The Performance Study of Hybrid-driving Differential Gear Trains

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Abstract

This article in view of the differential gear trains theory characteristic, carried on the analysis to the mechanism performance, and obtained the best design scheme of the Hybrid-driving two degree of freedom differential gear trains. Adopted programmable logic controller (short for PLC) component and frequency converter to control the constant speed motor and the variable speed motor, developed a laboratory bench of Hybrid-driving two degree of freedom differential gear trains and utilized a encoder to gather the experimental data under this condition. This laboratory bench will be used as a research platform for basic theory of textile machinery and providing the rationale to optimize its variable speed mechanism.

Keywords: Hybrid-driving, Two degree of freedom, Differential gear trains, PLC

1. Introduction

The Hybrid-driving mechanism both has the high efficiency and the high bearing capacity of traditional mechanism, and it has the merit of flexibility and adjustability of controllable drive mechanism. The system research of Hybrid-driving began in the early 90s by the British scholar Tokuz who first proposed the concept of Hybrid machine(Tokuz LC,1992,p.23-35). Interiorly, there also appeared some Hybrid-driving research(Liu,Jianqin,2000,p.157-198 &Cheng, Guangyun,1990,20(3),p.64-67),but the Hybrid-driving study of differential gear trains were few, in general to study most about the multi-degree of freedom link mechanism.

In domestic and abroad, differential gear trains as the gear trains of two degrees of freedom had been researched bounteously, and mainly concentrated in its transmission ratio, efficiency and power flow. Mathis Roland had carried on the theoretical analysis to differential gear trains about its each kind of computation; Heng Sun had a detailed study to the differential gear trains about its force and efficiency(Sun,heng,1990, p.247-264); Baoxian Jia and others analyzed the power flow and meshing efficiency of the 2K-H differential gear trains(Jia,Baoxian & Bian,Wenfeng, 1999, 16(05), p. 12-15); Shaolie Cao had a research about the movement of the 2K-H differential gear trains under the inotropic effect(Cao,Shaolie,1997,p.28-36).

At first, this article made a detailed description on kinematic analysis to the differential gear trains, and obtained the relationship between speed and torque of the input/output side of 2K-H type differential gear trains; Followed, this article conducted a systematic study of coordination and control; Finally, based on an analysis of the result ,this subject designed the experimental device parameter and a test-bench of Hybrid-driving differential gear trains, and accomplished the selection and debugging of hardware and software on the test-bench, and obtained some valuable data.

2. The overview of differential gear trains

The differential gear trains are a kind of epicyclic gear trains and are a two degree of freedom epicyclic gear trains. As shown in Figure 1 is a common 2K-H type differential gear trains. Among the three basic components (both center gears, a tie bar), two components must be given the track of its movement in order to derive the third component. The input components have a variety of combinations. The differential gear trains has a stable transmission and a high precision and many other merits, .it widely applies in textile machinery and vehicles and other engineering machinery.

3. Fundamental equations of differential gear trains

Shown in Figure 1 for the 2K-H type differential gear trains, if we take tie bar H for the reference system to observe this mechanism movement, then, relative to the tie bar H, the movement of planetary gear P is fixed axis rotation, and

because center gear A and B is coaxial line with tie bar H, therefore relative to the tie bar, the movement of two gears are also fixed axis rotation. Thus, when we analyze the epicyclic gear trains, if we take the tie bar as reference system to observe the movement of epicyclic gear trains, then this epicyclic gear trains on into "fixed axis gear trains" which is considerable with itself,this is known as conversion mechanism of epicyclic gear trains. In this case, the angular velocity of various components in conversion mechanism, namely, relative to the tie bar H, the angular velocity of various components are as follows:

$$\text{Gear A: } \omega_A^H = \omega_A - \omega_H$$

$$\text{Gear B: } \omega_B^H = \omega_B - \omega_H$$

$$\text{Tie bar H: } \omega_H^H = \omega_H - \omega_H = 0$$

Due to the transformation mechanism is fixed axis gear trains, therefore, its transmission ratio can be determined according to the calculation method of fixed axis gear trains, consequently:

$$i_{AB}^H = \frac{\omega_A^H}{\omega_B^H} = \frac{\omega_A - \omega_H}{\omega_B - \omega_H} = -\frac{z_B}{z_A} \tag{1}$$

In the formula: A, B are the central gear, H is tie bar; i_{AB}^H is the transmission ratio between the two center gears A and B, according to the transmission ratio of fixed axis gear trains to determine; ω_A is the rotational speed of center gear A, ω_B is the rotational speed of center gear B, ω_H is the rotation speed of tie bar H; ω_A^H is the relative velocity which is the central gear A relative to the tie bar H; ω_B^H is relative velocity which is the central gear B relative to the tie bar H.

By the formula (1), we can see, if we know any two Parameters among ω_A, ω_B, ω_H will be able to find the another. In other words, setting the two input rotational speed will be able to determine the output rotational speed .Therefore, the differential gear trains is two degree of freedom mechanism.

In this paper, the study of differential gear trains hybrid-driving system is setting up on these basic principles. Constant speed of main motor and control motor as two power input mechanism to realize controllable output of the third component of the differential mechanism.

4. Kinematic Analysis of Differential Gear trains

The Hybrid-driving differential gear trains have two inputs and one output. It respectively connect with the constant speed motor, the control motor and the output load, if according to the known parameters of constant speed motor, control motor and the load to design differential gear trains, first we must analyze the relationship between the torque and the speed of the I/O in the differential gear trains.

In this paper, the meaning of hybrid-drive differential gear trains is an entire system, which includes differential gear trains and other retarding mechanism as well as belt transmission. Accurately speaking, in the overall system, the transmission part and the mechanical parts are composed of the differential gear trains, constant speed motor, control motor and the decelerate gear cluster of output load.

4.1 The angular velocity analysis of differential gear trains

In order to clear the relationship of rotational speed of input and outputs, we set up a rotational speed model diagram to analyze the differential gear trains (as shown in Figure 2).

It is the part of structure diagram (shown in figure 3) of 2K-H type differential gear trains which had been studied in my subject. Thereinto, the planet carrier H is gear 5, gear 1 and gear 4 are center gears, gear 2 and gear 3 are symmetrical planetary gears. The main axis drive center gear 1 rotation, the axis 1 installs on planet carrier H, drive planetary gears 2 and 3 rotation. The center gear 4 is gear shaft, and set up at the main axis.

According to the basic principle of differential gear trains, we can educe its transmission ratio relationship (as shown in Figure 3):

$$i_{14}^h = \frac{\omega_1 - \omega_h}{\omega_4 - \omega_h} = \frac{z_4 \cdot z_2}{z_1 \cdot z_3} \tag{2}$$

In formula: i_{14}^h ------differential gear transmission ratio;

   ω_1 ------the rotational speed of center gear 1;

   ω_h ------the rotational speed of planet carrier H;

   ω_4 -------the rotational speed of center gear 4;

   z_1, z_2, z_3, z_4 for the gear;

We can see from the formula (2) that when the structure of differential gear trains is determined, In other words, when z_1, z_2, z_3, z_4 are known, the differential gear trains transmission ratio i_{14}^h is a constant. Once the structure is determined, the transmission ratio will be determined too, but has nothing to do with the input speed.For the sake of brevity, makes $i_{14}^h = k$, then ,there is:

$$\frac{\omega_1 - \omega_h}{\omega_4 - \omega_h} = k \tag{3}$$

1) center gear 1,4 as input and tie bar H as output, then,

$$\omega_h = f(\omega_1, \omega_4) = \frac{1}{1-k}\omega_1 - \frac{k}{1-k}\omega_4 \tag{4}$$

2) center gear 1 and tie bar H as input, center gear 4 as output, then,

$$\omega_4 = f(\omega_1, \omega_h) = \frac{1}{k}\omega_1 + \frac{k-1}{k}\omega_h \tag{5}$$

3) center gear 4 and tie bar H as input, center gear 1 as output, then,

$$\omega_1 = f(\omega_4, \omega_h) = k \cdot \omega_4 + (1-k) \cdot \omega_h \tag{6}$$

4.2 The torque analysis of differential gear trains

It is similar with the analysis of the rotational speed relationship of differential gear trains, to establish the torque analysis model diagram of differential gear trains (shown in Figure 4). This diagram appropriately transforms the reduction ratio to directly express the torque of the constant speed of main motor, control motor and output load at input / output three sides on differential gear trains. Only to analyse the torque relationship of the differential gear trains can obtain the torque relationship of the constant speed motor, the control motor and the output load.

Here do not consider the friction when we analyze torque relationship of the differential gear trains, the main purpose is to analyze the major questions. In order to solve the problem conveniently, we make the following hypothesis: (1) Only to consider the torque generated by the circumferential force which is the role by inter-gear; (2) No matter how many planetary gears to participate the transmission, regarding the transmission of all the load are finished by one planetary gear; (3) To consider all the force are on the same plane.

First, it is necessary to clarify a question: As mentioned above, the differentia gear trains is a two degree of freedom structure. The speed of the third client can be unique determined when we give the speed of arbitrary at both ends. Then, whether the torque of differential gear trains also has the similar nature?

After assigning the speed of both ends of differential gear train, we can only give one of the driving moment of one side, but the moment of the other end is possibly the driving moment, or is the resistance moment to nature counterbalance, this should accord to the specific circumstances of the mechanism themselves to confirmation, but can not be assigned casually. Generally, it is easy to have such illusion: the differential gear trains are two degrees of freedom, then you can give any one of two basic components with any angular velocity and moment. In fact, this idea is wrong. Because the three components of differential gear trains possess balance principle of outside moment. Consequently:

$$M_1 + M_4 + M_h = 0 \tag{7}$$

In formula: M_1 ----the torque of center 1; M_4 ----the torque of center 4;

   M_h ----the torque of planetary gear;

As for the uniform rotation of the differential gear trains, there exist a balanced relationship between the input power and output power,

$$M_1\omega_1 + M_4\omega_4 + M_h\omega_h = 0 \tag{8}$$

From the formulae (7), (8) and (3), we can obtain the torque relationship, as follows:

(1) the center gear 1 and 4 as input, tie bar H as output,

$$\begin{cases} M_1 = \frac{1}{k-1}M_h \\ M_4 = \frac{-k}{k-1}M_h \end{cases} \tag{9}$$

(2) the center gear 1 and the tie bar H as input, the center gear 4 as output,

$$\begin{cases} M_1 = -\frac{1}{k}M_4 \\ M_h = \frac{1-k}{k}M_4 \end{cases} \tag{10}$$

(3) the center gear 4 and the tie bar H as input, the center gear 1 as output,

$$\begin{cases} M_4 = -kM_1 \\ M_h = (k-1)M_1 \end{cases} \tag{11}$$

By formulae (9),(10) and (11) ,we can see that if we know the size of a differential gear trains mechanism, then we can determine the size of the other under known the two input rotational speed and a driving moment.

5. To determine the program of differential gear trains

The differential gear trains shows in the Figure 3, through the rotational speed analysis and torque analysis, we can see that there are three programs to achieve hybrid-driven. Which kind of program is reasonable and reliable? Next, from the aspect of torque to analyze the feasibility of every program, and put the results of the analysis as basis for designing differential gear trains and determining its structure.

Because this differential gear trains is outside mesh, its gear ratio must be greater than zero. This is an important prerequisite to analyse the mechanism.

(1) the center gear 1 and 4 as input, tie bar H as output,

By (9)-type,when $k > 0$,the two coefficient of $\frac{1}{k-1}$ and $\frac{-k}{k-1}$ are opposite, and also regarding one of the two torque M_1 and M_4 must be same direction with the output torque. In other words, one of the two torque M_1 and M_4 is impetus and the other is resistance, so we can see that this program is infeasible.

(2) the center gear 1 and the tie bar H as input, the center gear 4 as output,

By (10)-type, when 0<k<1, the input torque M_1 and M_4 are opposite direction, M_4 and M_h are same direction. In other words, M_1 is impetus and M_4 is resistance. When $k > 1$, M_1 and M_h are opposite direction with M_4. In other words, M_1 and M_h are impetus. So this program is feasible.

(3) the center gear 4 and the tie bar H as input, the center gear 1 as output,

By (11)-type, When $k > 1$, M_1 and M_h are opposite direction with M_4, M_1 and M_h are impetus. Under this condition, because of $M_4 = -kM_1$, Whether the input side connect with constant speed motor or control motor, we must provide K times the output torque, which is obviously infeasible.

From the above analysis, we can see that it is feasible for center gear and tie bar as input and the another center gear as output. This paper takes into account the factors of design, machining and cost of the deceleration gear trains of the I/O port. Finally, we determined the program 2 as our research object.

From the previous analysis, we can see that the transmission ratio $k > 1$ is to meet the requirement. Once the K is determined, the structure of differential gear train is determined too. According to the rotational speed model (shown in Figure 2), we can transform the (5)-type into the new form, as below:

$$i_o\omega_o = \frac{1}{k} \cdot \omega_c / i_c + \frac{k-1}{k} \cdot \omega_v / i_v \tag{12}$$

In formula: i_c ------the transmission ratio of constant speed input port;

   i_v ------the transmission ratio of variable speed input port;

   i_o ------the transmission ratio of output port;

   ω_o ------the rotational speed of output port;

   ω_c ------the rotational speed of constant input port;

   ω_v ------the rotational speed of variable input port;

   k ------- the transmission ratio of differential gear trains

Then, the rotational speed relationship between output port and constant speed motor and variable speed motor is as below:

$$\omega_o = \frac{1}{ki_o i_c}\omega_c + \frac{k-1}{ki_v i_c}\omega_v \tag{13}$$

According to the torque model shown in Figure 4, we can transform the (10)-type into the new form, as below:

$$\begin{cases} i_c M_c = -\dfrac{1}{k} M_o / i_o \\[2mm] i_v M_v = \dfrac{1-k}{k} M_o / i_o \end{cases} \tag{14}$$

In formula: M_C ---the torque of constant port; M_V ---the torque of variable port;

M_O ---the torque of output port;

Then, the output torque of constant speed motor: $M_c = -\dfrac{1}{k i_c i_o} M_o$

the output torque of variable speed motor: $M_v = \dfrac{1-k}{k i_v i_o} M_o$

6. The design of control system

6.1 The system hardware

The main hardware include a PLC, two frequency converters, two three-phase asynchronous motors, a transducer and a display. The concrete hardware are as follows: (1) DVP-20EX PLC as the core of system control, this PLC has two communication interfaces (rs-485 and rs-232).RS-485 can directly communicate with the frequency converters; RS-232 can communicate with the computer through the connecting line, which one side is a circular 8-pin interface, and the other side is needle-shaped 9-pin D-type interface. (2)The frequency converter selects VFD007B43A, its input: three-phase, 380-480VAC 50/60HZ; its output: the range of frequency is 0.1- 400HZ; the temperature of environment: -10 to 50 degree Centigrade. (3)Three-phase asynchronous motor, rated speed: 1380rad/min; Power: 0.6KW; Voltage: 380V; Electric current: 1.59A. (4)Sensor, it is to detect the actual rotational speed of output port. (5) display, it is to display the output port actual rotational speed.

6.2 The system of software

One PLC controls the two frequency converters to drive the two three-phase asynchronous motor respectively in this system. So that, one motor is constant speed and the other is variable speed. At the same time, installing a sensor on the output side to detect the actual rotational speed of the output port. PLC program flow diagram is in Figure 5, below:

7. Experimental results

The known transmission parameters of gear and belt of the test bench, as shown in the table 1.Based on these parameters, we can calculate the theoretical value of output-port of the hybrid-driving differential gear trains.

In the experimental process, the constant speed motor takes four fixed speed, they are 0 r / min, 330 r / min, 630 r / min and 930 r / min. The range of variable speed motor is from forward 1380r / min to reverse 1380 r/ min. When the two-motor run steadily, the constant speed motor takes a fixed speed, and change the speed of variable speed motor under this condition. At the same time, we record the actual speed of output-port. The actual value and theoretical value of the output-port are shown in the table 2 below. The negative mean reverse in the form 2. (Look from the right side of the test-bench, the output-port clockwise direction is forward.)

According to the experimental data in the table 2, we can see that the actual rotational speed of the motor is lower than its rated rotational speed, so it causes the actual rotational speed of output-port is less than its theoretical value. But generally speaking, the actual value and the theoretical value are quite close, this shows that the test-bench have achieved the anticipated request. This test-bench provides a good experimental platform for the basic theory research in the future.

8. Conclusion

Through researching the theory of hybrid-driving differential gear trains and carring out   experiment many times on the designed test-bench,finally, this article obtains two conclusions:

(1)This paper designed a test-bench of hybrid-driving two degree of freedom differential gear trains, and its mechanical properties are reliable and stable,low noise, smooth running. Generally speaking, it is able to achieve the anticipated purpose.

(2)This test-bench uses PLC component to enable system control more precise, easy operation, debugging easy, gathering the data accurately and conveniently. It provides a good experimental platform for the basic theory research in the future.

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Table 1. The transmission parameters of gear and belt

gear	m	z
1	1.75	67
2	1.75	23
3	1.75	30
4	1.75	60
5	2.5	20
6	2.5	100
7	2	20
8	2	35
belt transmission	2:1	

Table 2. the actual speed of output-port of hybrid-drive differential gear trains

contact speed (rad/min)	variable speed (rad/min)	actual speed (rad/min)	theoretical speed (rad/min)	contact speed (rad/min)	variable speed (rad/min)	actual speed (rad/min)	theoretical speed (rad/min)
930	1380	48.96	54.61	330	1180	-40.13	-43.65
930	1180	65.28	72.87	330	980	-18.58	-25.39
930	980	73.34	91.1304	330	780	-2.76	-7.13
930	780	102.52	109.39	330	580	5.64	11.13
930	580	125.10	127.65	330	380	24.98	29.39
930	380	132.86	145.91	330	-380	89.47	98.78
930	0	176.67	180.61	330	-580	108.52	117.04
630	1380	-1.93	-3.65	0	1380	-109.24	-126.00
630	1180	10.59	14.61	0	1180	-91.60	-107.74
630	980	22.64	32.87	0	980	-73.98	-89.48
630	780	46.28	51.13	0	780	-63.15	-71.26
630	580	61.12	69.39	0	580	-41.30	-52.96
630	380	78.23	87.65	0	380	-23.76	-34.69
630	-380	140.00	157.04	0	-380	33.40	34.69
630	-580	168.87	175.30	0	-580	48.65	52.96
330	1380	-53.81	-61.91	0	-780	66.82	71.22

Figure 1. planetary differential gear trains structure diagram

Figure 2. Hybrid drive differential gear train rotational speed analysis

Figure 3. structural diagram of differential gear

Figure 4. Hybrid-driven differential gear trains torque analysis

Figure 5. PLC program flow diagram