



Modeling the Bi-directional DC-DC Converter for HEV's

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Abstract

Hybrid Electrical Vehicles-HEV's are the important ways to improve vehicle performance. The transformer isolated bi-directional DC-DC converters are the key components of the traction system in HEV's. This paper presents a detail mathematic model of isolated bi-directional DC-DC converter for HEV's. Approximate models are important mathematic methods especially for analysis and closed-loop control design converter circuits. These differential equations, which govern the converter operation, change periodically among a set of linear differential equations because of the switch effect. Basing on the time-scale the state variables was separate as fast-scale and slow-scale variables. The fast changing variable of the leakage inductor was eliminated by substitute the fast-scale variable into slow-scale variable equations, resulting in reduced order differential equations. From this set of reduced order differential equations the completely averaged model of the isolated DC/DC converter was derived. The simulated results reveal that the circuit and mathematical model are consistent very well. The averaged state variables can be treated as a small component plus a DC component, so the averaged model can be separated a dynamic small signal part and a DC part. This linearized small signal model is suit for control design and analysis at a steady point that is decided by the DC component. As an example a PI controller was design basing on the linear model.

Keywords: HEV, Averaged method, Bi-directional converter, Linearization, Time-scale

1. Introduction

Hybrid Electrical Vehicles-HEV's are the important methods to improve vehicle efficiency, economize energy and reduce pollution (Su & Peng, 2002, pp.10-14. Peng, Li, & Su, 2004, pp.54-65). HEV's are the most prospective candidate to replace the conventional internal combustion engines-ICEs. In HEV's, it is required to have a relatively large power rate DC-DC converter for voltage matching and energy storage. Full bridge bi-directional DC-DC converters are a suitable choice for HEV's application because of their capabilities to deal with high power flow forward and backward according requirements, simultaneous they can realize the high power/volume ratio and high frequency easily.

Because of the switching effect, the converter circuits are strong non-linear systems from the viewpoint of control theory (Sun & Horst, 1992, pp.1165-1172. Chen, & Sun, 2006, pp.487-494. Vinod, Sun & Bonnie, 2003, pp. 381-389.). It is difficult to get the dynamic mathematical model to analyze and design the feedback control system basing on a uniform model, which depicts the whole system (Li, & Peng, 2004, pp.272-283.). The conventional analysis methods are very detailed, but these methods are usually to analyze the converter circuits themselves (Su & Peng, 2002, pp.10-14. Peng, Li, & Su, 2004, pp.54-65. Henry, Chung, Adrian & Cheung, 2003, pp.743-753.). Without the mathematical model it is also difficult to get the definite information of poles, zeros and the gain of the control system around the operating point. What's more important is how do the variations in the input voltage, the load current, or duty cycle affect the output voltage.

This paper is focus to develop the mathematical model for the bi-directional converter basing on the state space averaging theory (Sun & Horst, 1992, pp.1165-1172.). The converter circuits are divided into several operational modes in one period. Relaying on the semi conduct switch devices, such as the Mosfet/IGB, to force the converter from one operational mode to another. The whole circuits are discontinuous although in every operation mode it is possible continuous. Averaging approaches transform the discourteous converter circuits into continuous one (Sun & Horst, 1992 pp.1165-1172.). The averaged model is still a large-signal model however the dynamic small signal model can be derived from it. Taking advantage of the state space equations and transfer function of the small signal model, the control theories for the continuous system are applicable to converter circuits. The simulation experiment was carried out to validate the analysis.

2. Analysis of the operational modes

The transformer isolated bi-directional converter has dual active bridges with power transferred between them through an isolated high frequency transformer. Both two bridges consist H-bridge as shown in Figure1. From the first bridge to second bridge is voltage step up and on the contrary is voltage step down. The first bridge converts DC voltage to square wave AC voltage and feed into the primary side of the transformer. The second bridge rectified the AC to DC voltage, which feed to traction motor.

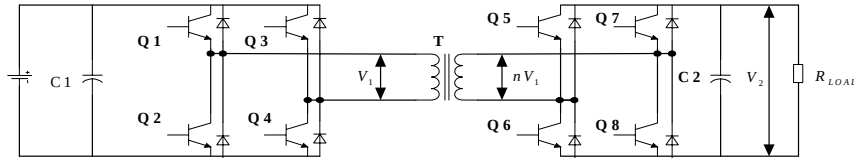


Figure 1. The bi-directional converter in HEV's

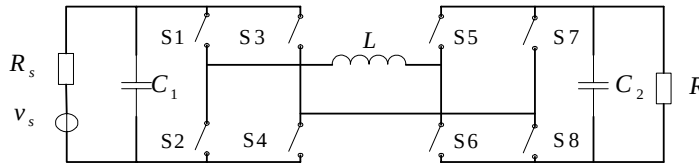


Figure 2. the simplification of the converter

In order to simplify the analysis, the device-level circuit is simplified as Figure 2. The switches are replaced with the ideal ones. The transformer is replaced with its leakage inductor L . The voltages on both sides of the transformer are v_1 and nv_1 . n is the ratio of the transformer. There are total four modes in one period^[11], the current wave in the leakage inductor is as Figure 3.

Figure 3. The current wave of leakage inductor

The operational modes have been analyzed detailed in (Dedoncker, Divan & Kheraluwala, 1991.). Here just give the results of the four operational modes. The state space equations are as (1), (2), (3) and (4) in corresponding operation mode.

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{4r_b}{L} & \frac{1}{L} & \frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{r_s C_1} & 0 \\ -\frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1 r_s} \\ 0 \end{bmatrix} v_s \quad (1)$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{4r_b}{L} & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{C_1 r_s} & 0 \\ \frac{1}{C_2} & 0 & \frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1 r_s} \\ 0 \end{bmatrix} v_s \quad (2)$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{4r_b}{L} & -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{C_1} & -\frac{1}{r_s C_1} & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{r_s C_1} \\ 0 \end{bmatrix} v_s \quad (3)$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{4r_b}{L} & -\frac{1}{L} & \frac{1}{L} \\ \frac{1}{C_1} & -\frac{1}{C_1 r_s} & 0 \\ -\frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1 r_s} \\ 0 \end{bmatrix} v_s \quad (4)$$

These differential equations change periodically between a set of linear differential equations at the switch points. In every sub-interval, the equation has the standard linear form as $\dot{x}(t) = A_i x(t) + B_i u(t)$.

The state variable i_L vary faster than v_1 and v_2 , so the system is belongs to two-time scale system^[5]. The problem is a time-scale separation between slow and fast variables that permits to define boundary conditions. The boundary conditions are allowed to be complex and it generates an averaged model of the slow state variables. Taking as the fast variables as the boundary condition, the averaged model is a reduce order model and is simple to deal with comparing with the full order model (Chen & Sun, 2006, pp.487–494).

3. Averaging the two time scale system

From the analysis above, the converter mode switches periodically inside a set of ordinary differential equations. This kind of functions also belongs to variable structure systems-VSR from the viewpoint of control theory (Su & Peng, 2002, pp.10-14). To derive an averaged model for the slow variables than accounting for the fast variables, there are three steps (Sun & Horst, 1992, pp.1165-1172. Chen & Sun, 2006, pp.487–494). First assuming the slow variable is constant while response to the fast variables. Second the solution of the fast variables was replaced in the slow equations. Third averaging the slow variable as show by (Chen & Sun, 2006, pp.487–494). The transformer isolated bi-directional converter, which includes current leakage inductor that varies fast than the voltage in capacitor C1 and C2 Corresponding the state variables are separated two parts as equation (5). State variables X is called slow variable because of the small parameter ε , whereas state variables Y are called fast variables.

$$\begin{aligned} \frac{dX}{dt} &= \varepsilon f(t, X, Y) & X(t_0) &= X_0 \\ \frac{dY}{dt} &= g(t, X, Y) & Y(t_0) &= Y_0 \end{aligned} \quad (5)$$

where, $X \in R^n, Y \in R^m$ and $\varepsilon > 0$ is small parameter and both function are vector valued continuous functions.

Assuming the converter consists of ideal switches and linear passive components (R, L, C), it state space equations have the linear form as equation (6).

$$\begin{aligned} \frac{dX}{dt} &= A_{sx}^{(i)} X + A_{sy}^{(i)} Y + B_s^{(i)} U \\ \frac{dY}{dt} &= A_{fx}^{(i)} X + A_{fy}^{(i)} Y + B_f^{(i)} U \end{aligned} \quad (6)$$

where, $i = 1, 2, 3, 4$.

Neglecting turning resistor r_b of power switch, the fast variables i_L is as equation (7).

$$L \frac{di_L}{dt} = \begin{cases} v_1 + v_2 & [0 \quad dT_s] \\ v_1 - v_2 & [dT_s \quad T_s] \\ -v_1 - v_2 & [T_s \quad (1+d)T_s] \\ -v_1 + v_2 & [(1+d)T_s \quad 2T_s] \end{cases} \quad (7)$$

The solutions of equation (7) are as equation (8), where the fast variable i_L is

$$\begin{cases} i_L(0) = -i_L(T_s) = -\frac{\pi}{\omega L} [v_1 + (2d-1)v_2] \\ i_L(dT_s) = -i_L[(1+d)T_s] = \frac{\pi}{\omega L} [(2d-1)v_1 + v_2] \end{cases} \quad (8)$$

According the equation (1), (2), (3) and (4), substitute i_L into the slow variable v_1 in the interval of corresponding mode as equation (9).

$$C_1 \frac{dv_1}{dt} = \begin{cases} \frac{v_s}{r_s} - \frac{v_1}{r_s} - \frac{v_1 + v_2}{L} t + \frac{\pi}{\omega L} [v_1 + (2d-1)v_2] & [0 \quad dT_s] \\ \frac{v_s}{r_s} - \frac{v_1}{r_s} - \frac{v_1 - v_2}{L} (t - dT_s) - \frac{\pi}{\omega L} [(2d-1)v_1 + v_2] & [dT_s \quad T_s] \\ \frac{v_s}{r_s} - \frac{v_1}{r_s} - \frac{v_1 + v_2}{L} (t - T_s) + \frac{\pi}{\omega L} [v_1 + (2d-1)v_2] & [T_s \quad (1+d)T_s] \\ \frac{v_s}{r_s} - \frac{v_1}{r_s} + \frac{-v_1 + v_2}{L} [t - (1+d)T_s] - \frac{\pi}{\omega L} [(2d-1)v_1 + v_2] & [(1+d)T_s \quad 2T_s] \end{cases} \quad (9)$$

Averaging the slow variable v_1 in one whole period $[0 \quad 2T_s]$, one can get the averaging value of variable v_1 as equation (10).

$$\frac{d\langle v_1 \rangle}{dt} = \frac{1}{C_1 T_s} \int_0^{2T_s} \langle v_1 \rangle dt = -\frac{2}{r_s C_1} \langle v_1 \rangle + \frac{2\pi}{\omega L C_1} [2d^2 - 2d] \langle v_2 \rangle + \frac{2}{r_s C_1} \langle v_s \rangle \quad (10)$$

where, $\langle v_1 \rangle = \frac{1}{2T_s} \int_0^{2T_s} v_1 dt$ denotes the average value in one period $[0 \quad 2T_s]$.

Similar to the state variable v_1 , the slow state variable v_2 are as equation (11).

$$C_2 \frac{dv_2}{dt} = \begin{cases} -\frac{v_1 + v_2}{L} t + \frac{\pi}{\omega L} [v_1 + (2D-1)v_2] - \frac{v_2}{R} & [0 \quad DT_s] \\ \frac{v_1 - v_2}{L} (t - DT_s) + \frac{\pi}{\omega L} [(2D-1)v_1 + v_2] - \frac{v_2}{R} & [DT_s \quad T_s] \\ \frac{-v_1 - v_2}{L} (t - T_s) + \frac{\pi}{\omega L} [v_1 + (2D-1)v_2] - \frac{v_2}{R} & [T_s \quad (1+D)T_s] \\ -\frac{-v_1 + v_2}{L} [t - (1+D)T_s] + \frac{\pi}{\omega L} [(2D-1)v_1 + v_2] - \frac{v_2}{R} & [(1+D)T_s \quad 2T_s] \end{cases} \quad (11)$$

Average the slow variable v_2 in $[0 \quad 2T_s]$. One can get the averaged equation (12).

$$\frac{d\langle v_2 \rangle}{dt} = \frac{1}{C_2 T_s} \int_0^{2T_s} \langle v_2 \rangle dt = \frac{2\pi}{\omega L C_2} (-2d^2 + 2d) \langle v_1 \rangle - \frac{2}{RC_2} \langle v_2 \rangle \quad (12)$$

Rewrite the averaged slow variables v_1 and v_2 as state space form as (13).

$$\begin{bmatrix} \frac{d\langle v_1 \rangle}{dt} \\ \frac{d\langle v_2 \rangle}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2}{r_s C_1} & \frac{2\pi(2d^2 - 2d)}{\omega L C_1} \\ \frac{2\pi(-2d^2 + 2d)}{\omega L C_2} & -\frac{2}{RC_2} \end{bmatrix} \begin{bmatrix} \langle v_1 \rangle \\ \langle v_2 \rangle \end{bmatrix} + \begin{bmatrix} \frac{2}{r_s C_1} \\ 0 \end{bmatrix} \langle v_s \rangle \quad (13)$$

The averaged model keep the input components, control signal, load resistance, slow state variables. It describes the averaged behavior of the original system and can be compared with the device level circuit to verify the conclusions. The device level circuit and the model are setup under Ansoft/Simplorer environment. The simulation parameters are as following $r_s = 0.1\Omega$, $C_1 = 30\mu F$, $C_2 = 100\mu F$, $R_1 = 2\Omega$, $L = 40\mu H$, $V_s = 100V$, $t_{delay} = 5\mu s$, $d = 0.2$, $f = 20kHz$.

The results are shown in Figure 4 and Figure 5. In the time domain, the trajectories of the averaged mode approximate the trajectories of the original system. The results of averaged model will be a useful approximation of the original

system although it is still nonlinear system because it includes the multiplication terms the complete method is to derive the linear small signal model.

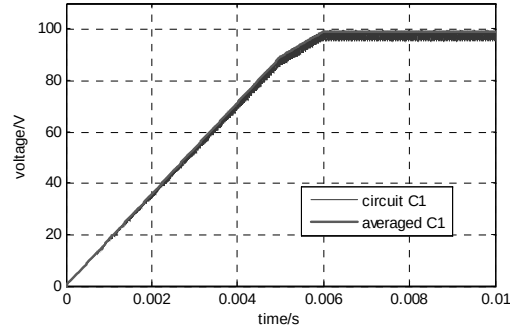


Figure 4. The comparison averaged and the device level variables in C1

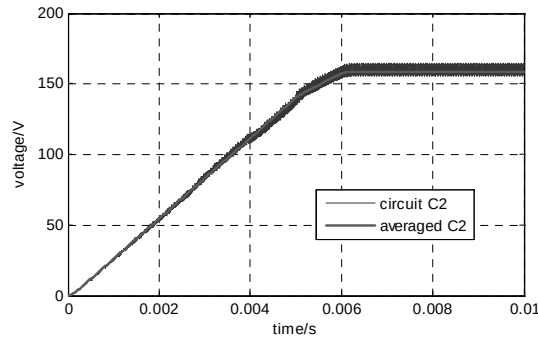


Figure 5. The comparison averaged and the level variables in C2

4. The AC small signal model

The averaged state variables consist of two parts, the DC component and the AC small component. These variables of the state variable and the duty cycle $d(t)$ can be separated as equation (14).

$$\begin{cases} \langle i_L(t) \rangle = I + \hat{i}(t) \\ \langle v_1(t) \rangle = V_1 + \hat{v}_1(t) \\ \langle v_2(t) \rangle = V_2 + \hat{v}_2(t) \\ \langle v_s(t) \rangle = V_s + \hat{v}_s(t) \\ d(t) = D + \hat{d}(t) \end{cases} \quad (14)$$

Substitute the equation (14) into the averaged equation (13), one can get the equation (15).

$$\begin{aligned} \frac{d(V_1 + \hat{v}_1)}{dt} &= -\frac{2}{r_s C_1} (V_1 + \hat{v}_1) + \frac{2\pi}{\omega LC_1} \left[2(D + \hat{d})^2 - 2(D + \hat{d}) \right] (V_2 + \hat{v}_2) + \frac{2}{r_s C_1} (V_s + \hat{v}_s) \\ \frac{d(V_2 + \hat{v}_2)}{dt} &= \frac{2\pi}{\omega LC_2} \left[-2(D + \hat{d})^2 + 2(D + \hat{d}) \right] (V_1 + \hat{v}_1) - \frac{2}{RC_2} (V_2 + \hat{v}_2) \end{aligned} \quad (15)$$

Equation (15) can be separated as three parts, that is the DC components, the first order components and the high order component. The DC term are as equation (16).

$$\begin{cases} -\frac{2}{r_s C_1} V_1 + \frac{2\pi}{\omega LC_1} (2D^2 - 2D) V_2 + \frac{2}{r_s C_1} V_s = 0 \\ \frac{2\pi}{\omega LC_2} (-2D^2 V_1 + 2D V_1) - \frac{2}{RC_2} V_2 = 0 \end{cases} \quad (16)$$

The first order term is as equation (17). The voltage $v_2(t)$ was selected as the output variable, so the output equation is as equation (18). The equation (17) and (18) is the linear equation of original system (13).

$$\begin{bmatrix} \frac{d\hat{v}_1}{dt} \\ \frac{d\hat{v}_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2}{r_s C_1} & \frac{2\pi}{\omega LC_1} (2D^2 - 2D) \\ \frac{2\pi}{\omega LC_2} (-2D^2 + 2D) & -\frac{2}{RC_2} \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} + \begin{bmatrix} \frac{2}{r_s C_1} & \frac{2\pi}{\omega LC_1} (4DV_2 - 2V_2) \\ 0 & \frac{2\pi}{\omega LC_2} (-4DV_1 + 2V_1) \end{bmatrix} \begin{bmatrix} \hat{v}_s \\ \hat{d} \end{bmatrix} \quad (17)$$

$$\hat{v}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} \quad (18)$$

From the state space form, the transfer functions are as equation (19).

$$\hat{v}_2(s) = C(sI - A)^{-1} B \hat{u}(s) = G_1(s) \hat{v}_s(s) + G_2(s) \hat{d}(s) \quad (19)$$

The transfer functions are as equation (20).

$$G_1(s) = \left. \frac{\hat{v}_2(s)}{\hat{v}_s(s)} \right|_{\hat{d}(s)=0} \quad \text{and} \quad G_2(s) = \left. \frac{\hat{v}_2(s)}{\hat{d}(s)} \right|_{\hat{v}_s(s)=0} \quad (20)$$

The parameters in state equation (17) and (18) are as following $r_s = 0.1\Omega$, $C_1 = 30\mu F$, $C_2 = 100\mu F$, $R_1 = 2\Omega$, $L = 40\mu H$, $V_s = 100V$, $d = 0.2$.

Then the transfer function is simplified as equation (21) and (22).

$$G_1(s) = \left. \frac{\hat{v}_2(s)}{\hat{v}_s(s)} \right|_{\hat{d}(s)=0} = \frac{26.68 \times 10^8}{s^2 + 6.77 \times 10^5 s + 6.67 \times 10^9 + 5.32 \times 10^7} \quad (21)$$

$$G_2(s) = \left. \frac{\hat{v}_2(s)}{\hat{d}(s)} \right|_{\hat{v}_s(s)=0} = \frac{1.5 \times 10^6 s + 10.005 \times 10^{11} - 33.32 \times 10^9}{s^2 + 6.77 \times 10^5 s + 6.67 \times 10^9 + 5.32 \times 10^7} \quad (22)$$

The bode diagrams of the $G_1(s)$ and $G_2(s)$ are as Figure 6 and Figure 7.

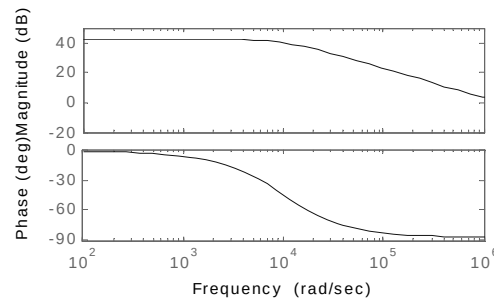
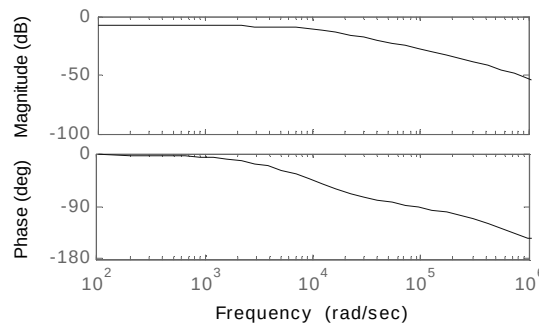


Figure 6. The bode diagram of $G_1(s)$



A prototype of 10Kw bi-directional dc-dc converter was setup in lab. Figure 8 is the voltage wave in the first and second side of the isolated transformer. Figure 9 is the voltage and current wave in the leakage inductor of transformer. The control strategy is phase shift control so between the first and the second side of the transformer there exist phase difference.

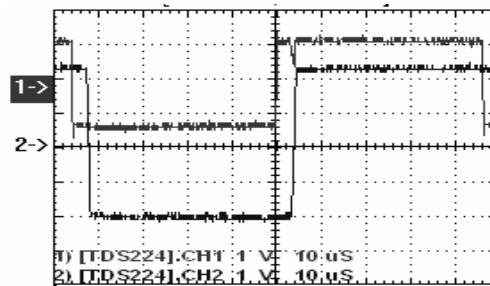


Figure 8. The first and second H-bridge waves

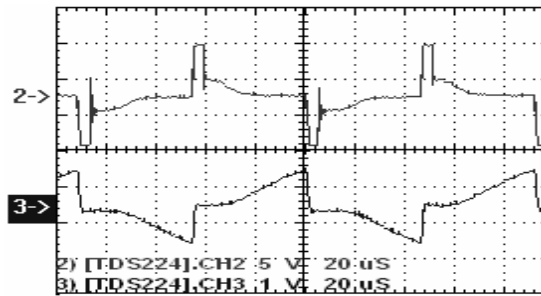


Figure 9. The voltage and current wave in leakage inductor

The reduced-order model of (17) and (18) can be used to predict the dynamic behaviors of bi-directional converter. The conventional analysis and design theory can be used to predict the characteristics of transformer-isolated bi-directional converter.

5. Conclusions

The transformer-isolated bi-directional DC-DC converter has four operation modes in a whole period because of the switching effect. Basing on the four differential equations the completely averaged model of the transformer isolated DC-DC converter was derived. The experiment results reveal that the device-level circuit and mathematical model are consistent very well. According the averaged model, the liberalized dynamic small signal components and the DC components of the averaged model are separated respectively. The static operation point can be decided by the DC equation. The linear small signal mode can be use to analyze and design control system using the control theories.

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