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Optimal Programming Models for Portfolio Selection with Uncertain Chance Constraint

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Abstract

The paper is concerned with the portfolio selection problem about how to assign one's money in security market in order to obtain the maximal profit. One type expected maximization programming model with chance constraint in which the security returns are uncertain variables are proposed in accordance with uncertainty theory. Since the provided models can not be solved by the traditional methods, the crisp equivalents of the corresponding models are discussed when the uncertain returns are chosen as some special cases such as linear uncertain variables, trapezoidal uncertain variables and normal uncertain variables. Two numerical examples with different types of uncertain variables are given in order to demonstrate the effectiveness and feasibility of the proposed programming models. Finally, the paper gives the conclusion.

Keywords: Chance constrain, Portfolio selection, Uncertain variable, Crisp equivalent programming

1. Introduction

Portfolio selection is concerned with an investor who is trying to allocate one's wealth among alternative securities so that the investment goal can be achieved. The problem was initialized by Markowitz (1952, p.77) and his mean-variance methodology has been regarded as the basis for the theory of modern portfolio selection. The pioneer work of Markowitz combined probability and optimization theory to model the investment behavior under uncertainty. An investor should always strike a balance between maximizing the return and minimizing risk for a predetermined return level. More importantly, Markowitz initially quantified investment return as the expected value of returns of securities and risk as variance from the expected value.

After Markowitz's work, scholars have been showing great enthusiasm in portfolio selection and tried to use different approaches to develop the theory of portfolio selection. Generally speaking, there are three models to deal with the portfolio selection problems with uncertain return rates. The first is expected value model (EVM), which optimizes the expected objective function subject to some expected constraints. The second chance-constrained programming (CCP) was proposed by Charnes and Cooper (1965, p.73) and developed by many scholars as means of dealing with uncertainty by specifying a confidence level at which the uncertain constraints hold. The employment and development of chance-constrained programming in portfolio selection with stochastic parameters can be found by Brockett (1992, p. 385), by Li (1995, p. 577) and by Williams (1997, p.77). Following the idea of stochastic chance-constrained programming, Liu (2002) developed a spectrum of general forms of fuzzy chance-constrained programming and a general of uncertain chance-constrained programming (2009). To use the theory of chance-constrained programming, the author tries to do something in portfolio selection problems when the return rates are assumed to be uncertain variable which is also proposed by Liu (2009). Two type of portfolio selection models are provided with uncertain return rates and the crisp equivalent programming of the corresponding models are given when the return rates are chosen as some special cases.

The rest of the paper is organized as follows. After recalling some definitions and results about uncertain measure and uncertain variable in section 2, two types of programming models for portfolio selection with chance constrain are introduced in section 3. Then section 4 discusses the crisp equivalents when the return rates are chosen as some special uncertain variables such as linear uncertain variable, trapezoidal uncertain variable and normal uncertain variable. In section 5, we provide two numerical examples to demonstrate the potential application and the effectiveness of the new models. Finally, we conclude the paper in section 6.

2. Preliminaries

Let Γ be a nonempty set, and let A be a σ-algebra over Γ. Each element $\Lambda \in$ A is called an event. In order to provide an axiomatic definition of uncertain measure, it is necessary to assign to each event Λ a number $M\{\Lambda\}$ which indicates the level that Λ will occur. In order to ensure that the number $M\{\Lambda\}$ has certain mathematical properties, Liu (2009) proposed the following five axioms:

Axiom 1 (Normality) $M(\Gamma)=1$;

Axiom 2 (Monotonicity) $M(\Lambda_1)\le M(\Lambda_2)$ whenever $\Lambda_1 \subseteq \Lambda_2$;

Axiom 3 (Self-duality) $M(\Lambda)+M(\Lambda^c)=1$ for every event Λ;

Axiom 4 (Countable subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have

$M(\bigcup_{i=1}^{\infty}\Lambda_i)\le \sum_{i=1}^{\infty} M(\Lambda_i)$.

The following is the definition of uncertain measure.

Definition 1 (Liu (2009)). The set function is called an uncertain measure if it satisfies the normality, monotonicity, self-duality and countable subadditivity axioms.

Example 1 Let $\Gamma=\{\gamma_1,\gamma_2\}$. For this case, there are only 4events. Define

$M\{\gamma_1\}=0.4,\ M\{\gamma_2\}=0.6,\quad M(\phi)=0,\ M(\Gamma)=1$,

then M is an uncertain measure because it satisfies the four axioms.

Definition 2 (Liu (2009)). Let Γ be a nonempty set, A a σ-algebra over Γ, and M an uncertain measure. Then the triplet (Γ, A, M) is called an uncertain space.

The product uncertain measure is defined as follows.

Axiom 5 (Liu (2009)). (Product Measure Axiom) Let Γ_k be nonempty sets on which M_k are uncertain measures, $k=1,2,\cdots,n$, respectively. Then the product uncertain measure on Γ is

$$M\{\Lambda\}=\begin{cases}\sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda}\ \min\limits_{1\le k\le n} M_k\{\Lambda_k\}, & \text{if } \sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda}\ \min\limits_{1\le k\le n} M_k\{\Lambda_k\}>0.5,\\ 1-\sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda^c}\ \min\limits_{1\le k\le n} M_k\{\Lambda_k\}, & \text{if } \sup\limits_{\Lambda_1\times\Lambda_2\times\cdots\times\Lambda_n\subset\Lambda^c}\ \min\limits_{1\le k\le n} M_k\{\Lambda_k\}>0.5.\end{cases}$$

For each event $\Lambda\in$ A, denoted by $M=M_1\wedge M_2\wedge\cdots\wedge M_n$.

Definition 3 (Liu (2009)). An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma,\mathrm{A},\mathrm{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$\{\xi\in \mathrm{B}\}=\{\gamma\in\Gamma \mid \xi(\gamma)\in \mathrm{B}\}$

is an event.

A random variable can be characterized by a probability density function and a fuzzy variable may be described by a membership function, uncertain variable can be characterized by identification function.

Definition 4 (Liu (2009)). An uncertain variable ξ is said to have a first identification function λ if

(1) $\lambda(x)$ is a nonnegative function on R such that

$\sup\limits_{x\ne y}\lambda(x)+\lambda(y)=1;$

(2) For any set B of real numbers, we have

$$M\{\xi\in \mathrm{B}\}=\begin{cases}\sup\limits_{x\in \mathrm{B}}\lambda(x), & \text{if } \sup\limits_{x\in\mathrm{B}}\lambda(x)<0.5,\\ 1-\sup\limits_{x\in \mathrm{B}^C}\lambda(x), & \text{if } \sup\limits_{x\in\mathrm{B}^C}\lambda(x)\ge 0.5.\end{cases}$$

Definition 5 (Liu (2009)). The uncertainty distribution $\Phi: R\to[0,1]$ of an uncertain variable ξ is defined by

$\Phi(x)=M\{\xi\le x\}$.

3. Uncertain programming models for portfolio selection

In Markowitz models, security returns were regarded as random variables. As discussed in introduction, there does exist situations that security returns may be uncertain variable parameters. In this situation, we can use uncertain variables to describe the security returns.

Let x_i denote the investment proportion in the ith security, ξ_i represents uncertain return of the ith security, $i = 1, 2, \cdots, n$, respectively, and a the minimum return level that the investor can tolerate. Following chance constraint idea, if we want to maximize the expected value or minimize risk of the total return subject to some chance constraints, to express it in mathematical formula, the models are as follows:

$$\begin{aligned} &\max\ E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\ &\text{Subject to:} \\ &M[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \le a] \le \alpha, \\ &x_1 + x_2 + \cdots + x_n = 1, \\ &x_i \ge 0,\ i = 1, 2, \cdots, n. \end{aligned} \tag{1}$$

where $\alpha \in (0,1)$ is a specified confidence level the investor given, and a is the minimum return that the investor can accept satisfying $M\{x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \le a\} \le \alpha$ in which $x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \le a$ means the investment risk. E is the expected value of uncertain variable. It is obvious that the combination of securities that can maximize $E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n]$ is the optimal portfolio the investor should select.

If the investor wants to minimize the investment risk with some chance constraints, then we have the following model,

$$\begin{aligned} &\min\ V[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\ &\text{Subject to:} \\ &M[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n \ge a] \ge \alpha, \\ &x_1 + x_2 + \cdots + x_n = 1, \\ &x_i \ge 0,\ i = 1, 2, \cdots, n. \end{aligned} \tag{2}$$

where V denotes the variance of the total return which represents the risk of the investment.

4. Crisp equivalents

In general, to solve the uncertain programming model the traditional solution methods require conversion of the objective function and the chance constraints to their respective deterministic equivalents. And this process is usually hard to perform and only successful for some special cases. In the next content, we will consider several special forms of uncertain return rate ξ_i, and convert the models (1) and (2) into their crisp equivalents.

4.1 Models for linear uncertain variable

An uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x < a \\ (x-a)/(b-a), & \text{if } a \le x \le b \\ 1. & \text{if } x > b \end{cases}$$

denoted by $L(a,b)$ where a and b are real number with $a < b$.

Suppose that the return rate ξ_i of the ith security is linear uncertain variable $\xi_i = L(a_i, b_i)$ with $a_i < b_i$, $i = 1, 2, \cdots, n$, then

$$x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n = L(\textstyle\sum_{i=1}^n x_i a_i, \sum_{i=1}^n x_i b_i).$$

In accordance with the propositions of linear uncertain variables, the expected value and variance of the total return are as follows,

$$E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] = \frac{1}{2}\textstyle\sum_{i=1}^n x_i(a_i + b_i),$$

$$V[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] = \frac{1}{12}(\textstyle\sum_{i=1}^n x_i b_i - \sum_{i=1}^n x_i a_i)^2 = \frac{1}{12}[\sum_{i=1}^n x_i(b_i - a_i)]^2.$$

Since the nonnegativity of the term $[\sum_{i=1}^{n} x_i(b_i - a_i)]^2$, to minimize $\frac{1}{12}[\sum_{i=1}^{n} x_i(b_i - a_i)]^2$ is equivalent to minimize $\sum_{i=1}^{n} x_i(b_i - a_i)$.

Theorem 1 Let $x_1, x_2, \cdots, x_n$ be nonnegative decision variables and ξ_i be linear uncertain variable $L(a_i, b_i)$ with $a_i < b_i, i = 1, 2, \cdots, n$. Suppose that $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables. Then for any scalar a and any confidence level $\alpha \in (0,1)$, the chance constraint

$$M\{\sum_{i=1}^{n} x_i\xi_i \geq a\} \geq \alpha$$

holds if and only if

$$\Phi(0) \leq 1 - \alpha$$

where $\Phi(x)$ is the uncertainty distribution of $x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n - a$.

Proof: Since ξ_i are assumed to be independent linear uncertain variables, the quantity

$$x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n - a$$

is also linear uncertain variable with parameters $a' = x_1a_1 + x_2a_2 + \cdots + x_na_n - a$ and $b' = x_1b_1 + x_2b_2 + \cdots + x_nb_n - a$. So the uncertainty distribution of $x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n - a$ is

$$\Phi(x) = \begin{cases} 0, & \text{if } x < a' \\ (x - a')/(b' - a'), & \text{if } a' \leq x \leq b' \\ 1. & \text{if } x > b' \end{cases}$$

Furthermore,

$$M\{\sum_{i=1}^{n} x_i\xi_i \geq a\} = M\{\sum_{i=1}^{n} x_i\xi_i - a \geq 0\} = 1 - M\{\sum_{i=1}^{n} x_i\xi_i - a \leq 0\}$$

Thus the inequality

$$M\{\sum_{i=1}^{n} x_i\xi_i \geq a\} \geq \alpha$$

is equivalent to the inequality

$$M\{\sum_{i=1}^{n} x_i\xi_i - a \leq 0\} \leq 1 - \alpha.$$

That is $\Phi(0) \leq 1 - \alpha$ which proves the theorem.

In this case, models (1) and (2) can be converted into its deterministic equivalents as follows,

$$\begin{aligned} & \max \sum_{i=1}^{n} x_i(a_i + b_i) \\ & \text{Subject to:} \\ & \Phi(0) \leq \alpha, \\ & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0,\ i = 1, 2, \cdots, n. \end{aligned} \tag{3}$$

or

$$\begin{aligned} & \min \sum_{i=1}^{n} x_i(b_i - a_i) \\ & \text{Subject to:} \\ & \Phi(0) \leq 1 - \alpha, \\ & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0,\ i = 1, 2, \cdots, n. \end{aligned} \tag{4}$$

4.2 Models for trapezoidal uncertain variable

If the return rates are all trapezoidal uncertain variables, Let ξ_i be (a_i, b_i, c_i, d_i), where

$a_i < b_i \leq c_i < d_i, i = 1, 2, \cdots, n$. Then $\sum_{i=1}^{n} x_i\xi_i$ is $(\sum_{i=1}^{n} x_ia_i, \sum_{i=1}^{n} x_ib_i, \sum_{i=1}^{n} x_ic_i, \sum_{i=1}^{n} x_id_i)$. In accordance with the properties of trapezoidal uncertain variable, we have

$$E[\sum_{i=1}^{n} x_i\xi_i] = (\sum_{i=1}^{n} x_ia_i + \sum_{i=1}^{n} x_ib_i + \sum_{i=1}^{n} x_ic_i + \sum_{i=1}^{n} x_id_i)/4 = \sum_{i=1}^{n} x_i(a_i + b_i + c_i + d_i)/4,$$

$$V[\sum_{i=1}^n x_i\xi_i] = \frac{4\alpha^2+3\alpha\beta+\beta^2+9\alpha\gamma+3\beta\gamma+6\gamma^2}{48} + \frac{[(\alpha-\beta-2\gamma)^+]^3}{384\alpha}$$

where

$$\alpha = (\sum_{i=1}^n x_ib_i - \sum_{i=1}^n x_ia_i) \vee (\sum_{i=1}^n x_id_i - \sum_{i=1}^n x_ic_i),\ \beta = (\sum_{i=1}^n x_ib_i - \sum_{i=1}^n x_ia_i) \wedge (\sum_{i=1}^n x_id_i - \sum_{i=1}^n x_ic_i),$$

and $\gamma = \sum_{i=1}^n x_ic_i - \sum_{i=1}^n x_ib_i$.

Theorem 2 Let $x_1, x_2, \cdots, x_n$ be nonnegative decision variables and ξ_i be trapezoidal uncertain variable (a_i, b_i, c_i, d_i) with $a_i < b_i \le c_i < d_i, i = 1, 2, \cdots, n$ and $\xi_1, \xi_2, \cdots, \xi_n$ be independent. Then for any scalar a and any confidence level $\alpha \in (0,1)$, the chance constraint

$$M\{\sum_{i=1}^n x_i\xi_i \ge a\} \ge \alpha$$

holds if and only if

$$\Phi(0) \le 1-\alpha$$

where $\Phi(x)$ is the uncertainty distribution of $\sum_{i=1}^n x_i\xi_i - a$.

Thus the models (1) and (2) can be changed into the following formulas,

$$\begin{aligned} &\max \sum_{i=1}^n x_i(a_i+b_i+c_i+d_i) \\ &\text{Subject to:} \\ &\Phi(0) \le \alpha, \\ &x_1+x_2+\cdots+x_n = 1, \\ &x_i \ge 0,\ i=1,2,\cdots,n. \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\min V[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n] \\ &\text{Subject to:} \\ &\Phi(0) \le 1-\alpha, \\ &x_1+x_2+\cdots+x_n = 1, \\ &x_i \ge 0,\ i=1,2,\cdots,n. \end{aligned} \tag{6}$$

4.3 Models for normal uncertain variable

An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = (1+\exp(\frac{\pi(e-x)}{\sqrt{3}\sigma}))^{-1}, x \in R ,$$

denoted by $N(e,\sigma)$ where e and σ are real number with $\sigma > 0$. Suppose that the return rate of ith security is normally distributed with parameters e_i and $\sigma_i > 0, i = 1, 2, \cdots, n$. Then we have

$$E[\sum_{i=1}^n x_i\xi_i] = \sum_{i=1}^n x_ie_i, \quad V[\sum_{i=1}^n x_i\xi_i] = (\sum_{i=1}^n x_i\sigma_i)^2.$$

Theorem 3 Assumed that $x_1, x_2, \cdots, x_n$ be nonnegative decision variables and $\xi_1, \xi_2, \cdots, \xi_n$ are independently uncertain variables with expected values $e_1, e_2, \cdots, e_n$ and variance $\sigma_1{}^2, \sigma_2{}^2, \cdots, \sigma_n{}^2$, respectively. Then for any scalar a and α , the chance constraint

$$M\{\sum_{i=1}^n x_i\xi_i \ge a\} \ge \alpha$$

Holds if and only if

$$\sum_{i=1}^n x_i(e_i + \sigma_i\Phi^{-1}(1-\alpha)) \ge a$$

where Φ is the standardized normal distribution , i.e.,

$$\Phi(x) = (1+\exp(-\frac{\pi x}{\sqrt{3}}))^{-1},\ x \in R .$$

Proof: Since ξ_i are assumed to be independently normal uncertain variables, the quantity

$$y = \sum_{i=1}^n x_i\xi_i - a$$

is also normal uncertain variable with the following expected value and variance,

$$E[y] = \sum_{i=1}^n x_ie_i - a,\ V[y] = (\sum_{i=1}^n x_i\sigma_i)^2.$$

We note that

$$\frac{(\sum_{i=1}^{n} x_i\xi_i - a) - (\sum_{i=1}^{n} x_i e_i - a)}{\sum_{i=1}^{n} x_i\sigma_i}$$

must be standardized normal uncertain variable. Since the inequality

$$\sum_{i=1}^{n} x_i\xi_i \ge a$$

is equivalent to

$$\frac{(\sum_{i=1}^{n} x_i\xi_i - a) - (\sum_{i=1}^{n} x_i e_i - a)}{\sum_{i=1}^{n} x_i\sigma_i} \ge \frac{0 - (\sum_{i=1}^{n} x_i e_i - a)}{\sum_{i=1}^{n} x_i\sigma_i}$$

We have

$$M\{\eta \ge -\frac{(\sum_{i=1}^{n} x_i e_i - a)}{\sum_{i=1}^{n} x_i\sigma_i}\} \ge \alpha$$

where η is the standardized normal uncertain variable. This inequality holds if and only if

$$\sum_{i=1}^{n} x_i (e_i + \sigma_i \Phi^{-1}(1-\alpha)) \ge a\ .$$

The theorem is proved.

Under the conditions, the models (1) and (2) may be formulated as the following linear equivalents,

$$\begin{aligned} &\max\ \sum_{i=1}^{n} x_i e_i \\ &\text{Subject to:} \\ &\sum_{i=1}^{n} x_i (e_i + \sigma_i \Phi^{-1}(\alpha)) \ge a, \\ &x_1 + x_2 + \cdots + x_n = 1, x_i \ge 0,\ i = 1,2,\cdots,n. \end{aligned} \tag{7}$$

and

$$\begin{aligned} &\min\ \sum_{i=1}^{n} x_i \sigma_i \\ &\text{Subject to:} \\ &\sum_{i=1}^{n} x_i (e_i + \sigma_i \Phi^{-1}(1-\alpha)) \ge a, \\ &x_1 + x_2 + \cdots + x_n = 1, \\ &x_i \ge 0,\ i = 1,2,\cdots,n. \end{aligned} \tag{8}$$

Thus we can solve the models (3)-(8) by traditional methods.

5. Numerical Examples

Example 2 Assume that there are 6 securities. Among them, returns of the six securities are all normal uncertain variables $\xi_i = N(e_i, \sigma_i),\ i = 1, 2, 3, 4, 5, 6$. Let the return rates be

$$\xi_1 = N(-1,1),\ \xi_2 = N(0,1),\ \xi_3 = N(1,2), \xi_4 = N(2,1), \xi_5 = N(3,2),\ \xi_6 = N(4,3)\ ,$$

Then

$$\sum_{i=1}^{6} x_i\xi_i = N(-x_1 + x_3 + 2x_4 + 3x_5 + 4x_6,\ x_1 + x_2 + 2x_3 + x_4 + 2x_5 + 3x_6)$$

Thus, we have

$$E[\sum_{i=1}^{6} x_i\xi_i] = -x_1 + x_3 + 2x_4 + 3x_5 + 4x_6\ ,$$

$$V[\sum_{i=1}^{6} x_i\xi_i] = x_1 + x_2 + 2x_3 + x_4 + 2x_5 + 3x_6\ .$$

Suppose that the confidence level $\alpha = 0.05$, and the minimum excepted return the investor can accept is $a = -0.1$, then $\Phi^{-1}(0.05) = -1.62.$ Thus the models (7) is the following:

$$\begin{aligned} &\max\ -x_1 + x_3 + 2x_4 + 3x_5 + 4x_6 \\ &\text{Subject to:} \\ &2.62x_1 + 1.62x_2 + 2.24x_3 - 0.38x_4 + 0.24x_5 + 0.86x_6 \le 0.1, \\ &x_1 + x_2 + \cdots + x_6 = 1, x_i \ge 0,\ i = 1,2,\cdots,6 \end{aligned} \tag{9}$$

By use of Matlab 7.0 on PC we obtain the optimal solution of model (9). The optimal solution of model (9) is

(0.0000,0.0000,0.0000,0.4907,0.2444,0.2649) and the optimal value of the objective function is 2.7742. This means that in order to gain maximum expected return with the risk not greater than -0.1 at the confidence level 0.05, the investor should assign his money according to the optimal. The corresponding maximum expected return is 2.7742.

In model (8), if the investor gives the preset total return rate is $a = 0.5$ and the preset confidence level is $\alpha = 0.9$, then the model (8) is the following,

$$\begin{array}{l} \min \ x_1 + x_2 + 2x_3 + x_4 + 2x_5 + 3x_6 \\ \text{Subject to:} \\ 2.2x_1 + 1.2x_2 + 1.4x_3 - 0.8x_4 - 0.6x_5 - 0.4x_6 \le -0.5, \\ x_1 + x_2 + \cdots + x_6 = 1, x_i \ge 0, \ i = 1, 2, \cdots, 6. \end{array} \qquad (10)$$

we obtain the optimal solutions of model (10) is

$$(0.0091, 0.1131, 0.0000, 0.8778, 0.0000, 0.0000),$$

and the value of objective function is 1.0000. This means that in order to minimize the risk with the return rate not less than 0.5 at the confidence level 0.9, the investor should assign his money according to the optimal. The corresponding minimum risk is 1.0000.

6. Conclusions

In this paper, uncertain variable is applied to portfolio selection problems, and two types of uncertain programming models for portfolio selection with uncertain returns on basis on the uncertain theory are provided. In order to solve the proposed models by traditional methods we discuss the crisp equivalents when the uncertain returns are chosen to be some special uncertain variables and give two examples to explain the efficiency of the method. The paper does not include the conditions when the return rates are general uncertain variables, this can be interesting areas for future researches.

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