Study on Error Measurement Method

for Actuator Frequency Characteristic Test System

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Abstract

System error measurement is key step in actuator frequency characteristic test and practical system error measurement methods must be studied to provide qualified results for system error compensation. Two error measurement methods (swept sine and FFT: fast Fourier transform) are explored, a strong anti-jamming frequency characteristic algorithm is perfected for swept sine and digital smoothing is introduced for improving accuracy of FFT. Numerical examples verified accuracy and robustness of swept sine and validity of FFT. These methods were applied to measure system error in missile actuator test system, the experiments showed that swept sine method could gain smooth and stable results and FFT method could achieve rather close results with swept sine. This study is part of general test system of some missile and provides valid technology analysis and reliable measurement results of system error for its actuator test.

Keywords: Frequency characteristic, System error, Swept sine, Curve fitting, Modulated frequency, Fast Fourier transform

1. Introduction

Actuator frequency characteristic test is an important part of general missile test system, and used for missile production, maintenance and launching test. In order to raise test accuracy and save precious hardware resources, for example, some PXI DAQ (data acquisition) card only has four channels for A/D, so it must be avoided to analyze and compensate all existent system error in test system under synchronous sampling on input and output; meanwhile, to reduce heat wastage of actuator and the effect on measurement, test efficiency must be raised. Therefore, the choice of measurement method directly decides the results of actuator frequency characteristic test, test accuracy and efficiency must be synthetically considered and how to measure system error with high accuracy and high efficiency becomes necessary step and important part of actuator frequency characteristic test.

In test system, system error mainly comes from A/D, D/A converter, input preprocessing, output processing, disturbance, noise and system nonlinearity(WANG Shou-kun, WANG Jun-zheng, 2007) . The paper emphasizes two methods for system error measurement; one is swept sine method which works according to swept sine principle, the other is FFT method (fast Fourier transform) which calculates spectrum of input and output according to fast Fourier transformation, figures out system error according to frequency characteristic definition, and then use digital smoothing to raise test accuracy.

The paper executes theoretical analysis and numerical simulation on the two methods, and then applies them to the measurement of system error in actuator frequency characteristic test system. Swept sine method grows maturely, and it has higher test accuracy but low efficiency. Taking the measurement of swept sine as criterion, swept sine method and FFT method are compared from the viewpoints of accuracy and efficiency, which demonstrates that, FFT method has rather high efficiency and completes measurement in ten seconds or a little more; furthermore, its accuracy is close to that of swept sine, it realizes high efficiency and high accuracy and can solve the problems of low efficiency and heat wastage from swept sine based frequency characteristic test.

2. System Error Analysis

2.1 System Error Distribution

Practical systems always have system error which can be divided into two types: analytic error and non-analytic error. The former expresses error analytically according to known test parameters and it comes from A/D, D/A, signal preprocessing and filtering; the latter includes noise, disturbance and nonlinearity which can not be described through analytic formula.

Based on system input and output, swept sine method and FFT method could raise accuracy of system error measurement because their results contain varieties of system error, even the error introduced by swept sine method and FFT method themselves. Because noise is random, there are always differences among multiple measurements from the two methods, but noise has low power and it will not influence measurements extraordinarily; electromagnetic disturbance is the principal disturbance factor in actuator test system, and it takes great effect on actuator frequency characteristic test.

According to the position relative to test plant, system error can be classified as prepositive error and postpositive error, which are described in Figure 1, where $H_1(\omega)$ is prepositive error, $H_2(\omega)$ is postpositive error and $H(\omega)$ is test plant.

Suppose frequency characteristic between input and output shown in Figure 1

$$H_c(\omega) = Y(\omega)/X(\omega) = H_2(\omega)H(\omega)H_1(\omega) \tag{1}$$

Therefore, the real characteristic $H(\omega)$ for plant

$$H(\omega) = \frac{H_c(\omega)}{H_2(\omega)H_1(\omega)} = H_c(\omega)/H_e(\omega) \tag{2}$$

where $H_e(\omega) = H_1(\omega)H_2(\omega)$ is just system error. Seen from (2), system error measurement is the necessary step for frequency characteristic test. Simulation and experiment show that, A/D and D/A will cause phase delay of 20° at least at $100Hz$ frequency point, and if signal processing loop exists, system error will grow increasingly.

2.2 Low-Pass Filter

As an electro-mechanic system, actuator are often disturbed by high frequency vibration and electromagnetic signal under test circumstances, therefore actuator output must be filtered through low pass filter so as to eliminate the effect from high frequency disturbance. In order to accommodate more types of actuator or plant and ensure high accuracy of frequency characteristic test in the range of actuator band width, it is required that the band width of low pass filter $H_F(\omega)$ is three times wider or more than that of test plant.

In some actuator frequency characteristic test system, both D/A and A/D in the applied DAQ card are realized as zero-order holder and their frequency characteristic are expressed as $H_{DA}(\omega)$ and $H_{AD}(\omega)$ which are referred to classical control theory. Contrasted with the distribution model shown in Figure 1, system error can be analytically described as

$$H_e(\omega) = H_{AD}(\omega)H_F(\omega)H_{DA}(\omega) \tag{3}$$

where postpositive error $H_2(\omega) = H_{AD}(\omega)H_F(\omega)$, and prepositive error $H_1(\omega) = H_{DA}(\omega)$. In later experiment, the expression is call analytic method.

3. System Error Measurement

3.1 Swept Sine Method

Swept sine method is mature and reliable for frequency characteristic test, but measurement noise existent in test system will make output apparently malformed and cause the test accuracy very low. Curve fitting (LIU Qiang, LIU De-peng, 2005) can overcome the above problems and it can be deduced from contradictory equations theory. Since curve fitting is a digital processing loop, it will introduce new system error inevitably.

Suppose system input at frequency f_i

$$u_i(t) = A_a \sin(2\pi f_i t + \phi_i) \tag{4}$$

where A_m is amplitude, ϕ_i is initial phase, often making $\phi_i = 0$.

According to swept sine principle, the response can be supposed as

$$y_o(t) = A_r \sin(2\pi f_i t + \phi_i + \Delta\phi) \tag{5}$$

where A_r is amplitude, $\Delta\phi$ is phase difference, namely the phase characteristic of test plant as frequency f_i.

Further decompose $y_0(t)$ as

$$y_0(t) = \sin(2\pi f_i t + \phi_i) A_r \cos(\Delta\phi) + \cos(2\pi f_i t + \phi_i) A_r \sin(\Delta\phi)$$
$$= \begin{bmatrix} \sin(2\pi f_i t + \phi_i) & \cos(2\pi f_i t + \phi_i) \end{bmatrix} \begin{bmatrix} A_r \cos(\Delta\phi) \\ A_r \sin(\Delta\phi) \end{bmatrix} \tag{6}$$

Suppose sampling period T, sampling time $t = 0, T, 2T, \ldots, nT$, the contradictory equations from (6) is

$$\begin{bmatrix} \sin(\phi_i) & \cos(\phi_i) \\ \sin(2\pi f_i T + \phi_i) & \cos(2\pi f_i T + \phi_i) \\ \cdots & \cdots \\ \sin(2\pi f_i nT + \phi_i) & \cos(2\pi f_i nT + \phi_i) \end{bmatrix} \begin{bmatrix} A_r \cos(\Delta\phi) \\ A_r \sin(\Delta\phi) \end{bmatrix} = \begin{bmatrix} y_0(0) \\ y_0(T) \\ \cdots \\ y_0(nT) \end{bmatrix} \tag{7}$$

According to contradictory equations theory, for contradictory equations $Ax = b$, its least squares solution is decided by

$$A^T Ax = A^T b \tag{8}$$

It is easily seen that, the coefficient matrix rank $rank(A) = 2$ in equations (7), so $A^T A$ is nonsingular, and the least squares solution is

$$x = (A^T A)^{-1} A^T b \tag{9}$$

where $b = [y_0(0), y_0(T), \cdots, y_0(nT)]^T$, and the solution $x = [A_r \cos(\Delta\phi), A_r \sin(\Delta\phi)]^T$, further get the magnitude $(= A_r / A_a)$ and the phase $\Delta\phi$ at frequency point f_i.

3.2 Fast Fourier Transformation Method

3.2.1 Modulated Frequency Signal Analysis

Proper selection of input signal is crucial to raise test accuracy, and when hardware permits, the signal which has strong power and wide band should be adopted as input. Modulated frequency signal $x(t) = A\cos(2\pi ft)$ has rather stable power spectrum in wider frequency range and choosing $f = \beta t + f_0$ will produce linear modulated frequency signal, where A is magnitude, β is frequency change rate, and f_0 is initial frequency.

Frequency change rate β indicates the spectrum distribution of modulated frequency signal, shown as Figure 2, and the power are equably centralized in the frequency range of 0.5β *Hz*. To adopt modulated frequency signal as input for frequency characteristic test can not only get higher accuracy in swept band, but also figure out frequency characteristic in this band through sweeping once, which avoids sweeping point by point in given frequency range and therefore accelerate test speed. But if the swept band is too wide and β is rather big, the power of modulated frequency signal will distribute dispersedly. In this case, the amplitude of modulated frequency signal should be amplified so as to get stronger spectrum, but the amplitude will be limited by hardware specification. So, modulated frequency signal is mainly used for frequency characteristic test of the plants whose band width is under 200*Hz*; for actuator with band with 30*Hz* or so, modulated frequency signal is adequate to provide satisfactory accuracy.

3.2.2 Key Procedures

Fast Fourier transformation (FFT) is a fast algorithm for discrete Fourier transformation and widely used to fault diagnosis(Z. Hameed, Y.S. Hong, Y.M. Cho, S.H. Ahn and C.K. Song, 2007)(V.K. Rai, A.R. Mohanty, 2007), frequency characteristic calculation (WANG Shou-kun, WANG Jun-zheng, 2006). Many improved fast algorithms have raised the calculation efficiency of FFT further (John D. Markel, 1971)(Fan Chih-Peng, Su Guo-An, 2007), and if data quantity is not too much, FFT already has adequate efficiency. Choose modulated frequency signal as test input(WANG Shou-kun, WANG Jun-zheng, 2006)(G. Gloth, M. Sinapius, 2004) .

Supposing input $x(t)$, its FFT is $X(k)$; sampled signal is $y(t)$ and its FFT is $Y(k)$, so get the measured system error

$$\begin{cases} |H_e(k)| = |Y(k)| / |X(k)| \\ \angle H_e(k) = \angle Y(k) - \angle X(k) \end{cases} \tag{10}$$

where $|\cdot|$ represents magnitude, $\angle \cdot$ is phase.

The key to apply FFT method is that the sampled data can effectively overcome the effect of limited time length. In example of some frequency characteristic test system, assigning modulated frequency signal $x(t) = A_a \sin[2\pi(\beta t + f_0)t]$, the band width of test plant is *B Hz*, then the following conditions had better be satisfied:

$$\begin{cases} \beta t + f_0 > \gamma B \\ \beta t + f_0 < (5 \sim 10)\gamma B \end{cases} \tag{11}$$

The sampling time length can be decided according to (11) where $\gamma > 0$ is an adjustable parameter.

1) Digital Smoothing

FFT transforms time-series data in time domain into frequency domain signal without choice, noise and disturbance are also included in transformed signals, so the transformed signal with noise and disturbance must be smoothed by digital smoothing.

Moving average filtering can effectively reject the components which are not smooth. Practice shows that, magnitude error is reduced from 2dB to 0.4dB and phase error decreases from 10º to 4º after applying moving average filtering, and the measured results is accurate and smooth.

4. Measurement Accuracy and Robustness

Since there are no ways to get analytic or real characteristic of system error in physical system, in order to evaluate measurement performance of swept sine method and FFT method, taking M-I: $G(s) = 200^2 /(s^2 + 280s + 200^2)$ and M-II: $G(s) = 1.0/(0.003s + 1.0)$ as test plants, simulation works out the differences between the characteristic in theory and that from swept sine and FFT methods.

The noise and disturbance are set as shown in Figure 3, and the calculated results are shown in Figure 4. In practical circumstances, the accuracy of some DAQ card is more than 0.001V (volt) and it can reach 0.0002V after calibration, so it is satisfied with 0.1V setting; real electromagnetic disturbance is not constant amplitude pulse signal, it is the most serious disturbance in actuator frequency characteristic test system and takes great effect on the measurement.

It can be seen from Figure 4 that, the calculated difference range from swept sine and FFT methods on different models (M-I and M-II) is stable; the results from swept sine method is smooth and that from FFT method is apparently fluctuating. Only considering the difference range, the two methods have close accuracy and even FFT method is better than swept sine method at high frequencies.

Furthermore, swept sine method is more robust to noise and disturbance than FFT method and it is a good way for frequency characteristic test. Besides, simulation also demonstrates that, the two methods have close performance in case that noise and disturbance both are small, and the magnitude difference is under 0.2dB while the phase difference is under 1.5º.

5. Experiments

In some actuator frequency characteristic test system, adopt DAQ-2006 multifunction DAQ card in ADLINK which has sample rate up to 250*k/s* and update rate up to 1*M/s*, and the card supports single and double buffer DMA mode which effectively alleviates the occupation of test computer resources. In experiment, A/D sample rate f_s and D/A update rate f_a both are 10*kHz*, and single buffer DMA mode is adopted. The achieved system error from the three methods (swept sine, FFT, analytic) is shown in Figure 5~Figure 7.

Figure 5 demonstrates the measurement comparison from swept sine method with different swept steps 0.5*Hz* and 1.0*Hz* respectively. We can see from the two measurements that, although least squares based curve fitting is applied, their difference is under 0.02dB on magnitude and under 0.15º on phase and at few points, the phase difference is up to 0.5º, which indicates that swept sine method has high accuracy on one side and noise and disturbance have taken effect on measurement on the other side.

It can be seen from Figure 6 that, FFT method has close measurement with swept sine method (S-Sine) and its magnitude difference is under 0.2dB and the phase difference at higher frequencies is under 1.8º; but to complete the same test, swept sine method under swept step 1.0*Hz* consumes 4.3 minutes while FFT method only requires 14 seconds, and FFT method raises test efficiency remarkably. The band width of system error is 65*Hz* from swept sine method and 64.926*Hz* from FFT method, and the difference is 0.074*Hz*. So we can deem that, FFT method achieves comparative test accuracy with swept sine method, but the former works much more efficiently than the latter; furthermore, the frequency resolution of the former is 0.08*Hz* but that of the latter is 1.0*Hz* which is just swept step.

It can be seen from Figure 7 that, relative to FFT method, the difference between analytic method and swept sine method is much big, the magnitude difference reaches 1.2dB at high frequencies, and the phase difference is 1.8º or so, which demonstrates that, the nonlinearity, noise, disturbance and the non-ideal implementation of some loops in practical system will cause the bigger difference.

6. Conclusions

System error measurement in missile actuator test system is the necessary procedure, and its test accuracy and efficiency directly affects that of actuator frequency characteristic test system. The paper focuses two system error measurement methods; swept sine method applies curve fitting to calculate the characteristic of system error according to swept output while FFT method calculates spectrum of input and output signal through fast Fourier transformation, and attains the characteristic of system error according to frequency characteristic definition and digital smoothing algorithm.

Simulation studies the produced difference from the above methods, compares and analyzes anti-noise and anti-disturbance ability, verifies the high accuracy and robustness of swept sine method and the efficacy of digital smoothing introduced in FFT method. And the two methods are applied to some missile actuator test system, the experiment indicates that, swept sine method can get smooth, stable and accurate characteristic, and FFT method can achieve high magnitude characteristic and relatively close phase characteristic. Relative to swept sine method, FFT method has rather high test efficiency and can avoid wastage of actuator performance because of heating during test.

The study is part of some general missile test system, provides reliable analysis for actuator frequency characteristic test and can be used as the principle of choosing the method for system error measurement and compensation in frequency characteristic test system.

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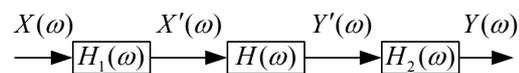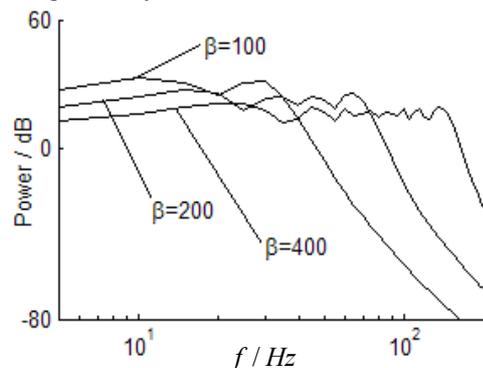
Figure 1. System error distribution model

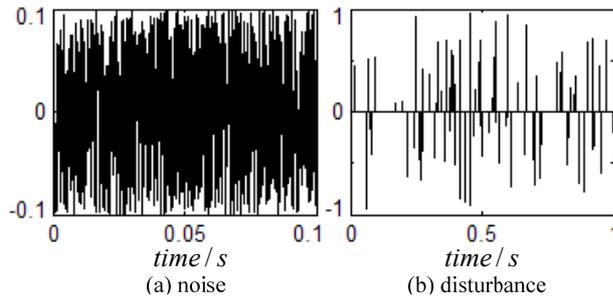Figure 2. Influence of β on power distribution

(a) noise   (b) disturbance

Figure 3. Noise and electromagnetic disturbance

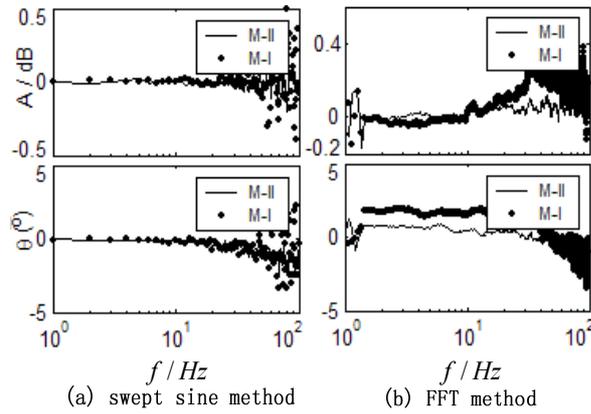(a) swept sine method   (b) FFT method
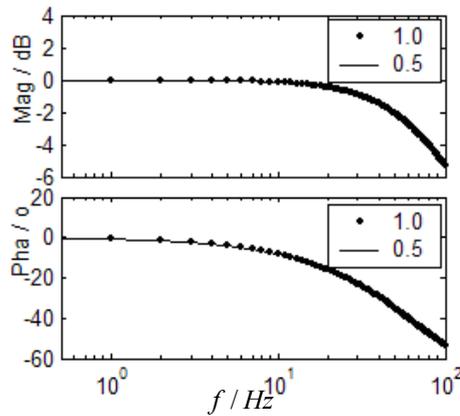
Figure 4. Measurement Error of Swept Sine and FFT

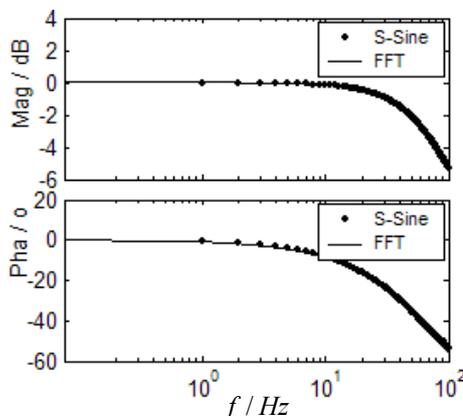Figure 5. Swept sine under different swept steps
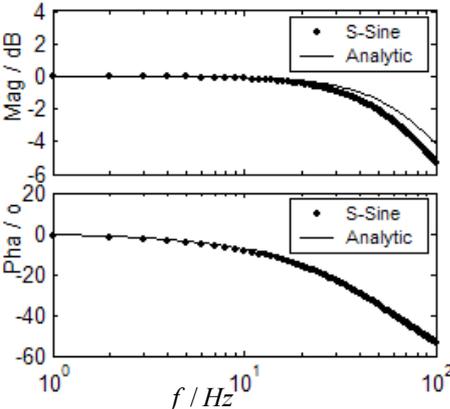

Figure 6. Comparison of FFT and swept sine methods

Figure 7. Comparison of analytic and swept sine methods