahedral mesh. In more details of the used crack block are described in the article of Gloger, et al (2012). If it is necessary, the developed template can be optimized to the required crack size. The crack block is inserted in a small solid plate with the size (400×400mm), which is cut from the global model. This loads and boundary conditions are applied to the submodel: displacement of the global model, dead load (the weight of the steel sheet) and hydrostatic pressure.

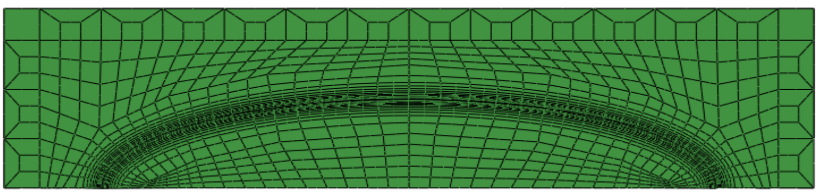


Figure 3. Template of crack

*2.3 Verification of the SIF Calculation*

The SIF of crack ($a$ = 9.96 mm, $b$ = 3.32 mm) in a straight plate with the thickness $t$ = 8.7 mm is calculated for verification. The solutions for solid – solid and shell – solid submodeling are compared with the result obtained by Raju and Newman (1979), see Figure 4. The parametric angle $\boldsymbol{\varphi}$ describes the position of the elliptical crack.

This angle is equal to $\pi/2$ at the surface and 0 at the middle of the crack.

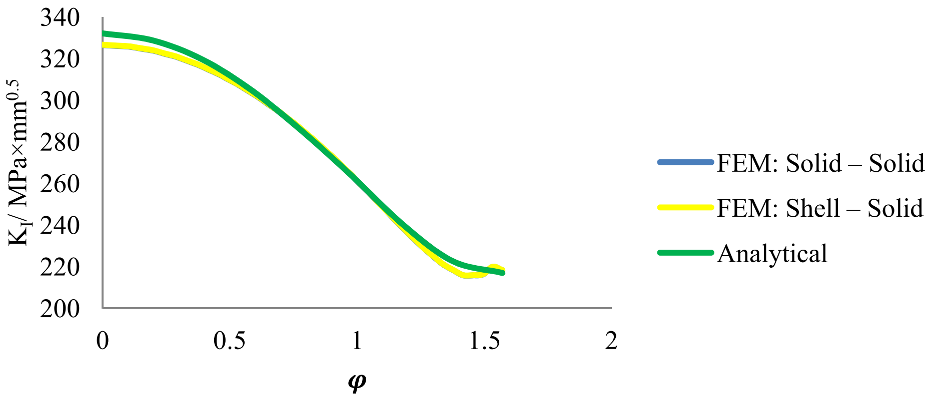



Figure 4. Distribution SIF along the crack front

The SIFs of the submodels have a very good agreement with the analytical solution by Raju and Newman (1979). The error is less than 1.68% for the most dangerous and deepest point of the crack front. However, there is a considerable error in the surface point.

## 3. Results and Discussion

### *3.1 Stress – Strain State of the Tank*

The distributions of hoop (maximum in-plane principal stress) and axial (minimum in-plane principal stress) stresses in first course for various filling levels of the tank are shown in Figures 5 – 6. According to the results (Figure 5-6), it is clear, that the defects far from the nozzles are more dangerous than flaws between the nozzles. Bending caused by the fixed bottom of the tank wall can influence the SIF values. The obtained data coincide with known results (Hoefakker, 2010).

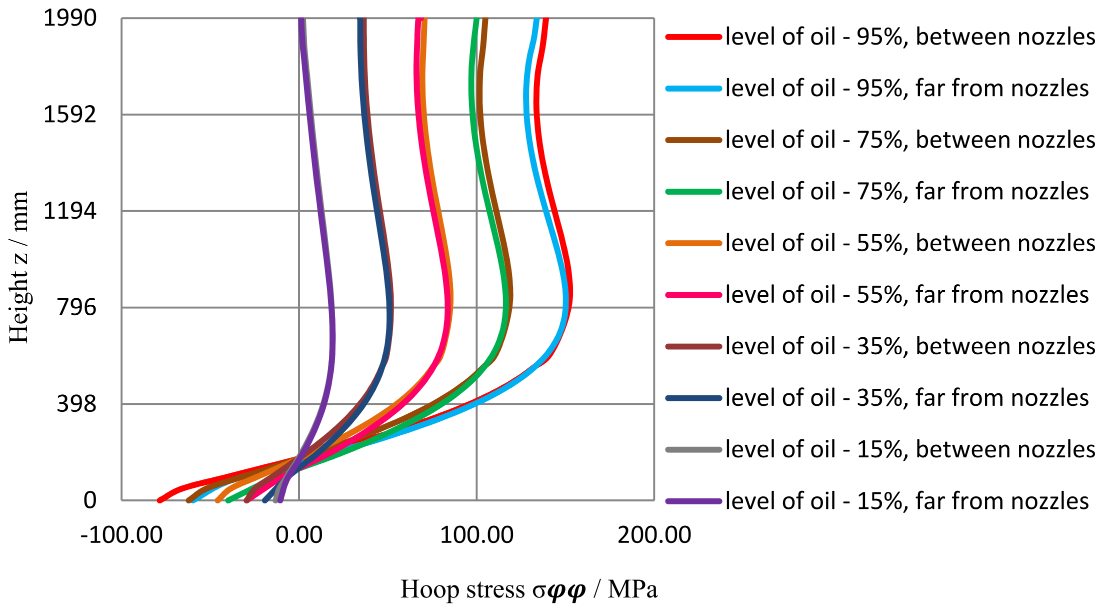



Figure 5. Hoop stress over height of the first tank course

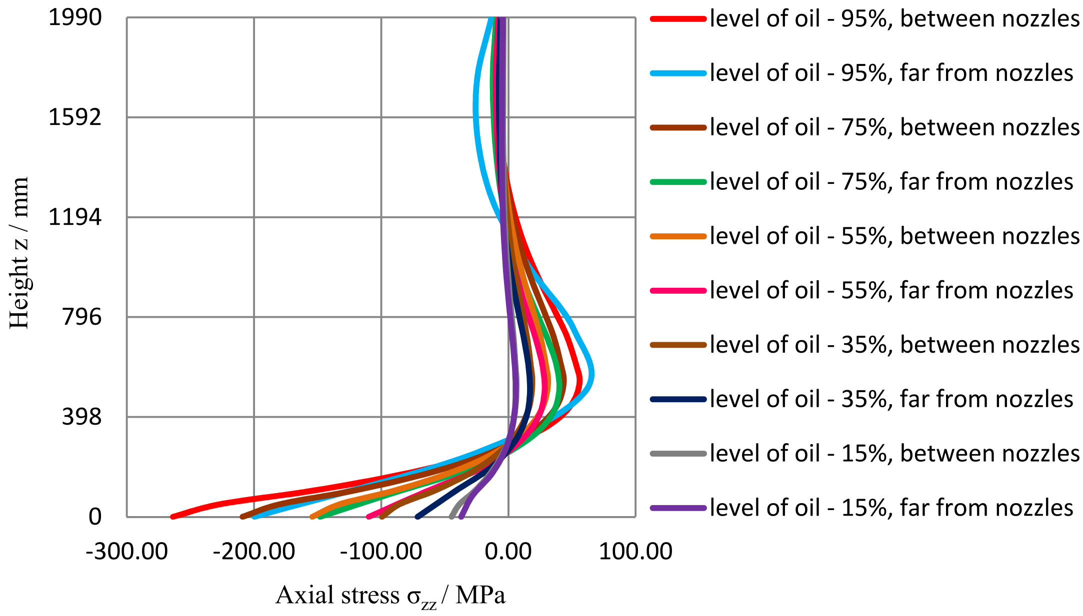



Figure 6. Axial stress over height of the first tank course

### *3.2 Stress Intensity Factor for Longitudinal External Surface Cracks in Tanks*

Wellknown fact, that maximum depth point of crack front has a higher values of SIF than others points. So the SIF of the depth point is important for prediction residual life of vertical steel tanks with external surface cracks.

The SIF of the middle of the crack front is plotted over the height in first belt (*c/l*) for various ratios *b/t*, see Figures 7 – 10. It should be noted, that increasing the filling level, the size of the crack and its depth in wall leads to higher SIFs.

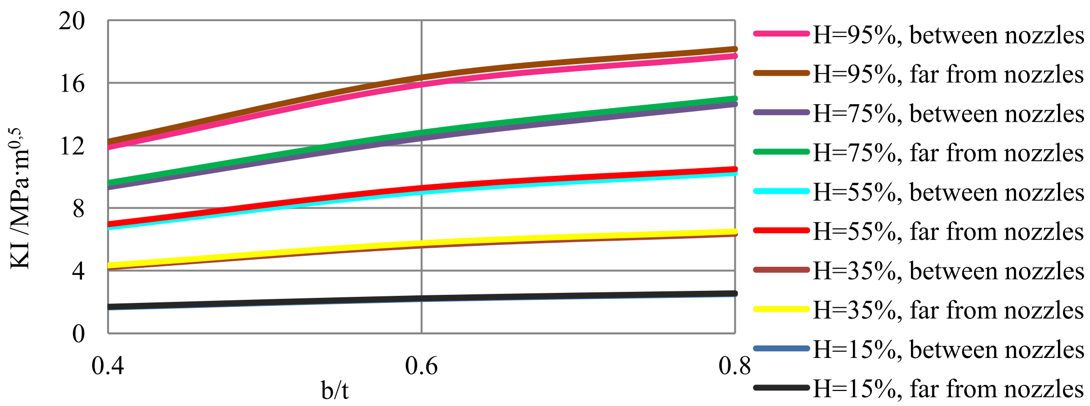



Figure 7. Stress intensity factors for $\varphi = 0$, $b/a = 1/3$, $c/l = 0.2$

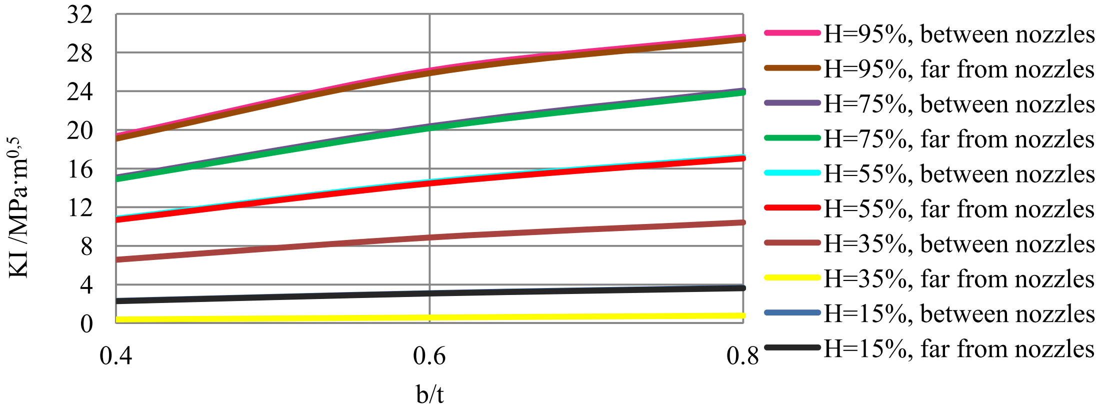



Figure 8. Stress intensity factors for $\boldsymbol{\varphi} = 0$, $b/a = 1/3$, $c/lh = 0.4$

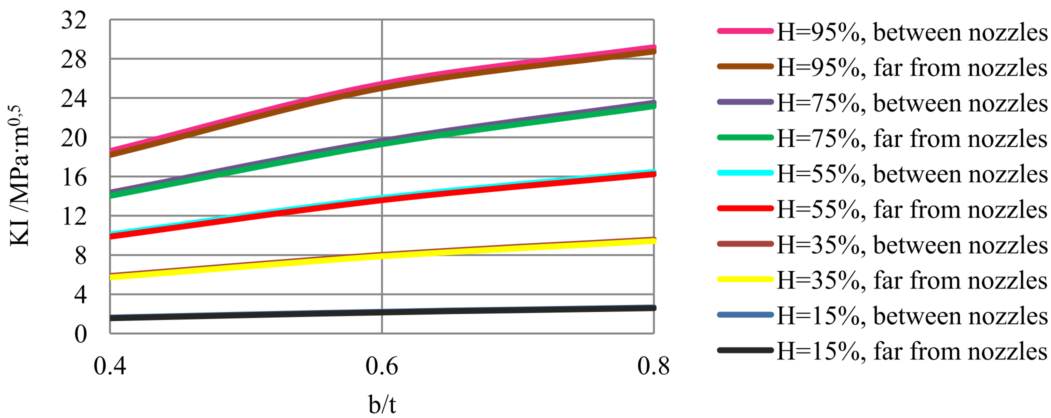



Figure 9. Stress intensity factors for $\boldsymbol{\varphi} = 0$, $b/a = 1/3$, $c/l = 0.6$

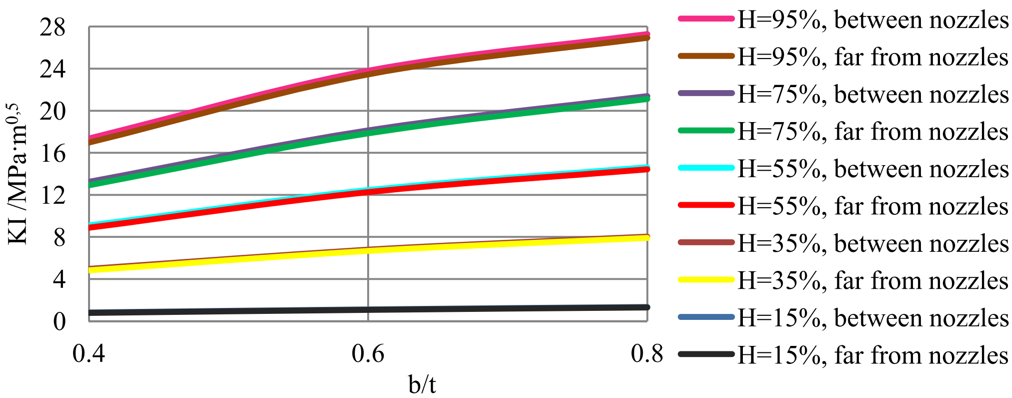



Figure 10. Stress intensity factors for $\boldsymbol{\varphi} = 0$, $b/a = 1/3$, $c/l = 0.8$

### *3.3 SIF Analytical Expression*

For practical convenience, the SIF is written as (1)

$$K_I = \sigma^{ANALYTICAL} \cdot \sqrt{\pi b} \cdot g\left(\frac{b}{t}, \frac{H}{h}, c\right) \cdot f(c, H), \tag{1}$$

$$\sigma^{ANALYTICAL} = [p \cdot g \cdot (H - c) + 1.2 \cdot P]\frac{R}{t} \tag{2}$$

where $\sigma^{ANALYTICAL}$ is hoop stress, $p$ is oil density, $P$ is evaporation pressure, $R$ is tank radius, $g$ and $f$ is, respectively, geometric and load - correction functions for the surface crack.

Function $f$ should be used in equation (1) if the nominal stress calculated without FEM. The expression for $f$ is obtained from the exact FEM hoop stress solution normalized by analytical stress (2). The latter is given by the Russian recommendations Russian Federation Direct Document 153-112-017-97 (1997) and Organization standard OS-03-001-06 (2006). The function can be expressed with the polynomial dependences for area between nozzles (3) and far from (4). The unit of $c$ and $H$ is a meter.

$$f(c,H)=-0.5472+3.481\cdot c+0.0603\cdot H-1.035\cdot c^2+0.03523\cdot c\cdot H-0.02283\cdot H^2-1.981\cdot c^3-0.1745\cdot c^2\cdot H+ \\ +0.01788\cdot c\cdot H^2+0.003014\cdot H^3+1.458\cdot c^4+.08195\cdot c^3\cdot H+0.01765\cdot c^2\cdot H^2-0.004058\cdot c\cdot H^3- \\ -0.0001291\cdot H^4-0.2174\cdot c^5-0.05867\cdot c^4\cdot H+0.009261\cdot c^3\cdot H^2-0.001893\cdot c^2\cdot H^3+0.0002513\cdot c\cdot H^4 \tag{3}$$

$$f(c,H)=-0.4088+3.048\cdot c +0.01061\cdot H +0.02682\cdot c^2-0.01046\cdot c\cdot H -0.001998\cdot H^2-3.318\cdot c^3-0.09748\cdot c^2\cdot H \\ +0.01177\cdot c\cdot H^2+0.0001005\cdot H^3+2.184\cdot c^4+0.109\cdot c^3\cdot H -0.006932\cdot c^2\cdot H^2-0.0006968\cdot c\cdot H^3-0.4235\cdot c^5- \\ -0.01792\cdot c^4\cdot H -0.002235\cdot c^3\cdot H^2+0.0005696\cdot c^2\cdot H^3 \tag{4}$$

Function g is found for deepest point of crack front. It is dependent from the ratio of the crack depth $b$ to the wall thickness $t$, crack location in the belt and level of oil filling. The expression for $g$ is obtained from the exact FEM SIF normalized by $\sigma^{FEM} \cdot \sqrt{\pi b}$ according to the Alturi and Kathiresan approach (1980). The function can be expressed with the polynomial dependences for various location of the crack, see Table 2.

Table 2. Geometric correction function for the surface crack in vertical cylindrical steel tanks with capacity 10000 m$^3$

| | *c/l* | Geometric correction function |
|---|---|---|
| Between nozzles | 0.2 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.4466 + 0.1598\left(\frac{H}{h}\right) + 2.037\left(\frac{b}{t}\right) - 0.1284\left(\frac{H}{h}\right)^2 + 0.03877\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.586\left(\frac{b}{t}\right)^2$ |
| | 0.4 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.341 - 0.06197\left(\frac{H}{h}\right) + 2.815\left(\frac{b}{t}\right) + 0.001809\left(\frac{H}{h}\right)^2 + 0.158\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 2.335\left(\frac{b}{t}\right)^2$ |
| | 0.6 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.3904 + 0.05686\left(\frac{H}{h}\right) + 2.48\left(\frac{b}{t}\right) - 0.03673\left(\frac{H}{h}\right)^2 - 0.00399\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.824\left(\frac{b}{t}\right)^2$ |
| | 0.8 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.3701 + 0.2171\left(\frac{H}{h}\right) + 2.368\left(\frac{b}{t}\right) - 0.1639\left(\frac{H}{h}\right)^2 + 0.03075\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.738\left(\frac{b}{t}\right)^2$ |
| Far from nozzles | 0.2 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.4443 + 0.1587\left(\frac{H}{h}\right) + 2.076\left(\frac{b}{t}\right) - 0.1267\left(\frac{H}{h}\right)^2 + 0.03729\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.626\left(\frac{b}{t}\right)^2$ |
| | 0.4 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.3937 + 0.0581\left(\frac{H}{h}\right) + 2.447\left(\frac{b}{t}\right) - 0.04658\left(\frac{H}{h}\right)^2 + 0.02314\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.879\left(\frac{b}{t}\right)^2$ |
| | 0.6 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.4102 - 0.03115\left(\frac{H}{h}\right) + 2.529\left(\frac{b}{t}\right) - 0.02048\left(\frac{H}{h}\right)^2 + 0.02738\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.846\left(\frac{b}{t}\right)^2$ |
| | 0.8 | $g\left(\frac{b}{t},c,\frac{H}{h}\right) = 0.3665 - 0.007006\left(\frac{H}{h}\right) + 2.719\left(\frac{b}{t}\right) + 0.02583\left(\frac{H}{h}\right)^2 - 0.04983\left(\frac{H}{h}\right)\left(\frac{b}{t}\right) - 1.949\left(\frac{b}{t}\right)^2$ |

The determined SIF value of crack in first belt is verified with solution (5) obtained by Russian Federation Direct Document 153-112-017-97 (1997) and Organization standard OS-03-001-06 (2006). Also SIF is compared with the result obtained by formula (6) (Raju & Newman, 1979).

$$K_I = \sigma \cdot \sqrt{\pi b} \tag{5}$$

$$K_I = \sigma \cdot F \cdot \sqrt{\pi b} \tag{6}$$

where σ is FEM hoop stress, $F$ is geometrical function dependent from the ratio of the crack depth $b$ to the thickness $t$.

It is found that using polynomial geometric functions ($g$) for the surface crack in the vertical cylindrical steel tanks gives opportunity to improve accuracy of analytical SIF calculation on the 10-20% in comparison with decision (5) and by a mean of 5-15% with the analytical solution by Raju and Newman (1979).

Obtained analytical solution of SIF (1) should be used in Paris-Erdogan equation for calculation cycles before failure. For this purpose Paris-Erdogan equation should be integrated as

$$N = \int_{b_0}^{0.8t} \frac{db}{C(\sigma^{ANALYTICAL} \cdot \sqrt{\pi b} \cdot g\left(\frac{b}{t}, \frac{H}{h}, c\right) \cdot f(c,H))^n} \tag{7}$$

where $C$, $n$ are the empirical coefficients of fatigue crack growth, the crack depth $b$ change from initial size $b_0$ to 0.8 of wall thickness $t$.

## 4. Conclusion

In this paper longitudinal, external surface crack in the vertical cylindrical steel tanks is studied. The SIF values are given for various oil filling levels as well as different sizes and locations of the crack. The calculations are performed using the FEM and the submodeling technique. All results are presented for a wide range of geometrical parameters of the crack and the service loads. The interpolation could be used for other crack sizes or filling levels.

The obtained SIF data can be used for residual life predictions of steel tanks. Firstly, it is necessary to compare obtained SIF values with the critical SIF. If brittle fracture can be excluded, the analytical expressions (1) of the SIF can used in the Paris-Erdogan equation take into account special geometrical and loading conditions.

The obtained $g$ geometric and $f$ load - correction functions could be used only for vertical steel tank with capacity 10000 $m^3$.For other volumes of tanks, geometric and $f$ load - correction functions should obtain using above mentioned submodeling technique and FEM analysis. In the future, the analytical expression for SIF will be found for tanks with volume 20000 and 5000 $m^3$.

## References


Alturi, S. N., & Kathiresan, K. (1979). 3-D Analyses of Surface Flaws in Thick-Walled Reactor Pressure-Vessels Using Displacement-Hybrid Finite Element Method. *Nuclear Engineering and Design*, *51*(2), 163-176. http://dx.doi.org/10.1016/0029-5493(79)90088-8

Alturi, S. N., & Kathiresan, K. (1980). Influence of Flaw Shapes on Stress Intensity Factors for Pressure Vessel Surface Flaws and Nozzle Corner Cracks. *Journal of Pressure Vessel Technology*, *102*(3), 278-286. http://dx.doi.org/10.1115/1.3263332

Anderson, R. B., & Sullivan, T. L. (1966). Fracture Mechanics of Through-Cracked Cylindrical Pressure Vessels. *NASA TN D-3252*. Ohio, USA: Levis Research Center Cleveland.

Atluri, S. N., Kathiresan, K., Kobayashi, A. S., & Nakagaki, M. (1977). Inner Surface Cracks in an Internally Pressurized Cylinder Analyzed by a Three-Dimensional Displacement Hybrid Finite Element Method. *Proceedings of the Third International Conference on Pressure Vessel Technology—Part II: Materials and Fabrication,* 527-533. Tokyo, Japan.

Gloger, D., Enderlein, M., & Kuna M. (2012). Error analysis for FEM analysis of cracks using submodels. *Proceedings of the ASME 2012 Pressure Vessels and Piping Division Conference PVP2012-78522,* 41-47. http://dx.doi.org/10.1115/PVP2012-78522

Hoefakker, J. H. (2010). *Theory review for cylindrical shells and parametric study of chimneys and tanks*. (Doctoral thesis, Delft University of Technology). Delft: Eburon Academic Publishers.

Kuna, M. (2013). *Finite Elements in Fracture Mechanics: Theory - Numerics - Applications (Solid Mechanics and Its Applications)*. http://dx.doi.org/10.1007/978-94-007-6680-8

Murakami, Y. (1987). *Stress intensity factors handbook*. Oxford: Pergamon Press.

Newman, J. C., & Raju, I. S. (1979). Stress-intensity factors for internal surface cracks in cylindrical pressure vessel, *NASA Technical Memorandum 80073*. Washington, DC: National Aeronautics and Space Administration.

Newman, J. C. (1976). Fracture analysis of surface and through cracks in cylindrical pressure vessels, *NASA TN D-8325*. Washington, DC: National Aeronautics and Space Administration.

*Organization standard OS-03-001-06.* (2006). *Industrial safety expertise of welded steel vertical tanks for oil and petroleum products.* Moscow: Korina-Ofset.

Raju, I. S., & Newman, Jr. J. C. (1979). Stress-intensity factors for a wide range of semi-elliptical surface cracks in finite-thickness plates. *Engineering Fracture Mechanics*, *11*(4), 817-829. http://dx.doi.org/10.1016/0013-7944(79)90139-5

*Russian Federation Direct Document 153-112-017-97. Diagnostics and residual life assessment of vertical steel tanks: instruction.* (1997). Moscow: Neftemontagdiagnostika.

SIMULIA Abaqus/CAE User's Manual v 6.12 2008.

Yahsi, O. S., & Erdogan, F. A. (1985). A pressurized cylindrical shell with a fixed end which contains an axial part-through or through crack. *International Journal of Fracture*, *28*(3), 161-187. http://dx.doi.org/10.1007/BF00018491

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