



## Performance Analysis on Equivalent Elasticity of 3D 4-directional Braided Composites

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### Abstract

Based on the small parameter asymptotic homogenization theory, this article adopts the technology of digitized cell-based finite element method (DCB-FEM) to establish the digital cell-based mechanics analysis model of three-dimensional 4-directional braided composite material, analyzes and calculates the equivalent elasticity performance of 3D braided composite material using homogenization method, and the results show a good agreement with the experimental data.

**Keywords:** Three-dimensional braided composite material, Homogenization, Microstructure, Digital model, Finite element method

Three-dimensional braided composite material are the engineering structure composite material which are developing very fast for recent years. These materials have many advantages such as good integrated performance, reasonable mechanical structure and good abilities to resist damage limits and crackle extension, which offer wider prospects of applications on main load-bearing structure for 3D braided composite material (Yang, 1992, p.87-91). The mechanical performances of 3D braided composite material depend on not only performances of various part materials, but also their microstructures. The yarn spaces in 3D braided structure present multi-directional distribution and form seasonal yarn structure by completing each other, and just because of the complexity of this microstructure, it becomes very difficult to make the mechanical performance analysis for 3D braided composite material. Based on the finite element method of the digital cell-based model, this article establishes the numerical computation method of homogenization for 3D 4-directional braided composite material, analyzes and calculates the equivalent elasticity constants of these materials, and makes the comparative research with the test results.

### 1. Cell-based model of 3D 4-directional braided composite material

3D 4-directional braided composite material are superposed by single seasonal microstructure unit cells (Chen, 1999, p.391-404). The interior unit cell model is seen in Figure 1. The interior unit cell model of 3D 4-directional braided composite material presents cube shape. If taking braid direction as axis x, so width direction of unit cell is axis y and thickness direction is axis z. The included angle between braid yarn and braid axis direction x is the interior braid angle  $\gamma$ , and the included angle between projection of braid yarn on workpiece cross section (y-z section) and thickness direction (axis z) is  $\theta$  which is the interior tropism angle of interior yarn.

Supposed that the cross section of braid yarn is ellipse which long axis and short axis respectively are  $2a$  and  $2b$ , the section shape along axis direction of braid yarn is not changeable, the braiding process is stable and even to ensure the equality and consistency of braid structure, all braid yarns have same geometry characters, braid yarns have enough tensions to make braid yarns produce bends only on the surface of the preforms, we can describe the relations between geometry structure of unit cell and geometry shape of braid yarn as follows.

$$\text{Width of interior unit cell: } W_i = \frac{4b}{\cos \varphi} \quad (1)$$

$$\text{Thickness of interior unit cell: } T_i = \frac{4b}{\sin \theta} \quad (2)$$

Relation between interior braid angle  $\gamma$  and braid angle  $\alpha$ :  $tg\gamma = \frac{1}{\sin\theta} tg\alpha$  (3)

Braided pitch length of interior unit cell h:  $h = \frac{8b}{tg\gamma \sin 2\theta}$  (4)

1.1 Fiber volume content

The fiber volume content is one of main performance indexes for 3D braid composite material, which is higher and the performances of composite material are better. The fiber volume content lies on two aspects, one aspect is the interspace among interior braid yarns of 3D braid composite material, the other one is the interspace among interior fibers of braid yarns, that is the yarn filling factor. The fiber volume content  $V_f$  can be calculated by the following formula.

$V_f = \text{volume of fibers} / \text{volume of unit cell} = \frac{8A \sec \gamma}{h^2 tg^2 \gamma \sin 2\theta} \epsilon$  (5)

Where, A is the cross section acreage of yarn, h is the braided pitch length,  $\theta = 45^\circ$  and  $\epsilon$  is the yarn filling factor.

1.2 Tightening status

In the braid shaping process, because of the function of yarn tension and “tightening” working procedure after every machine cycle, conterminous braided yarns in preform contact each other and are in tightening estate. The completing estate of interior yarns of 3D 4-directional braid composite material is seen in Figure 2. Define the shape change factor of yarns cross section  $k=a/b$ , according to the tightening estate of yarns, the relation between shape change factor of yarns cross section and various tropism angle of yarns can be described as follows.

When  $\theta \geq 45^\circ$ ,  $k = \cos \gamma \sqrt{(3 \csc 2\theta + ctg 2\theta)(\csc 2\theta - ctg 2\theta)}$  (6)

When  $\theta < 45^\circ$ ,  $k = \cos \gamma \sqrt{(3 \csc 2\theta - ctg 2\theta)(\csc 2\theta + ctg 2\theta)}$  (7)

1.3 Yarn surface equation

Supposed the position of one braided yarn in 3D 4-directional composite material in the whole coordinate system (x, y, z) is seen in Figure 3, define the local coordinate system of yarns as ( $x''$ ,  $y''$ ,  $z''$ ), axis  $y''$  and axis  $z''$  as the cross section of braided yarn.

Through the transform from the local coordinates to the whole coordinates, considering the origin of local coordinates is located at the point ( $x_0$ ,  $y_0$ ,  $z_0$ ) in the whole coordinate system, the yarn surface equation can be obtained as follows.

$$\frac{((y - y_0) \sin \theta + (z - z_0) \cos \theta) \cos \gamma - (x - x_0) \sin \gamma)^2}{a^2} + \frac{((z - z_0) \sin \theta - (y - y_0) \cos \theta)^2}{b^2} = 1$$
 (8)

The interior of 3D 4-directional braid composite material has 4 directional braid yarns, and the tropism angles respectively are ( $\gamma, \theta$ ), ( $-\gamma, \theta$ ), ( $\gamma, -\theta$ ), and ( $-\gamma, -\theta$ ). Taking these angles in the equation (8), four surface equations of tropism braid yarn can be obtained. Levelly move various yarns to the corresponding positions of unit cell, we can obtain the interior unit cell entity model of 3D 4-directional braid composite material, which is seen in Figure 4.

2. Cell-based digitalization of 3D braided composite material

Based on the interior unit cell entity model of 3D 4-directional braid composite material, this article adopts 3D rasterizing technology to digitalize the unit cell entity model (Wang, 2001), that is to say, when the resolution ratio is  $30 \times 30 \times 30$ , the unit cell of 3D 4-directional braid composite material is dispersed as the form of space element. Four kinds of glue braid yarns and one kind of substrate material are represented as 0 to 4, where, 0 represents the substrate material and 1 to 4 respectively represent four kinds of tropism glue braid yarn. In the scanning process, first the attribute values of all space elements are endowed as 0, that is 0 represents the space element material, then according to the relations between the position of space element point and the braid yarn surface equation, the attribute value of every space element is quantified, and if the space element point is located in the interior of one yarn surface equation, so this space element point is activated and endowed the corresponding material attribute. When the digitalization of unit cell of 3D 4-directional braid composite material is completed, the corresponding data collection of space element is formed, and the distribution of component material in unit cell is detailedly recorded, and the digital unit cell model obtained is seen in Figure 5.

3. Numerical computation of equivalent performance

Based on the finite element method of the digital cell-based model (Guedes, 1990, p.143-198 & Chen, 1999, p.2383-2391 & Peng, 2002, P.45-56 & Wang, 2003 & Ma, 2006 & Chen, 2007, p.1-5), this article analyzes the

equivalent elasticity modulus, and compares it with the test results. And the workpiece is made by the 3D 4-directional braid technology and pitch transfer mould technics (RTM), where, the strengthened fiber is fiberglass of 1440Tex, the substrate material adopts the epoxy resin of TDE-86#, the braid technical parameters are seen in Table 1, and the component material performances are presented in Table 2.

### 3.1 Glue braid yarn

When braid yarns in the 3D composite material infiltrate the substrate material, it can be considered as the unilateralism fiber strengthened composite material, which fiber volume content is the filling factor of yarn and is called glue braid yarn. The stress-change relation of glue braid yarn in the material principle axis direction can be represented as the following equation.

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & \text{symmetry} & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix} \quad (9)$$

Where,  $\sigma_1 = \sigma_{xx}, \sigma_2 = \sigma_{yy}, \sigma_3 = \sigma_{zz}, \sigma_4 = \gamma_{yz}, \sigma_5 = \gamma_{xz}, \sigma_6 = \gamma_{xy}$

$\varepsilon_1 = \varepsilon_{xx}, \varepsilon_2 = \varepsilon_{yy}, \varepsilon_3 = \varepsilon_{zz}, \varepsilon_4 = \tau_{yz}, \varepsilon_5 = \tau_{xz}, \varepsilon_6 = \tau_{xy}$

, and [C] = rigidity matrix of unilateralism fiber strengthened material.

For the glue braid yarn with four kinds of tropism in 3D 4-directional composite material, the material principle axis direction is not consistent with the whole coordinate of the composite material, which needs be uniformed into the whole coordinate system. Rotating axis formula of material rigidity is

$$[C(\gamma, \theta)] = [T(\gamma, \theta)][C][T(\gamma, \theta)]^T \quad (10)$$

Where, [C(γ, θ)] is the rigidity matrix in the material principle axis, [T(γ, θ)] is the stress conversation matrix, and the superscript T represents the transpose of the matrix.

And [T(γ, θ)] has the following form.

$$[T(\gamma, \theta)] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2m_1n_1 & 2n_1l_1 & 2l_1m_1 \\ l_2^2 & m_2^2 & n_2^2 & 2m_2n_2 & 2n_2l_2 & 2l_2m_2 \\ l_3^2 & m_3^2 & n_3^2 & 2m_3n_3 & 2n_3l_3 & 2l_3m_3 \\ l_2l_3 & m_2m_3 & n_2n_3 & m_2n_3 + m_3n_2 & n_2l_3 + n_3l_2 & l_2m_3 + l_3m_2 \\ l_3l_1 & m_3m_1 & n_3n_1 & m_3n_1 + m_1n_3 & n_3l_1 + n_1l_3 & l_3m_1 + l_1m_3 \\ l_1l_2 & m_1m_2 & n_1n_2 & m_1n_2 + m_2n_1 & n_1l_2 + n_2l_1 & l_1m_2 + l_2m_1 \end{bmatrix} \quad (11)$$

Where,  $l_1 = \cos \gamma, l_2 = \sin \gamma \sin \theta, l_3 = \sin \gamma \cos \theta$

$m_1 = -\sin \gamma, m_2 = \cos \gamma \sin \theta, m_3 = \cos \gamma \cos \theta$

, and  $n_1 = 0, n_2 = -\cos \theta, n_3 = \sin \theta$ .

### 3.2 Numerical computation

This article adopts the development software of Visual C++ to implement the numerical computation. First taking the fiber filling factor obtained by computation as the fiber volume content, the homogenization equivalent elasticity modulus is calculated, then input the above results as the material performances, calculate the homogenization equivalent modulus of 3D 4-directional braid composite material. The input parameters include elasticity constants of substrate material and glue braid yarn, diameter and braiding angle of braid yarn, fiber volume content and cavalcade line yarn coefficients. The comparative results between the theory values and test values of portrait elasticity modulus E11 are seen in Figure 6, and the computation results show a good agreement with the test data, which indicates that this method is feasible.

## 4. Influences of braid technical parameters

### 4.1 Braiding angle

The factors which influence the mechanical performances of braid structure composite materials mainly include the interior braiding angle γ and fiber volume content V<sub>f</sub>. Keeping the performances of component material and supposed

Figure 7, we can see that with the increase of the interior braiding angle  $\gamma$ , the extension modulus E11 sharply decrease, E22 and E33 gradually increase, and the transverse extension modulus E22 and E33 are much smaller than the portrait extension modulus E11 and the changing current is reverse with the portrait, and with the increase of the interior braiding angle  $\gamma$ , the clipping modulus G32 gradually increase, the increasing current of G12 and G13 is from fast to slow, and when  $\gamma = 40^\circ$ , the portrait clipping modulus G12 and G13 achieve maximal, and the Poisson's ratio  $\nu_{12}$  and  $\nu_{13}$  is from increase to decrease with the increase of the interior braiding angle, and when  $\gamma < 25^\circ$ ,  $\nu_{12}$  and  $\nu_{13}$  increase with the increase of  $\gamma$ , then decrease with the increase of  $\gamma$ , and the Poisson's ratio  $\nu_{32}$  is from decrease to increase with the increase of  $\gamma$ , and when  $\gamma < 30^\circ$ ,  $\nu_{23}$  decreases with the increase of  $\gamma$  and achieves minimal when  $\gamma = 30^\circ$ , and when  $\gamma > 30^\circ$ ,  $\nu_{23}$  increases with the increase of  $\gamma$ . In a word, the interior braiding angle  $\gamma$  observably influences the extension modulus E11.

#### 4.2 Fiber volume content

Keeping the performances of component material and supposed the interior braiding angle  $\gamma$  is 20%, if the fiber volume content  $V_f$  changes, we will obtain Figure 8 which presents the values of effective elasticity performance forecast composite material change with the fiber volume content  $V_f$ .

Form Figure 8, we can see that all extension modulus and clipping modulus increase with the increase of the fiber volume content  $V_f$ , but the change of Poisson's ratio is relative complicated, with the increase of the fiber volume content  $V_f$ , Poisson's ratio  $\nu_{23}$  gradually decrease, but  $\nu_{12}$  and  $\nu_{13}$  is from the increase to decrease, when the fiber volume content  $V_f$  achieves about 45%, they get the maximal values, and when  $V_f < 45\%$ ,  $\nu_{12}$  and  $\nu_{13}$  increase with the increase of  $V_f$ , and when  $V_f > 45\%$ ,  $\nu_{12}$  and  $\nu_{13}$  decrease with the increase of  $V_f$ . Anyway, the Poisson's ratio  $\nu_{12}$  and  $\nu_{13}$  are mainly controlled by the interior braiding angle  $\gamma$  and the fiber volume content  $V_f$  has few influences to them.

### 5. Conclusions

Based on the small parameter asymptotic homogenization theory, this article adopts the technology of 3D graph digital processing to establish the digital cell-based mechanics analysis model of 3D 4-directional braided composite material, analyzes and calculates the equivalent elasticity performance of 3D braided composite material using homogenization method, and the results show a good agreement with the experimental data. Therefore the forecast method of the elasticity performance of 3D 4-directional braided composite material is established, and this article offers an effective analysis method for the technical parameter choosing, performance design and structure optimizing of this type of material in the developing process, promotes the further application of 3D composite material in the area of aviation and spaceflight, and establishes the bases for further analyzing the damage, intension and non-linearity activity of this type of material.

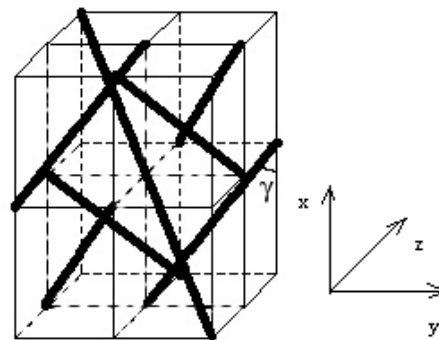


Figure 1. Schematic Illustration of the Spatial Traces of Interior Yarns

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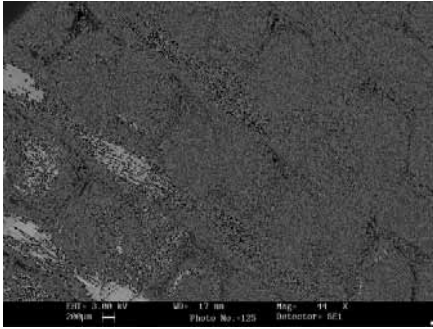


Figure 2. The Cross Section Photograph of Preform

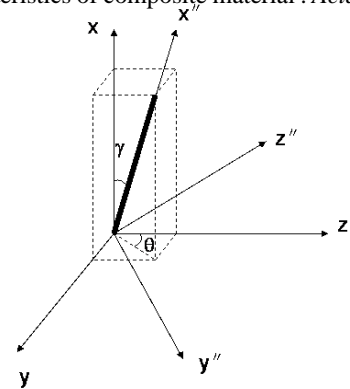


Figure 3. The Directions of Braided Yarn

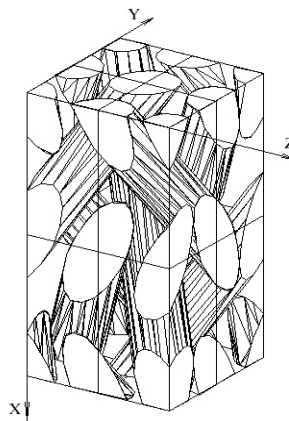


Figure 4. Interior Unit Cell Model of 3D 4-directional Braided Composites

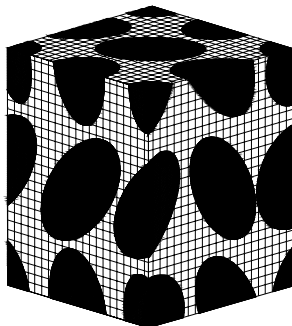


Figure 5. The Digitized Cell Model of 3D Four-directional Composites

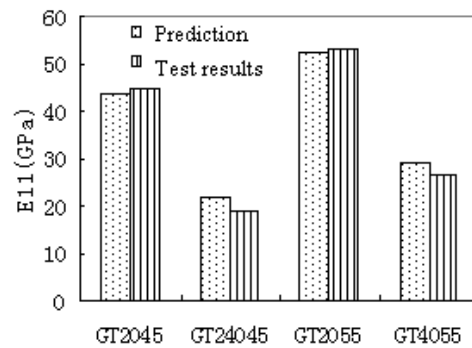


Figure 6. Prediction and Test Results of E11

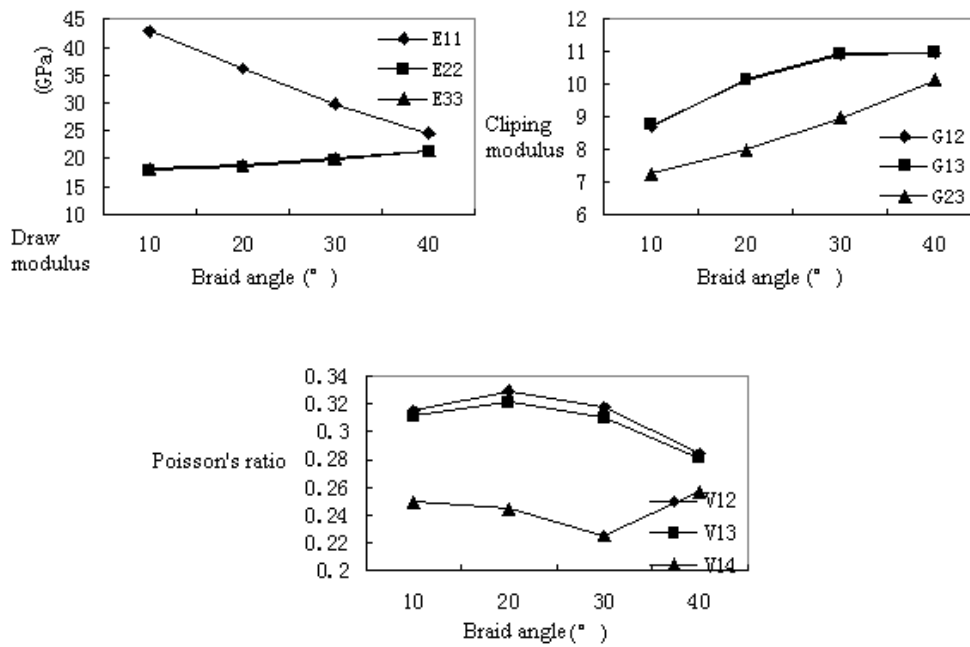


Figure 7. Elastic Constants Variation with Braiding Angle

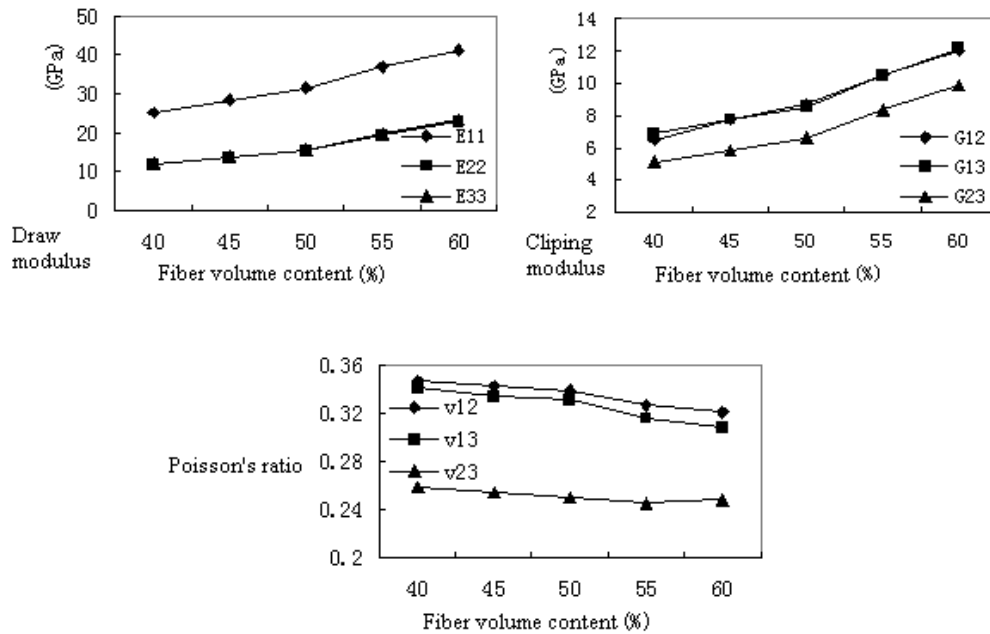


Figure 8. Elastic Constants Variation with Fiber Volume Content

Table 1. Technical parameters of the specimens

No.	Materials	Fiber	Size (mm×mm×mm)	Main body yarn	Braid angle (Deg.)	Fiber volume content (%)
GT2045	Glass	1440Tex	5×25×250	4×23	9.18	52.84
GT4045	Glass	1440Tex	5×25×250	3×22	33.21	48.77
GT2055	Glass	1440Tex	5×25×250	5×25	9.57	63.44
GT4055	Glass	1440Tex	5×25×250	4×23	31.19	60.61

Table 2. Mechanical property parameters of components properties

	Extension modulus (GPa)	Poisson's ratio	Density (g/cm <sup>3</sup> )
Glass fiber	82.9	0.30	2.54
Epoxy resin	3.5	0.35	1.17