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Relationship Between Maximum Basal Aarea

Carrying Capacity and Maximum Size-density Rrelationships

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Abstract

Two important concepts in forest stand density management — the maximum basal area carrying capacity and the maximum size-density relationship (MSDR) for individual stands — are mathematically related. For stands with a MSDR slope less than 2, maximum basal area carrying capacity will occur after a stand has reached its MSDR stage of stand development, or after Reineke's SDI has been maximized. Maximum basal area carrying capacity and the MSDR stage of stand development will occur simultaneously in a stand that has a MSDR slope equal to 2. If a MSDR's slope is greater than 2, a stand will reach its maximum basal area carrying capacity prior to reaching its MSDR stage of stand development.

Keywords: Reineke, SDI, Stand density, Stand density index

1. Introduction

Maximum basal area carrying capacity and Maximum Size-Density Relationships (MSDRs) are commonly applied concepts in the management of forest stand density. This note elucidates the relationship between individual stand maximum basal area carrying capacity and individual stand MSDRs — what Weller (1990) called a dynamic thinning line. For this paper, our definition of maximum basal area carrying capacity is a modification of The Dictionary of Forestry's (Helms 1998) definition for carrying capacity:

"The maximum amount of basal area of a given species that can be sustained on a long-term basis within a stand"

Herein the tree per hectare — average stem diameter MSDR, commonly referred to as Stand Density Index (Reineke 1933), is examined rather than the tree per hectare — average tree volume MSDR, which is commonly referred to as the Self-Thinning rule. Stand Density Index (SDI) is expressed as:

$$SDI = TPH*[QMD/25.4]^{b} \tag{1.1}$$

Where:

TPH – trees per hectare,

QMD – quadratic mean diameter (cm), and

b – the MSDR dynamic thinning line boundary slope

Reineke (1933) originally determined *b* to be 1.6. However, studies have showed substantial variation in the MSDR dynamic thinning line *b* (Tang et al. 1994, del Rio et al. 2001) and reported that *b* could exceed 2.

Some growth and yield models have constrained stand development using basal area (Wykoff et al. 1982, Somers and Farrar 1991, Zhang et al. 1993) while others have constrained stand development using MSDRs (Maguire et al. 1990, Cao et al. 2000, Donnelly et al. 2001, Mack and Burk 2002). An interesting question is whether a stand that has reached its maximum basal area carrying capacity will also simultaneously have reached it's MSDR dynamic thinning line. If the two points occur simultaneously then limiting stand development using either of the two measures will produce the same constraints on yields (assuming both measures are correctly quantified). However, if the two concepts don't occur simultaneously, then using one measure to constrain stand development could produce different estimates of stand development across a rotation than if the other measure was used to constrain stand development.

Others have briefly commented on the relationship between the two measures but, to date, a clear and reasonably complete discussion of the relationship does not appear to have been documented. The observation of Strub and Bredenkamp (1985) that "… asymptotic basal area implies that the SDI must eventually decrease and asymptotic SDI implies that the basal area must always increase…" applies only when the *b* is less than 2.

In their model describing MSDR dynamic thinning lines, Lloyd and Harms (1986) fixed the slope at 0.5. By rotating the axes from lnQMD over lnTPH to lnTPH over lnQMD, the *b* becomes 2 (same as basal area) where ln is the natural logarithm ($\log_{10}$ can also be used). The authors stated:

"Further, 0.5 is the value [the MSDR dynamic thinning line *b*] must take for basal area to remain constant, which is the expected response of stands after they have reached the carrying capacity of the site."

In order for the maximum basal area carrying capacity to be maintained across time, for a given change in tree density, QMD must change consistent with a *b* value of 2. Stated differently, the change in TPA given a change in QMD on the ln-ln scale must be -2.

Usually, basal area per hectare is a measure of the total stem cross-sectional area at breast height for a stand of trees. Basal area per hectare is a composite measure of two components; individual tree basal areas and numbers of trees per hectare. MSDRs express the relationship of the maximum number of trees per hectare that can be obtained for a given QMD (conversely, MSDRs can also be the maximum QMD that can be obtained for a given number of trees per hectare). MSDR dynamic thinning lines quantify how maximum numbers of trees per hectare for a given stand change with a change in QMD. Maximum basal area carrying capacity provides no information about limiting tree size-tree density relationships — only the maximum of the combination of the tree of average basal area and tree density. So, what are the mathematical and temporal relationships between these two measures of limiting stand density?

The answer depends on the MSDR dynamic thinning line *b*. Since it can be assumed that MSDR dynamic thinning lines are linear on the ln-ln scale, the value of SDI is constant when a stand is at its MSDR dynamic thinning line stage of stand development. Assuming a *b* of 1.6 and that the stand is at its MSDR dynamic thinning line stage of stand development, for a given change in tree density the change in QMD is consistent with a *b* of 1.6. Since basal area is composed of QMD raised to the second power:

$$BA = TPH*[0.00007854*QMD^2] \tag{1.2}$$

Where:

BA – square meters of basal area per hectare

basal area would continue to increase in this stand at the MSDR dynamic thinning line stage of stand development (Figure 1). Alternatively, by taking logarithms of eqn. [1], eqn. [3] is obtained:

$$\ln SDI = \ln TPH + 1.6*\ln QMD - 1.6*\ln[25.4] \tag{1.3}$$

And taking the logarithm of both sides of eqn. [2] results in:

$$\ln BA = \ln TPH + \ln[0.00007854] + 2*\ln QMD \tag{1.4}$$

For a MSDR dynamic thinning line *b* of 1.6, in order to keep lnSDI constant for any change in lnTPH, lnQMD must change consistent with a slope of 1.6. Thus, during the MSDR stage of stand development, or when lnSDI is constant, lnBA cannot remain constant since the slope on lnQMD is equal to 2 in eqn. [4].

If a stand's MSDR dynamic thinning line *b* is equal to 2, then the period of maximum basal area carrying capacity and the period when the stand is at its MSDR dynamic thinning line stage of stand development would coincide. Some MSDR models have fixed the *b* at 2 (Lloyd and Harms 1986, Voit 1988, Cao 1994). If a stand has a MSDR dynamic thinning line boundary *b* greater than 2, then basal area will decline when the stand is at its MSDR dynamic thinning line stage of stand development. Thus, for a stand with a MSDR dynamic thinning line *b* greater than 2, the period of maximum basal area carrying capacity would occur prior to the MSDR dynamic thinning line stage of stand development. Analogous to this, if a *b* value was 2, for the ages at which a particular stand would be at its MSDR dynamic thinning line stage of stand development SDI would be flat when plotted over age. If the *b* was not 2, then basal area per hectare would either increase or decrease depending on the value of *b*.

2. Conclusion

Maximum basal area carrying capacity and MSDR dynamic thinning lines are commonly used stand density measures and are also used to verify and regulate growth and yield models. In this note, the relationship between the two concepts for an individual stand was examined. For those stands that have a MSDR dynamic thinning line *b* of 2, the period at which the two measures occur will be the same. Thus, using either of the two concepts to constrain stand development would produce the same limits. If an individual stand's *b* is less than 2, a stand will reach its MSDR dynamic thinning line prior to reaching its maximum basal area carrying capacity. Conversely, if a stands MSDR dynamic thinning line *b* is greater than 2, then a stand will reach its maximum basal area carrying capacity prior to reaching its MSDR dynamic thinning line stage of stand development. For either of the two latter cases, when exclusively using one of the two measures as a limit, the two density measures will produce different constraints on stand development and estimated yields should be different across a rotation.

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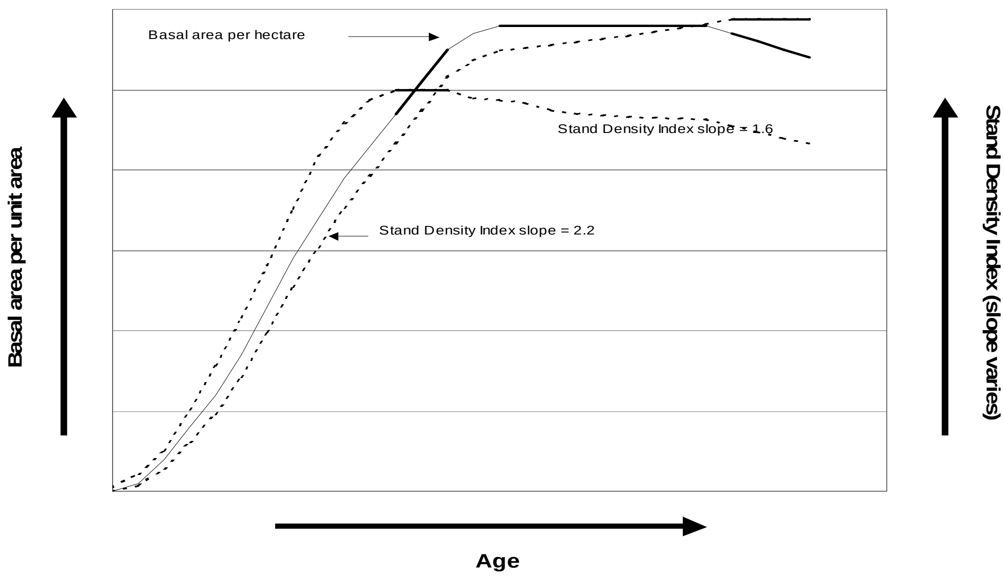

Figure 1. The Trend of Stylized curves of basal area per unit area (BA — solid line) and Stand Density Index (where *b* = 1.6, *b* = 2, and *b* = 2.2 — dashed lines) over age. Stand Density Index (SDI) with a *b* = 2 basically mimics the growth pattern of BA and for clarity was not included in the figure. For the BA growth trajectory, the first black bolded component corresponds to the maximum SDI value using a *b* = 1.6, the second bolded component corresponds to the maximum SDI value using a *b* = 2, and the final bolded component corresponds to the maximum SDI value using a *b* = 2.2. For the SDI growth trajectories with *b* = 1.6 and *b* = 2.2, the bolded components correspond to the period of maximum SDI.

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