Modern Applied Science
www.ccsenet.org/journal.html
Vol. 2, No. 2
March 2008

# The $\varepsilon$-Core of a $n$-person Stochastic Cooperative Game

Zuofeng Gao
College of Science, Yanshan University, Qinhuangdao 066004, China
Hongxin Bai (Corresponding author)
College of Science, Yanshan University, Qinhuangdao 066004, China
Department of math and computer, Baoding College, Baoding 071051, China
E-mail: baihongxin1@126.com
Suting Zhang, Yongbo Yu, Chunyan Han, Hua Zhang
College of Science, Yanshan University, Qinhuangdao 066004, China

*Supported by the Foundation for the natural science of He Bei province of China (A2005000301)*

**Abstract**

In this paper, based on the core of the stochastic cooperative game (Oviedo, 2000, pp. 519-524), We define the $\varepsilon-$core of the stochastic cooperative game. Thus, we recuperate the defect in theory that the core is empty usual. And we introduce some characters and properties about the kind $\varepsilon$-core.

**Keywords:** Stochastic game, $n$-person stochastic cooperative game, Core, $\varepsilon$-core

**Introduction**

It is well known, there is difficult that the core of cooperative game, as a solution of the game because of the core of the game is empty usual. The core of the stochastic cooperative game which Suijs et al. developed in 1995. In order to solve the problem, this paper import the concept of $\varepsilon$-core and discuss the properties of the $\varepsilon$-core.

**1. Basic definitions**

The stochastic cooperative game which Suijs introduced denoted by $\Gamma=\left(N,\{X_S\}_{S\subset N},(\succ_i)_{i\in N}\right)$, $N$ is the set of players, $X_S$ is the payoff function of the coalitions $S$ ,and every stochastic payoff has finite expectation.

We denote the stochastic payoff has $X_S$ of $S$ as $(d^S,r^S)\in R^S\times R^S$ , and for all of $i$ in $S$ it satisfies $\sum_{i\in S}d_i\le 0$, $\sum_{i\in s}r_i=1, r_i\ge 0$. According to $(d^S,r^S)$ the stochastic payoff of player $i$ in $S$ is equal to $d_i+r_iX_S$ ,and denoted by $(d^S,r^S)_i$. It is noted that $(d^S,r^S)_i$ is a stochastic variable .The set of all the payoff of $S$ , we denoted by $Z(S)$, and the set all of the individually rational payoff $S$ we denote by $IR(S)$ .So

$IR(S)=\left\{(d^S,r^S)\in Z(S)\,\middle|\,\forall i\in S: d_i+r_iX_S \succ_i X_{\{i\}}\right\}$.

Let $(d^N,r^N),(\tilde{d}^N,\tilde{r}^N)\in Z(N)$ are two stochastic payoff. If exist is a coalition $S$, for all of $i\in S$ ,we have $\tilde{d}_i+\tilde{r}_iX_S \succ_i d_i+r_iX_S$ ,and $\sum_{i\in S}(d^N,r^N)_i\le X_S$ then we call $(\tilde{d}^N,\tilde{r}^N)$ dominate $(d^N,r^N)$ with respect to $S$, and denoted by $(\tilde{d}^N,\tilde{r}^N)\succ_S(d^N,r^N)$.

We define the core of stochastic cooperative game $\Gamma$ as the sets of payoffs which are undominated, and denoted by $core(\Gamma)$ .i.e.

$$core(\Gamma)=\left\{\begin{array}{l}(d^N,r^N)\,\middle|\,(d^N,r^N)\in IR(N),\text{there are no exist S}\\ \text{and } (\tilde{d}^N,\tilde{r}^N)\in Z(N)\text{ , satisfy : }(\tilde{d}^N,\tilde{r}^N)\succ_S(d^N,r^N)\end{array}\right\}$$

**2. The $\varepsilon$ core of a repeater $n$ -person stochastic cooperative game**

Definition 1 Let $\Gamma=\left(N,\{X_S\}_{S\subset N},(\succ_i)_{i\in N}\right)$ be a $n$ -person stochastic cooperative game, $\varepsilon\in R$ ,we denote the $\varepsilon$-core of $\Gamma$ by

$$C_\varepsilon(\Gamma)=\left\{(d^N,r^N)\in IR(N)\left|\begin{array}{l}\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon,\\ \forall S\subseteq N,\forall i\in N,\forall(\tilde{d},\tilde{r})\in Z(S),\sum_{i\in S}(d,r)_i=X_N\end{array}\right.\right\}$$

When $\varepsilon=0$, $C_\varepsilon(\Gamma)=C(\Gamma)$, $C(\Gamma)$ is the 0-core of $\Gamma$. When $\varepsilon$ is sufficient min, $C_\varepsilon(\Gamma)=\phi$;

When $\varepsilon$ sufficient enough, $C_\varepsilon(\Gamma)\neq\phi$. Let $\varepsilon_0=\inf\{\varepsilon\,|\,C_\varepsilon(\Gamma)\neq\phi\}$, then we call $C_\varepsilon(\Gamma)$ is the least core of $\Gamma$ and denoted by $LC(\Gamma)$.

It is noted that when the $\varepsilon$ -core is not empty, the elements of the $\varepsilon$ -core are not always stochastic payoffs of the stochastic cooperative game.

**3. Result**

Theorem 1. Let $(\Gamma^t)_{t=0}^m=\left(H,(\{X_S\}_{S\subset N}^t)_{t=0}^m\right)$ be a repeater $n$ -person stochastic

cooperative game,if $\varepsilon<\varepsilon'$,then $C_\varepsilon\left((\Gamma^t)_{t=0}^m\right)\subset C_{\varepsilon'}\left((\Gamma^t)_{t=0}^m\right)$.

Proof. Let $(d^N,r^N)\in C_\varepsilon(\Gamma)$,be a stochastic payoff, for $\forall S\subseteq N$, $\forall(\tilde{d},\tilde{r})\in Z(S)$, $\forall i\in N$,

$$\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon$$

and $\varepsilon<\varepsilon'$, so

$$\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon$$

Then the stochastic payoff $(d^N,r^N)\in C_{\varepsilon'}(\Gamma)$,i.e. $C_\varepsilon\left((\Gamma^t)_{t=0}^m\right)\subset C_{\varepsilon'}\left((\Gamma^t)_{t=0}^m\right)$.

Theorem 2. Let $\Gamma=\left(N,\{X_S\}_{S\subset N},(f_i)_{i\in N}\right)$ and $\Gamma'=\left(N,\{X'_S\}_{S\subset N},(f_i)_{i\in N}\right)$ be two $n$ -person stochastic cooperative game, there exist $\varepsilon$ and $\varepsilon'$,satisfy:

$C_\varepsilon(\Gamma)=C_{\varepsilon'}(\Gamma')\neq\phi$,then $\forall\ \delta>0$, $C_{\varepsilon-\delta}(\Gamma)=C_{\varepsilon'-\delta}(\Gamma')$.

Proof. First, we make proof $C_{\varepsilon-\delta}(\Gamma)\subseteq C_{\varepsilon'-\delta}(\Gamma')$. Let $(d^N,r^N)\in C_\varepsilon(\Gamma)$ be a stochastic payoff , and $(d^N,r^N)\in C_\varepsilon(\Gamma)$,

It is known

$C_\varepsilon(\Gamma)=C_{\varepsilon'}(\Gamma')$,

i.e. for $\forall S\subseteq N$, $\forall(\tilde{d},\tilde{r})\in Z(S)$, $\forall(\tilde{d},\tilde{r})\in Z'(S)$, $\forall i\in N$,

we have

$$\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon \tag{1}$$

$$\sum_{i\in S}(\tilde{\tilde{d}}^N,\tilde{\tilde{r}}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon' \tag{2}$$

For $(d^N,r^N)\in C_{\varepsilon-\delta}(\Gamma)$, we have

$$\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon-\delta$$

$$\sum_{i\in S}(\tilde{d}^N,\tilde{r}^N)_i-\left(\sum_{i\in S}(d^N,r^N)_i-\delta\right)\le\varepsilon$$

From (1) and (2):

$$\sum_{i\in S}(\tilde{\tilde{d}}^N,\tilde{\tilde{r}}^N)_i-\left(\sum_{i\in S}(d^N,r^N)_i-\delta\right)\le\varepsilon$$

i.e. $$\sum_{i\in S}(\tilde{\tilde{d}}^N,\tilde{\tilde{r}}^N)_i-\sum_{i\in S}(d^N,r^N)_i\le\varepsilon-\delta$$

i.e. $(d^N,r^N)\in C_{\varepsilon'-\delta}(\Gamma')$, so $C_{\varepsilon-\delta}(\Gamma)\subseteq C_{\varepsilon'-\delta}(\Gamma')$.

Similarly

$C_{\varepsilon'-\delta}(\Gamma') \subseteq C_{\varepsilon-\delta}(\Gamma)$,

So

$C_{\varepsilon-\delta}(\Gamma) = C_{\varepsilon'-\delta}(\Gamma')$.

Theorem 3. Let $\Gamma = \left(N, \{X_S\}_{S\subset N}, (f_i)_{i\in N}\right)$ and $\Gamma' = \left(N, \{X'_S\}_{S\subset N}, (f_i)_{i\in N}\right)$ be two $n$-person stochastic cooperative game, there are $\varepsilon$ and $\varepsilon'$,satisfy: $C_{\varepsilon}(\Gamma) = C_{\varepsilon'}(\Gamma') \neq \phi$,then for $\forall\ \delta > 0$,we have $LC(\Gamma) = LC(\Gamma')$.

Proof. According to theorem 2,because of the randomlity of $\delta$,so

$LC\left(\left(\Gamma^t\right)_{t=0}^{m}\right) = LC\left(\left(\Gamma'^t\right)_{t=0}^{m}\right)$.

**References**


Gao, Z.F. (2007).The core of $n$-person repeated stochastic cooperative game. *Applied mathematics transaction of senior*. 22 (1), 1-8.

Gao, Z.F. (2006).Game theory and the economy. *China forestry*, (Chapter 4).

Liu, D.M. (1995).*Game theory and its application, defence-related science and of technology university,* (Chapter 4).

Oviedo J. (2000).The core of a repeated n-person cooperative game.European *Journal of Operational Research.* 127, 519-524.

Suijs J, Borm P, & De Waegenaere A et al.(1999). Cooperative game with stochastic payoffs. *European Journal of Operational Research*. 113, 193-205.

Suijs J, & Borm P. (1999). Stochastic cooperative game: superadditivity, convexity, and certainty equivalents.*Game and Economic Behavior*. 27, 331-345.

Suijs J, De Waegenaere A, & Borm P. (1998).*Stochastic cooperative game in insurance: Mathematics & Economics*. 22, 209-228.

Xie, Z.(2004).Game theory, *Publishing company of defence-related science and of technology university* (Chapter 6).