# Extraction of Inherent Frequency Components of Multiway EEG Data Using Two-Stage Neural Canonical Correlation Analysis 

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#### Abstract

This paper presents an algorithm for extracting underlying frequency components of massive Electroencephalogram (EEG) data. Frequency components of these data play a vital role to realize brain-body condition. Usually, a huge amount of time and specially built computers are essential to process these EEG data having different subjects. It also restricts to visualize inherent frequency of EEG for a general practitioner. An algorithm is developed using two-stage cascaded architecture of canonical correlation analysis with neural network named multiway neural canonical correlation analysis (MNCCA) to address three major challenges for extracting frequency components from EEG data, such as: (a) It processes multiway data which are feed sequentially into neural network, rather than feeding whole data at a time, (b) It uses the conventional personal computer instead of special computer built for such application, (c) It spends very short time for a moderate data set consisting of several ways (time, trials and channels). The experimental results are obtained with three different kinds of networks having linear, nonlinear and nonlinear feedback structures. The inherent dominant frequency of 1 Hz having a quite resemblance with EEG landscape has been found. This provides a great opportunity in analyzing brain-body function.


Keywords: Electroencephalogram (EEG), canonical correlation analysis (CCA), steady-state visual evoked potential (SSVEP), neural network (NN), multiway data

## 1. Introduction

The human brain generates electrical signals called electroencephalogram (EEG) which are related to body functions. These signals are roughly less than $100 \mu \mathrm{~V}$ and can be measured with electrodes placed on the scalp, noninvasively. The EEG is typically described in terms of rhythmic activity. The rhythmic activity is divided into frequency bands. These rhythmic activities within a certain frequency range were noted to have a certain distribution over the scalp or a certain biological significance. Frequency bands are usually extracted using spectral methods (for instance Welch) as implemented for instance in freely available EEG software (EEGLAB, 2013). A series of operation is repetitively required to get final frequency components if a conventional software or frequency analyzer algorithm is used. Moreover, there has been computational intractability if the data are massive and multiway.
The processing of massive EEG data is a great challenge for computational scientists. The multiway EEG data is configured using different ways such as time, channels and trials - usually termed as tensor and the order of the tensor is the number of dimensions, also known as ways or models (Cichocki et al., 2009). Steady-state visual evoked potential (SSVEP) is one kind of potential of brain signal. It is evoked over occipital scalp areas with the same frequency as the visual stimulus and may also include its harmonics when subject focuses on the repetitive flicker of a visual stimulus (Scherer, Brauneis, \& Pfurtscheller, 2005; Zhu et al., 2010). Recent approaches try to find inherent underlying frequency components in EEG signals. In addition, EEG signal may be contaminated by noise and it is still a challenge to detect the rhythmic activities of such signals especially at low stimulus frequency (Zhang et al., 2011). Hence, this requires high capacity machine which are specially built for a
particular EEG application and are not available for general users or practitioners. Therefore, a comfortable and user friendly faster method is essential to process multiway data with limited computational resources such as an ordinary personal computer.
There have been a number of approaches to recognize frequency of EEG signals. A traditional and widely used method for EEG signal recognition is power spectral density analysis (PSDA). PSD is estimated from the EEG signals within a time window typically by Fast Fourier Transform (FFT), and its peak is detected to recognize the target stimulus. It takes longer time window to estimate spectrum with sufficient resolution (Cheng et al., 2002). Some studies also took the PSDs as features and applied linear discriminate analysis (LDA) or support vector machine (SVM) classifier to classify the desired frequency (MÄuller-Putzet et al., 2005; Zhu et al., 2010) which may limit the real-time implementation.
Lin has found out the correlations between a set of EEG signals of multiple channels and a set of reference sine-cosine signals with different stimulus frequencies using statistical Canonical Correlation Analysis (CCA) (Lin et al., 2006). The desired stimulus is then recognized from conversed correlations by maximization process. It provides better recognition performance than that of the PSDA since it delivers an optimization for the combination of multiple channels and improves the noise tolerance. A comparative analysis between the CCA and PSDA was also discussed in (Hakvoort, Reuderink, \& Obbink, 2011). They also adopted the sine-cosine waves as reference signals used in the CCA for SSVEP recognition. Tensor CCA is an extension of the statistical CCA, which addresses on inspecting the correlation between two multiway data groups, instead of two sets of variables (Kim et al., 2009). Multiway CCA (MCCA) (Zhang et al., 2011) has been proposed to address the real time implementation of brain computer interface system. They remove inter subject variability and trial to trial variability in finding the optimized reference signals, although it requires specially built computers which are not available for general purpose. The matlab program for MCCA requires very long time to execute. However, the detail insight realization of correlation profile is still missing.
In this paper we implement CCA using neural network (NN), since NN is well known for their powerful capacity (Oja, 1982; Sanger, 1990; Lai \& Fyfe, 1999). We demonstrate two-stage "Multiway Neural Canonical Correlation Analysis" (MNCCA), which maximizes correlation between a set of sine cosine signals and a set of EEG signals. As a result, an optimized reference signal is obtained in the first stage. In the second stage, a test set of EEG signals and optimized reference signal are applied to the same network to find another optimized signal. Finally, frequency components of EEG data set are determined from above two optimized signals where their correlation becomes maximum. This does not require high capacity machine and it performs better than others since special NN cascade architecture is incorporated. According to our best knowledge this is the first application of neural CCA for extracting frequency components from EEG data.
This approach provides several advantages such as i) it does not require high capacity machine, ii) it uses NN that exhibits improve correlation in comparison to standard statistical methods (Lai \& Fyfe, 1999), iii) EEG input data are presented sequentially in the MNCCA network instead of presenting entire data at once. As a result, MNCCA does not require huge computer memory at a time. It is usually seen that an ordinary machine cannot process such huge data with MATLAB environment. In addition, nonlinear networks are used to reflect on nonlinear correlation of EEG data.
This paper is structured as follows. Section 2 describes characteristics of EEG data. In section 3, entire methodology is configured and described. In addition, three consecutive subsections - reference signals generation, outline of MNCCA and extraction of frequency components are discussed. Experimental results and discussion are presented in section 4 . We conclude the paper in section 5.

## 2. EEG Data Characteristics and Collection

The EEG dataset have been collected from SSVEP database (SSVEP DATA, 2013). For the clarity, we describe first about the database. Brain signal acquisition in SSVEP was performed with 128 active electrodes (channels) at a sampling rate of $2,048 \mathrm{~Hz}$ (Biosemi Inc., Amsterdam) (Hovagim, Toshihisa, \& Andrzej, 2010). Four healthy subjects with normal or corrected-to-normal vision participated in this study. The subjects were fully informed of all procedures and having no neurological disorders. Before each experiment they were briefly tested for photosensitive epilepsy. Subjects, who did not have any prior training except for a short practice run during the briefing, were seated 0.9 m from a 21" CRT computer display operated at a high vertical refresh rate.
SSVEP stimulation was achieved using small reversing black and white checkerboards with $6 \times 6$ checks. The checkerboards had dimensions $1.8^{\circ} \times 1.8^{\circ}$ arc, so that the diameter $\left(2.5^{\circ}\right.$ arc $)$ would just cover the size of the fovea. A single small checkerboard stimulus was displayed for three frequencies sequentially ( 8,14 and 28 Hz ), covering each of the three SSVEP response regions (low, medium, and high frequency) (Regan, 1977). The EEG
data was preprocessed using a demodulation procedure to remove the interference (Müller \& Hillyard, 2000). There were 5 trials of each subject for each frequency. Each subject had total 15 trials at above 3 different frequencies. Therefore, a total of 60 trials of four subjects were found in the database. There were 128 channels (rows) with more than 6000 columns (sampling points) for a trial.

## 3. Methodology

The general framework of the entire methodology is shown in Figure 1. Our aim is to extract the frequency components from large EEG data. In order to make it possible we devise two stage cascade CCA. The first stage determines optimized reference signal from a set of sine-cosine reference signals and a set of EEG data. The second stage determines another optimized signals from the first stage's optimized signals and a new set of EEG data. A correlation coefficient is computed from above two optimized signals. This process is repeated at different stimulus frequency or time window. We determine the desired frequency at which the correlation is maximum. Each part of the method is briefly described in the following sub-sections.


Figure 1. Framework of the two-stage cascaded MNCCA

### 3.1 Reference Signal Generation

Usually a signal can be represented in terms of sine-cosine waveform according to theory of Fourier transform. Therefore we consider sine-cosine as reference signals in order to determine underlying frequency components in the EEG data. The reference signals are constructed by sine-cosine waves at the $m$-th stimulus frequency $f_{m}$ ( $m$ $=1,2 \ldots M$ ) as follows (Zhang et al., 2011), where $H$ denotes the number of used harmonics, $J$ is the number of sampling points and $f_{s}$ represents the sampling rate.

$$
Y_{m}=\left(\begin{array}{c}
\sin \left(2 \pi f_{m} 1 / f_{s}\right) \ldots \ldots \ldots \ldots  \tag{1}\\
\cos \left(2 \pi f_{m} 1 / f_{s}\right) \ldots \ldots \ldots . \sin \left(2 \pi f_{m} J / f_{s}\right) \\
\\
\cdot \\
\cdot \\
\\
\sin \left(2 \pi H f_{m} 1 / f_{s}\right) \ldots \ldots \ldots \ldots \cos \left(2 \pi f_{m} J / f_{s}\right) \\
\cos \left(2 \pi H f_{m} 1 / f_{s}\right) \ldots \ldots \ldots \ldots \sin \left(2 \pi H f_{m} J / f_{s}\right) \\
\hline
\end{array}\right)
$$

These signals are used in first stage of MNCCA to find an optimized reference signal. Since pure sine-cosine reference signals do not contain any information on EEG data, we generate this kind of optimized reference signals with MNCCA.

### 3.2 MNCCA

CCA is a multivariable statistical method to reveal the underlying correlation between two sets of data (Hotelling, 1936). It finds two bases, one for each variable, that are optimal with respect to correlations and it finds the corresponding correlations at the same time. The standard statistical CCA avoids nonlinear relationship between datasets. Extension of standard CCA with NN can overcome this problem (Lai \& Fyfe, 1999). A brief description of neural CCA is presented here for the sake of simplicity. In this paper we use three kinds of neural networks -linear, nonlinear and nonlinear with feedback and they are described briefly below. Consider $\boldsymbol{x}_{\boldsymbol{1}}$ set of reference signals and $\boldsymbol{x}_{2}$ set of EEG signals, and then we attempt to find the linear combination of the signals that gives us maximum correlation between the combinations as described in Figure 2, let

$$
\begin{align*}
& \boldsymbol{y}_{\mathbf{1}}=\boldsymbol{w}_{\mathbf{1}} \boldsymbol{x}_{\mathbf{1}}=\sum_{j} w_{1 j} x_{1 j}  \tag{2}\\
& \boldsymbol{y}_{\mathbf{2}}=\boldsymbol{w}_{\mathbf{2}} \boldsymbol{x}_{\mathbf{2}}=\sum_{j} w_{2 j} x_{2 j} \tag{3}
\end{align*}
$$

where $j$ is the number of column in every row, there were total 128 rows for 128 electrodes. Then we wish to find those values of $\boldsymbol{w}_{1}$ and $\boldsymbol{w}_{2}$ that maximize the correlation between $y_{1}$ and $y_{2}$.


Figure 2. Standard CCA network

The input data comprises two vectors $\boldsymbol{x}_{1}$ and $\boldsymbol{x}_{2}$. A complete column for a row of a particular subjects/reference signals are entered in the CCA network at a time as input $\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right)$. In this way, every row is presented in the network sequentially. Activation is fed forward from each input to the corresponding output through the respective weights, $\boldsymbol{w}_{1}$ and $\boldsymbol{w}_{2}$. We use the joint learning rules for linear correlation, where $\lambda_{1}$ and $\lambda_{2}$ are Lagrange multipliers, $\mathrm{w}_{1 \mathrm{j}}$ is the $j$ th element of weight vector, $\boldsymbol{w}_{1}$, etc.

$$
\begin{gather*}
\Delta w_{1 j}=\eta x_{1 j}\left(y_{2}-\lambda_{1} y_{1}\right)  \tag{4}\\
\Delta \lambda_{1}=\eta_{0}\left(1-y_{1}^{2}\right)  \tag{5}\\
\Delta w_{2 j}=\eta x_{2 j}\left(y_{1}-\lambda_{2} y_{2}\right)  \tag{6}\\
\Delta \lambda_{2}=\eta_{0}\left(1-y_{2}^{2}\right) \tag{7}
\end{gather*}
$$

The frequencies of different subjects are also recognized with nonlinear and nonlinear with feedback networks. These networks search the nonlinear combination of the signals that gives us maximum correlation between the reference signals set $\boldsymbol{x}_{\boldsymbol{1}}$ and EEG data set $\boldsymbol{x}_{2}$ as described in Figure 2. For nonlinear networks, let

$$
\begin{align*}
& \boldsymbol{y}_{\mathbf{1}}=\boldsymbol{w}_{\mathbf{1}} \boldsymbol{f}_{\mathbf{1}}=\sum_{j} w_{1 j} \tanh \left(v_{1 j} x_{1 j}\right)  \tag{8}\\
& \boldsymbol{y}_{\mathbf{2}}=\boldsymbol{w}_{\mathbf{2}} \boldsymbol{f}_{2}=\sum_{j} w_{2 j} \tanh \left(v_{2 j} x_{2 j}\right) \tag{9}
\end{align*}
$$

The joint learning rules for nonlinear correlation are

$$
\begin{gather*}
\Delta w_{1 j}=\eta f_{1}\left(y_{2}-\lambda_{1} y_{1}\right)  \tag{10}\\
\Delta v_{1 j}=\eta x_{1 j} w_{1 j}\left(y_{2}-\lambda_{1} y_{1}\right)\left(1-f_{1}^{2}\right)  \tag{11}\\
\Delta \lambda_{1}=\eta_{0}\left(1-y_{1}^{2}\right)  \tag{12}\\
\Delta w_{2 j}=\eta f_{2}\left(y_{1}-\lambda_{2} y_{2}\right) \tag{13}
\end{gather*}
$$

$$
\begin{gather*}
\Delta v_{2 j}=\eta x_{2 j} w_{2 j}\left(y_{2}-\lambda_{1} y_{1}\right)\left(1-f_{2}^{2}\right)  \tag{14}\\
\Delta \lambda_{2}=\eta_{0}\left(1-y_{2}^{2}\right) \tag{15}
\end{gather*}
$$

The joint learning rules for nonlinear feedback network are

$$
\begin{gather*}
\Delta w_{1 j}(t)=\eta f_{1}\left(y_{2}(t)-\lambda_{1} y_{1}(t)\right)+0.5 y_{1}(t-1)  \tag{16}\\
\Delta v_{1 j}(t)=\eta x_{1 j} w_{1 j}\left(y_{2}(t)-\lambda_{1} y_{1}(t)\right)\left(1-f_{1}^{2}\right)+0.5 y_{1}(t-1)  \tag{17}\\
\Delta w_{2 j}(t)=\eta f_{2}\left(y_{1}(t)-\lambda_{2} y_{2}(t)\right)+0.5 y_{2}(t-1)  \tag{18}\\
\Delta v_{2 j}(t)=\eta x_{2 j} w_{2 j}\left(y_{2}(t)-\lambda_{1} y_{1}(t)\right)\left(1-\boldsymbol{f}_{2}^{2}\right)+0.5 y_{2}(t-1) \tag{19}
\end{gather*}
$$

The correlation is very important in order to assess the relationship between two time series. Experiments were done using time series of EEG signals of different subjects and sine-cosine reference signals with different harmonics. Borrowing the idea of multiway CCA, we introduce two-stage multiway neural CCA to find frequency components of EEG data set. We consider a three-way EEG data (channel $\times$ time $\times$ trial) and a sine-cosine reference signal matrix (harmonic $\times$ time) with stimulus frequency and its higher harmonics. Our aim is to find underling frequency components of multiway EEG data set, based on the optimized reference signals of sine-cosine and multiway data groups. A brief description of two-stage MNCCA algorithm is outlined in A1 below.

## Al: Algorithm for computing correlations using two-stage MNCCA

Input: EEG data $x_{1}, x_{2}, x_{3}, \quad x_{m} \in R^{I x J x K}$ and reference sine-cosine signals $Y_{m}(m=1,2, \ldots M) \in R^{2 H x J}$ corresponding to $M$ stimulus frequencies. Here $I, J, K$ and $H$ indicate the total number of channels, sampling points, trials and harmonics respectively.

## Output: Correlation $S_{m}$

## Begin loop

for $m=1$ to $M d o$
begin stage 1
Input: Training subjects (EEG data set) and reference sine-cosine signals $Y_{m}$
Output: Optimized signals $y_{1}$ and $y_{2}$
Random initialization of $\boldsymbol{w}_{1}$ and $\boldsymbol{w}_{2}$ for $i=1$ to $I d o$
repeat
Update $\boldsymbol{w}_{1}$ and $\boldsymbol{w}_{2}$
until the maximum number of iteration is reached

## end

Compute the optimized signals $y_{1}$ and $y_{2}$

$$
\text { end stage } 1
$$

begin stage 2
Input: Testing subject (EEG data set) and optimized reference signal $y_{2}$ Output: Optimized signals $y_{3}$ and $y_{4}$

Random initialization for weight $\boldsymbol{w}_{3}$ and $\boldsymbol{w}_{4}$
for $i=1$ to $I d o$
repeat
Update $\boldsymbol{w}_{3}$ and $\boldsymbol{w}_{4}$
until the maximum number of iteration is reached
end
Compute the optimized signals $y_{3}$ and $y_{4}$

$$
\text { end stage } 2
$$

Compute correlation $S_{m}$ from $y_{2}$ and $y_{4}$

## End loop

### 3.3 Extraction of Frequency Components

We extracted the frequency components of EEG signals using MNCCA. Since there is no information of EEG signals on sine-cosine signals, we optimize reference signals from sine-cosine and EEG signals. We use four-fold cross validation i.e. three subjects consisting of 45 trials are taken for training in the first stage and remaining subject ( 15 trials) is used for finding final optimize reference signal in the second stage. We concatenated 45 trials for first stage and 15 trials for second stage before presenting in the neural network. In this way channel wise EEG patterns are presented in the network and updated as visualize in Figure 3. We use three different types of networks such as linear, nonlinear and nonlinear with feedback for different frequency settings.


Figure 3. Illustration of two-stage MNCCA approach for extraction of frequency components

The correlation coefficient $S_{m}$ which reflects the relationship between $y_{2}$ and $y_{4}$ is calculated by Equation (20), where $\|$.$\| denotes norm.$

$$
\begin{equation*}
S_{\mathrm{m}}=\sqrt{1-\frac{\left\|\mathrm{y}_{2}-\mathrm{y}_{4}\right\|^{2}}{\left\|\mathrm{y}_{2}-\mathrm{E}\left[\mathrm{y}_{2}\right]\right\|^{2}}} \tag{20}
\end{equation*}
$$

Larger $S_{m}$ implies more significant relationship between $y_{2}$ and $y_{4}$ (Zhang et al., 2011).
From correlation $S_{m}$, we find desired frequency $f_{\text {desired }}$ by using the Equation (21), where, $f_{m}$ is the stimulus frequency of sine-cosine reference signal.

$$
\begin{equation*}
f_{\text {desired }}={ }_{\max _{m}} S_{m} \tag{21}
\end{equation*}
$$

We consider time window $(T W)=1 / f_{m}$. at different $f_{m}$ we have a correlation profile. The maximum correlation converges at one or more $f_{m}$ value(s) which is (are) our desired frequency component(s).

## 4. Results and Discussion

The MNCCA algorithm is used to extract underlying frequency components of multiway EEG data. The idea is to find maximum correlation points between reference sine-cosine signals and EEG signals of different subjects. Initially random weights are generated for both stages of the networks. Weights are updated according to update rules and normalized after presenting each data into the network. Three harmonic settings are utilized for each types of network. The harmonic settings of reference sine-cosine signals are regarded as $H 1 \in$ Fundamental and multiple of fundamental, $H 2 \in 2^{\text {nd }}$ Harmonic and multiple of $2^{\text {nd }}$ Harmonic, $H 3 \in 3^{\text {rd }}$ Harmonic and multiple of $3^{\text {rd }}$ Harmonic. Subjects 1 to 4 are denoted as S1, S2, S3 and S4 respectively. The programs are implemented in MATLAB environment using a computer with Intel (R) Core ${ }^{\text {TM }}$ i5-2450MA $2.50 \mathrm{GHz}, 4.00 \mathrm{~GB}$ of RAM and 64-bit Operating system.
Figures 4,5 and 6 describe correlation profiles against time $\left(1 / f_{m}\right)$ in seconds. At $H 1$, maximum correlation occurs at 1 Hz stimulus frequency for different subjects as observed in Figure 4. It is seen that maximum correlation occurs at 5 Hz and 1 Hz with linear network for $\mathrm{S} 1, \mathrm{~S} 2, \mathrm{~S} 3$ and S4 as shown in Figure 4(a). Similar results are also observed with nonlinear feedback network as shown in Figure 4(c). However, maximum correlation becomes 1 Hz only with feedback free nonlinear network across all subjects as shown in Figure 4(b).
At H2, when $2^{\text {nd }}$ Harmonic and multiple of $2^{\text {nd }}$ Harmonic are used as sine-cosine reference signals set, it is found that maximum correlation is found at $0.5,1,2.5$ and 5 Hz with linear network, as shown in Figure 5(a). We also use nonlinear and nonlinear feedback networks, as observe in Figures 5(b) and 5(c) respectively. In the last two cases, maximum correlation is found at 1 Hz stimulus frequency only.
At H3, when $3^{\text {rd }}$ Harmonic and multiple of $3^{\text {rd }}$ Harmonic are used as sine-cosine reference signals set, it is found that maximum correlation is found at $0.625,0.71,1,1.672$ and 5 Hz with linear network, these are shown in Figure 6(a). Nonlinear and nonlinear feedback networks are also studied. The results are plotted in Figures 6(b) and 6(c). Maximum correlation is found at 1 Hz stimulus frequency for both cases. In this sense, we claim that a frequency of 1 Hz is dominant for these experimental EEG data of different subjects. The appearance of maximum correlations at frequencies other than 1 Hz may be due to the harmonic and subjects variations.
MNCCA networks involve a number of used specified parameters such as learning rate ( $\eta, \eta_{0}$ ) and Lagrange multipliers $\left(\lambda_{1}, \lambda_{2}\right)$. The values of them are selected with few initial trial runs. It has been found empirically that best results are achieved when, $\eta_{0} \gg \eta$. We choose $\lambda_{1}=0.0005, \lambda_{2}=0.000002, \eta_{0}=0.0005$ and $\eta=0.00000015$ for representative result. Correlation profile does not change significantly if these values of constants are increased or decreased. Iteration is one of important factors for the convergence of NN. If iteration is increased the correlation profile is improved, resulting no change in frequency characteristic.
One can determine brain condition of a particular subject using this approach which is easy to program in an ordinary machine. Usually massive parallel EEG data requires a number of days to observe the final result. Our approach is simple to execute within a minute and does not require special machine.


(c)


Figure 4. Correlation coefficient of MNCCA with (a) linear network, (b) nonlinear network, and (c) nonlinear network with feedback for H 1 settings




Figure 5. Correlation coefficient of MNCCA with (a) linear network, (b) nonlinear network, and (c) nonlinear network with feedback for H 2 settings


(c)


Figure 6. Correlation coefficient of MNCCA with (a) linear network, (b) nonlinear network, and (c) nonlinear network with feedback for H 3 settings

## 5. Conclusion

In this study, two-stage MNCCA approach is proposed to recognize the stimulus frequency. MNCCA is implemented between the EEG data and reference sine-cosine signals to get optimized reference signals in the first stage. MNCCA is again applied in the second stage to inspect the correlation between the test EEG data and optimized reference signals of the first stage. Finally frequency components are extracted from two optimized signals. Both optimized signals include the information of subject-specific and trial-to-trial variability meaning that the MNCCA converges to underlying frequency components. Among different frequencies for different subjects, a frequency of 1 Hz is dominant across the subjects. The propriety of dominant frequency has been confirmed by observing correlation profile. The proposed method takes about a minute with user friendly ordinary computer, whereas the statistical CCA takes several hours to execute. It is indicative that nonlinear network shows an improved correlation profile leaving the overall performance similar. The algorithm is simple to program, implement and useful for finding underlying frequency of SSVEP.

## References

Cichocki, A., Zdunek, R., Phan, A., \& Amari, S. (2009). Nonnegative matrix and tensor factorizations: Applications to exploratory multi-way data analysis and blind source separation. New York, NY: Wiley. http://dx.doi.org/10.1002/9780470747278
Cheng, M., Gao, X., Gao, S., \& Xu, D. (2002). Design and implementation of a brain-computer interface with high transfer rates. IEEE Trans. Biomed. Eng., 49, 1181-1186. http://dx.doi.org/10.1109/TBME.2002.803536
EEGLAB News. (2013). Retrieded from http://scen.ucsd.edu/eeglab/
Hakvoort, G., Reuderink, B., \& Obbink, M. (2011). Comparison of PSDA and CCA detection methods in a SSVEP-based BCI-system. Technical Report, TR-CTIT-11-03, EEMCS. ISSN:1381-3625.
Hotelling, H. (1936). Relations between two sets of variates. Biometrika, 28, 321-377.
Hovagim, B., Toshihisa, T., \& Andrzej, C. (2010). Optimization of SSVEP brain responses with application to eight-command Brain-Computer Interface. Neuroscience Letters, 469, 34-38. http://dx.doi.org/10.1016/j.neulet.2009.11.039
Kim, T. K., \& Cipolla, R. (2009). Canonical correlation analysis of video volume tensor for action categorization and detection. IEEE Trans. PAMI., 31, 1415-1428. http://dx.doi.org/10.1109/TPAMI.2008.167
Lai, P. L., \& Fyfe, C. (1999). A neural implementation of canonical correlation analysis. Neural Networks, 12, 1391-1397. http://dx.doi.org/10.1016/S0893-6080(99)00075-1
Lin Z., Zhang, C., Wu, W., \& Gao, X. (2006). Frequency recognition based on canonical correlation analysis for SSVEP-based BCIs. IEEE Trans. Biomed. Eng., 53, 2610-2614. http://dx.doi.org/10.1109/TBME.2006.886577
MÄuller-Putz, G. R., Scherer, R., Brauneis, C., \& Pfurtscheller, G. (2005). Steady-state visual evoked potential (SSVEP)-based communication: impact of harmonic frequency components. J. Neural. Eng., 2, 123-130. http://dx.doi.org/10.1088/1741-2560/2/4/008
Müller, M. M., \& Hillyard, S. (2000). Concurrent recording of steady-state and transient event related potentials as indices of visual-spatial selective attention. Clin. Neurophysiol., 111, 1544-1552. http://dx.doi.org/10.1016/S1388-2457(00)00371-0
Oja, E. A. (1982). Simplified neuron model as a principle component analyzer. Journal of Mathematical Biology, 16, 267-273. http://dx.doi.org/10.1007/BF00275687
Regan, D. (1977). Steady-state evoked potentials. J. Opt. Soc. Am., 67, 1475-1489. http://dx.doi.org/10.1364/JOSA.67.001475
Sanger, T. (1990). Analysis of the two-dimensional receptive fields learned by the generalized hebbian algorithm in response to random dot input. Biological Cybernetics, 63, 221-228. http://dx.doi.org/10.1007/BF00195861
SSVEP DATA. (2013). Retrieved from http://www.bakardjian.com/work/ssvep_data_Bakardjian.html/
Zhang, Y., Jin, J., Qing, X., Wang, B., \& Wang, X. (2011). LASSO based stimulus frequency recognition model for SSVEP BCIs. Biomedical Signal Processing and Control, 7(2), 104-111. http://dx.doi.org/10.1016/j.bspc.2011.02.002

Zhang, Y., Zhou, G., Zhao, Q., Onishi, A., Wang, J. J., \& Cichocki, A. (2011). Multiway Canonical Correlation Analysis for Frequency Components Recognition in SSVEP-based BCIs. In Neural Information Processing (pp. 287-295). Springer Berlin Heidelberg. http://dx.doi.org/10.1007/978-3-642-24955-6_35
Zhu, D. H., Bieger, J., Molina, G. G., \&Aarts, R. (2010). A survey of stimulation methods used in SSVEP-based BCIs. Computational intelligence and neuroscience, 2010, 1. http://dx.doi.org/10.1155/2010/702357
Zhu, D., Molina, G. G., Mihajlovic, V., \& Aarts, R. M. (2010). Phase synchrony analysis for SSVEP-based BCIs. 2nd International Conference on Computer Engineering and Technology (ICCET 2010), April 16-18, China.

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