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Effect of Thermal Slip on the Falkner-Skan Stretching and Shrinking Wedge Flow of a Power-Law Fluid and Heat Transfer

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Abstract

Studies on boundary layer flow of the Falkner-Skan stretching and shrinking wedge flow of a power-law fluid and heat transfer have received much attention because of their extensive applications in engineering. In this paper, the heat transfer analysis of Falkner-Skan stretching and shrinking wedge flow of a power-law fluid with thermal slip and variable consistency is studied. Scaling group of transformation is used to map the governing nonlinear system of partial differential equations (i.e. the boundary layer equations) into a system of nonlinear ordinary differential equations. The resulting equations contain thermophysical parameters, namely: variable consistency parameter *M*, suction/injection parameter f_w, wedge velocity parameter λ, thermal slip parameter b, Falkner-Skan flow parameter *m* and power-law index *n*. The transformed equations are solved numerically to study the behavior of the dimensionless velocity, the temperature, the wall shear stress as well as the rate of heat transfer for different values of the parameters. The results are compared with those known from the literature and good agreement between the results is obtained.

Keywords: non-Newtonian fluid, variable consistency, stretching/shrinking wedge, thermal slip, group-theoretic method

1. Introduction

Non-Newtonian fluids such as butter, jams, jellies, soup, blood, saliva etc have many biological and industrial applications (Postelnicu & Pop, 2011). The fundamental theory and areas of applications of non-Newtonian fluids may be found in several texts, for example texts by Bird et al. (1987) and Crochet et al. (1984). Due to the many applications of non-Newtonian fluids, many researchers have investigated the flow field and heat transfer over various geometries for various boundary conditions. Abel et al. (2009) has found that power-law index *n* decreases momentum boundary layer. The effects of variable wall heat and mass fluxes on the free convective flow along a vertical plate which is in a porous medium filled with a non-Newtonian fluid was studied by Cheng (2011). Li et al. (2011) studied heat transfer in a pseudo-plastic non-Newtonian fluid which is aligned with a semi-infinite plate, and it was found that there is an increase in the thermal diffusion ratio with the increases the power-law index. Afify (2009) highlighted that the suction and injection parameter has a significant effect on the flow and on the rate of heat transfer. Abel et al. (2010) investigated the boundary layer flow as well as heat transfer characteristics for a second grade, non-Newtonian fluid in a porous medium. They have found that suction had the effect of decreasing the magnitude of heat transfer.

In 1931, Falkner and Skan studied the similarity solutions of boundary layer flow over a static wedge which is immersed in a viscous fluid. In the last few years, many researchers have investigated Falkner-Skan flow considering different effects. Liu and Chang (2008), for example, presented a simple but accurate method to estimate the unknown initial boundary conditions by utilizing the Lie-group shooting method on the Blasius and Falkner-Skan equations. Alizadeh et al. (2009) studied the same problem by the Adomian decomposition method, which is a semi analytical method. Afzal (2010) discussed suction/injection effects on the tangential movement of a nonlinear power-law stretching surface. Parand et al. (2011) applied Hermite functions pseudospectral method to find an approximate solution for the Falkner-Skan equation. The Falkner-Skan wedge flow for a non-Newtonian fluid with a variable free stream condition was studied numerically by Postelnicu and Pop (2011). Jalil et al. (2012) studied analytical solutions of Falkner-Skan wedge flow for a non-Newtonian power-law fluid using perturbation. They concluded that the skin friction coefficient is decreased with the increase of power-law exponent (n) in this analytical (perturbation) study of Falkner-Skan wedge flow for a non-Newtonian power-law fluid.

Many investigators investigated the effect of variable consistency (variable viscosity for Newtonian fluid) on the flow over different geometries. Tsai et al. (2009) explored the effects of variables viscosity and thermal conductivity on the heat transfer characteristics for flow which is hydromagnetic. They found that the fluid temperature increases whilst the fluid velocity and heat transfer rate decrease with the increase in viscosity parameter. Rahman and Salahuddin (2010) studied the effect of temperature dependent viscosity on MHD flow over a radiating inclined surface. They concluded that velocity and temperature for the case constant viscosity are higher than for corresponding case of variable viscosity. Hassanien and Rashed (2011) investigated the effects of variable viscosity and thermal conductivity on coupled heat and mass transfer in porous media. They found that variable viscosity have significant influence on the velocity, temperature as well as the rate of heat transfer at the wall. Mukhopadhyay (2011) presented an analysis of heat transfer over a stretching surface with thermal slip condition and the conclusion drawn was that the heat transfer reduces with an increase in thermal slip parameter.

A number of authors have applied group-theoretic methods to find the similarity solutions of several transport problems. For example, Hamad et al. (2011) applied a one-parameter group to free convective flow to study the magnetic field effects of a nanofluid over a semi-infinite vertical flat plate. Mutlag et al. (2012) used this method to study the effect of thermal radiation of a non-Newtonian power-law fluid on the velocity temperature profile. Uddin et al. (2012) have used scaling group transformation to study the problem of viscous in compressible MHD laminar boundary layer slip flow for a nanofluid over a convectively heated permeable moving linearly stretching sheet.

In our present paper, the Falkner-Skan stretching and shrinking wedge flow of a non-Newtonian power-law fluid is considered. The effect of thermal slip and variable consistency are included. The similarity representation of the problem is presented and investigated via scaling group of transformations and the transformed equations are then solved numerically to show the effects of the governing parameters, namely, the Falkner–Skan power-law index, consistency, suction/injection, and wedge velocity parameters on the dimensionless velocity, the temperature as well as on the wall shear stress and the rate of heat transfer.

2. Mathematical Formulation of the Problem

We consider steady 2-D laminar incompressible boundary layer flows due to non-Newtonian fluid over a porous stretching and shrinking wedge. It is assumed that free stream velocity is of the form $\bar{u}_e(\bar{x}) = U_o(\bar{x}/L)^m$. It is further assumed (as also done by others) that wedge velocity is of the form $\bar{u}_w(\bar{x}) = \lambda U_o(\bar{x}/L)^m$. The physical configuration is shown in Figure 1.

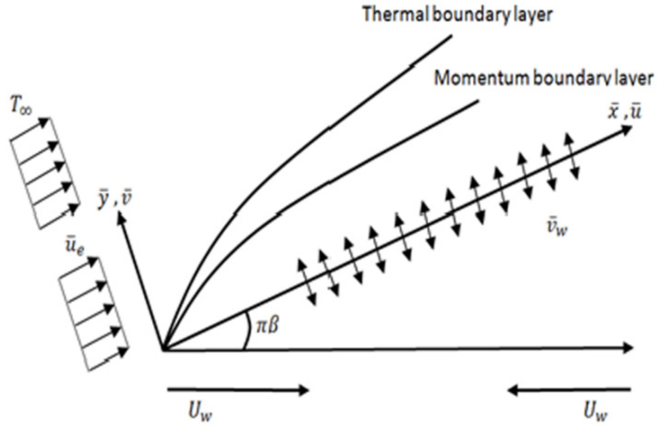

Figure 1. Physical model and Coordinates system

The relevant boundary layer equations can be written as (Postelnicu & Pop, 2011)

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}}+\frac{1}{\rho}\frac{\partial}{\partial \bar{y}}\left[\bar{K}\left|\frac{\partial \bar{u}}{\partial \bar{y}}\right|^{n-1}\frac{\partial \bar{u}}{\partial \bar{y}}\right], \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

where $\alpha = k/\rho\, c_p$ is the thermal diffusivity, ρ is the fluid density, c_p is the specific heat, k is the thermal conductivity of the fluid, x, y are the coordinates along and normal to the surface of the wedge. $\bar{u}$ and v are the boundary layer velocity components along the $\bar{x}$ and y -axes respectively, and n is the index in the power-law variation of of a non-Newtonian fluid. It was pointed out by Postelnicu and Pop (2011) that Equation (2) governs the flow of a shear-thinning or pseudoplastic fluid for the case $n<1$ and a shear-thickening or dilatants fluid for the case $n>1$. T is the temperature inside the boundary layer, K is consistency of the fluid.

We assumed the Reynold's model for the variation of consistency with temperature be (Szeri, 1998; Massoudi & Phuoc, 2004)

$$\bar{K}(\theta)=K_\circ \exp(-M\theta) \tag{4}$$

Here $K_\circ$ is the ambient fluid dynamic consistency, M is a consistency variation parameter. The following appropriate boundary conditions on the velocity and the temperature are employed:

$$\begin{aligned}&\bar{u}=-\lambda U_\circ\left(\frac{\bar{x}}{L}\right)^m,\bar{v}=\bar{v}_w(\bar{x}),T=T_w+D_1(\bar{x})\frac{\partial T}{\partial \bar{y}} \quad \text{at} \quad \bar{y}=0,\\ &\bar{u}\to\bar{u}_e(\bar{x}),T\to T_\infty \quad \text{as} \quad \bar{y}\to\infty\end{aligned} \tag{5}$$

where λ is a constant, with $\lambda>0$ and $\lambda<0$ corresponding to a moving wedge in a direction similar to and opposite from the free stream, respectively, whereas $\lambda=0$ is the case of a static wedge (see Yacob et al., 2011). $\bar{v}_w>0$ is the suction velocity while $\bar{v}_w<0$ is the injection velocity, D_1 is thermal slip factor with dimension length.

3. Method of Solution

The following dimensionless variables are now utilized

$$x=\frac{\bar{x}}{L},y=\frac{\bar{y}}{L}\text{Re}^{\frac{1}{n+1}},u=\frac{\bar{u}}{U_\circ},v=\frac{\bar{v}}{U_\circ}\text{Re}^{\frac{1}{n+1}},\theta=\frac{T-T_\infty}{T_w-T_\infty},u_e=\frac{\bar{u}_e}{U_\circ} \tag{6}$$

where $\text{Re}=U_\circ^{2-n}L^n/\nu$ is the generalized Reynolds number based the characteristic length L. We have

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=mx^{2m-1}+\frac{K_\circ\exp(-M\theta)}{\rho\nu}\left[n\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3}-M\frac{\partial\theta}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^n\right], \tag{7}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{\alpha\,\text{Re}^{\frac{2}{n+1}}}{LU_\circ}\frac{\partial^2\theta}{\partial y^2}. \tag{8}$$

The boundary conditions become

$$\begin{aligned}&\frac{\partial\psi}{\partial y}=-\lambda x^m,\frac{\partial\psi}{\partial x}=\frac{-v_w(x)\text{Re}^{\frac{1}{n+1}}}{U_\circ},\theta=1+D_1(x)\frac{\text{Re}^{\frac{1}{n+1}}}{L}\frac{\partial\theta}{\partial y} \quad \text{at} \quad y=0,\\ &\frac{\partial\psi}{\partial y}\to x^m,\theta\to 0 \quad \text{as} \quad y\to\infty.\end{aligned} \tag{9}$$

The stream function $\psi(x,y)$ defined by $u=\partial\psi/\partial y, v=-\partial\psi/\partial x$ satisfy the continuity Equation (1) automatically. In order to find similarity transformations, we consider following simplified form of the one-parameter group (Uddin et al., 2012; Mutlag et al., 2012; Na, 1979)

$$\begin{aligned}&x=e^{-\varepsilon c_1}x^*,y=e^{-\varepsilon c_2}y^*,\psi=e^{-\varepsilon c_3}\psi^*,\theta=e^{-\varepsilon c_4}\theta^*,\\ &D_1=e^{-\varepsilon c_5}D_1^*,v_w=e^{-\varepsilon c_6}v_w^*,c_p=e^{-\varepsilon c_7}c_p^*\end{aligned} \tag{10}$$

where $c_1, c_2, c_3, ..., c_7$ are constant and ε is the parameter of the group. Substituting Equation (10) into the Equations (7)-(9), we will then obtain the following relationship among $c's$

$$\begin{aligned} &c_1 = c_3\left(\frac{n+1}{2mn-m+1}\right), c_2 = c_5 = c_3\left(\frac{mn-2m+1}{2mn-m+1}\right), c_4 = 0 \\ &c_6 = c_3\left(\frac{2mn-m-n}{2mn-m+1}\right), c_7 = c_3\left(\frac{3m-3mn+n-1}{2mn-m+1}\right) \end{aligned} \tag{11}$$

Note that $\theta^* = \theta$, i.e., θ is invariants. The characteristic equations are as follows:

$$\begin{aligned} &\frac{dx}{\left(\frac{n+1}{2mn-m+1}\right)x} = \frac{dy}{\left(\frac{mn-2m+1}{2mn-m+1}\right)y} = \frac{d\psi}{\psi} = \frac{dD_1}{\left(\frac{mn-2m+1}{2mn-m+1}\right)D_1} \\ &= \frac{dv_w}{\left(\frac{2mn-m-n}{2mn-m+1}\right)v_w} = \frac{dc_p}{\left(\frac{3m-3mn+n-1}{2mn-m+1}\right), c_p} \end{aligned} \tag{12}$$

Solving the above equations, the following similarity transformations are obtained

$$\begin{aligned} &\eta = x^{\left(\frac{-mn+2m-1}{n+1}\right)} y, \psi = x^{\frac{2mn-m+1}{n+1}} f(\eta), \theta = \theta(\eta) \\ &D_1 = x^{\frac{mn-2m+1}{n+1}} (D_1)_\circ, v_w = x^{\frac{2mn-m-n}{n+1}} (v_w)_\circ, c_p = x^{\frac{3m-3mn+n-1}{n+1}} (c_p)_\circ \end{aligned} \tag{13}$$

where $(D_1)_\circ, (v_w)_\circ, (c_p)_\circ$ are constant thermal slip, velocity of suction/injection and the specific heat.

Using Equation (18) in Equations (7)-(9), we obtain following nonlinear system of ordinary differential equations

$$f'''(f'')^{n-1} - \frac{M}{n}\theta'(f'')^n + \exp(M\theta)\left(\frac{2mn-m+1}{n(n+1)}ff'' - \frac{m}{n}f'^2 + \frac{m}{n}\right) = 0, \tag{14}$$

$$\theta'' + \frac{\Pr_m(2mn-m+1)}{n+1}f\theta' = 0. \tag{15}$$

The modified Prandtl number (for power law fluids) is $\Pr_m = LU_\circ \operatorname{Re}^{\frac{-2}{1+n}} / \alpha$. The corresponding boundary conditions are now as follows:

$$\begin{aligned} &f' = -\lambda, f = -\frac{n+1}{2mn-m+1}f_w, \theta = 1 + b\theta' \quad \text{at} \quad \eta = 0, \\ &f' \to 1, \theta \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{16}$$

where $b = (D_1)_\circ \operatorname{Re}^{\frac{1}{n+1}} / L$ and $f_w = (n+1)(v_w)_\circ \operatorname{Re}^{\frac{1}{n+1}} / U_\circ$ are the dimensionless thermal slip parameter and the suction/injection parameter respectively. Note that all parameters are free from *x* which confirms the true similarity solution of the Equations (14)-(15) subject to the boundary conditions given in Equation (16). Here primes denote differentiation with respect to η.

It is to be noted that for $M = 0, n = 1$ our problem reduces to Jiji (2009) in this case the Equations (14)-(15) are

$$f''' + \frac{m+1}{2}ff'' - mf'^2 + m = 0, \tag{17}$$

$$\theta'' + \frac{\Pr_m(m+1)}{2}f\theta' = 0. \tag{18}$$

Note that when $m = M = 0, n = 1$ then Equation (14) conforms to Equation (11) in Ishak and Bachok (2009) and the system of ordinary differential Equations (14)-(15) is the same as that obtained by Aziz (2009) when $m = M = 0, n = 1$.

4. Physical Parameters

Quantities of thermal and other engineering design are skin friction factor $C_{f\bar{x}}$ and the Nusselt number $Nu_{\bar{x}}$, which take the form:

$$C_{f\bar{x}} = \frac{-K}{\rho\bar{u}_e^2(\bar{x})}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^n_{\bar{y}=0}, \quad Nu_{\bar{x}} = \frac{-\bar{x}}{T_w - T_\infty}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0}, \tag{19}$$

Using (6) and (13) we have from (19)

$$C_{f_{\bar{x}}}\left(\mathrm{Re}_{\bar{x}}\right)^{\frac{1}{1+n}}=\left[f''(0)\right]^{n},\ Nu_{\bar{x}}\left(\mathrm{Re}_{\bar{x}}\right)^{\frac{-1}{1+n}}=-\theta'(0), \tag{20}$$

where $\mathrm{Re}_{\bar{x}}=\bar{u}_e^{2-n}\,\bar{x}^n/\nu$ is the local Reynolds number.

5. Results and Discussions

The governing partial differential equations have been successfully transformed into ordinary differential equations. The transformations (Equations 14-16) resulted in a two point boundary value problem which is highly nonlinear and their closed form analytical solution may not exist and hence we will solve them numerically using the Runge-Kutta-Fehlberg fourth-fifth order numerical method (in conjunction with shooting method). In the present study, the numerical solutions for the effect of thermal slip on the heat transfer Falkner-Skan stretching wedge flow of a power-law fluid with variable consistency are presented. To compare the present results with those of Aziz (2009), Bognar and Hriczo (2011), the values of $-\theta'(0)$ for $f_w=\lambda=M=m=0, n=1$, and $\mathrm{Pr}_m=0.7,10$ and those of Jiji (2009) for the values of $f''(0)$ for $f_w=\lambda=M=0, n=1$, are considered as shown in Tables 1, 2, 3. In Table 1, we compare the present results with those of Ishak et al. (2007), Postelnicu and Pop (2011). Table 2 shows that our results are in agreement with those obtained by Aziz (2009), Bognar and Hriczo (2011), Table 3 show that the present results are in agreement with those of Jiji (2009). We cannot compare with regard to $-\theta'(0)$ because the boundary conditions in our study differ from those in Jiji (2009).

Table 1. The values of $f''(0)$ when $f_w=\lambda=M=0, m=n=1$

Ishak et al. (2007)	Postelnicu and Pop (2011)	Present study
1.2326	1.23259	1.232587

Table 2. The values of $\theta(0)$ for values of b and Pr_m when $f_w=\lambda=M=m=0, n=1$

	Pr = 0.72			Pr = 10		
b	Aziz (2009)	Bognar and Hriczo (2011)	Present study	Aziz (2009)	Bognar and Hriczo (2011)	Present study
20	0.1447	0.144661	0.144661	0.0643	0.064256	0.064255
10	0.2528	0.252758	0.252758	0.1208	0.120752	0.120752
5	0.4035	0.403523	0.403522	0.2155	0.215484	0.215484
2.5	0.575	0.575014	0.575013	0.3546	0.354566	0.354565
1.66	0.6699	0.669916	0.670801	0.4518	0.451759	0.452752
1.25	0.7302	0.73017	0.730169	0.5235	0.523512	0.523511
1	0.7718	0.771822	0.771822	0.5787	0.578656	0.578656
0.2	0.9441	0.944173	0.944173	0.8729	0.872883	0.872883
0.1	0.9713	0.971285	0.971285	0.9321	0.932128	0.932127
0.05	0.9854	0.985434	0.985433	0.9649	0.964872	0.964871

Table 3. The values of $f''(0)$ for values of m when $f_w=\lambda=M=0, n=1$

m	Jiji (2009)	Present study
0.0	0.3206	0.33205
0.111	0.5120	0.51169
0.333	0.7575	0.75713
1.0	1.2326	1.23258

Numerical results are obtained to study the effect of the parameters f_w, b, λ, M, m and n on the dimensionless velocity, dimensionless temperature, friction factor coefficient and rate of heat transfer. The numerical results are shown in Figures 2-17.

Figures 2-3 present the effects of *M, b* and *n* on the friction factor and the rate of heat transfer. In Figure 2 we see that the shear stress increases with an increase in consistency variation parameter for both non-Newtonian and Newtonian fluid. It is further found that shear stress decreases with increasing *n*. Postelnicu and Pop draw a similar conclusion (2011). It is further noticed that the shear stress decreases with thermal slip parameter *b* for both non-Newtonian and Newtonian fluid. In Figure 3 it can be seen that the Nusselt number rises with increasing the values of *M* and *b*, decreases with increasing *n*.

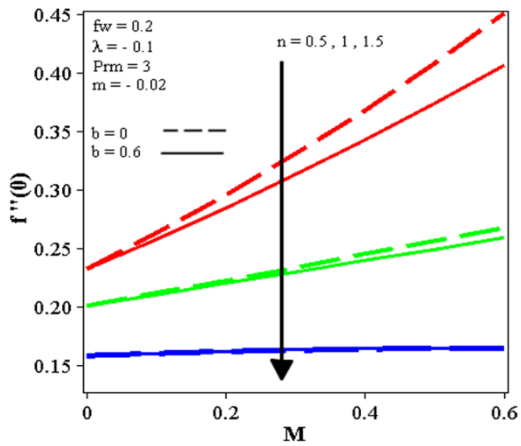

Figure 2. Variation of friction factor with *M*, *n* and *b*

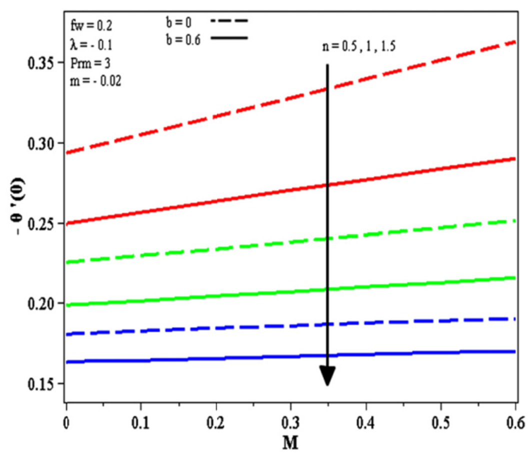

Figure 3. Variation of Nusselt number with *M*, *n* and *b*

From Figures 4 and 5 we see that the profile of the shear stress coefficient and the rate of heat transfer with *n*, λ and *m*. From Figure 4 we notice that the shear stress coefficient decreases as *n* decreases, increases as with increasing values of *m*, λ. From Figure 5 we observe that the Nusselt number reduces with increasing the values of *n*, λ and increases with an increase in values of *m*.

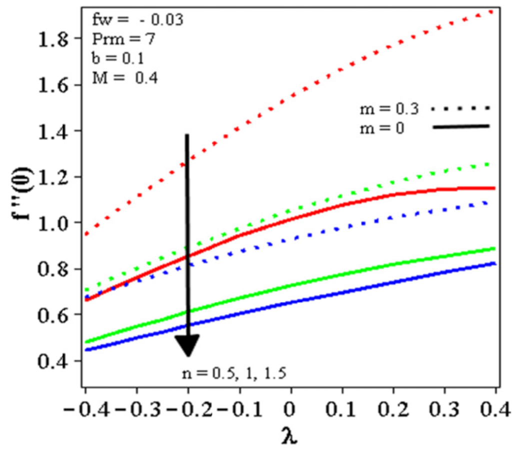

Figure 4. Variation of friction factor with λ , n and m

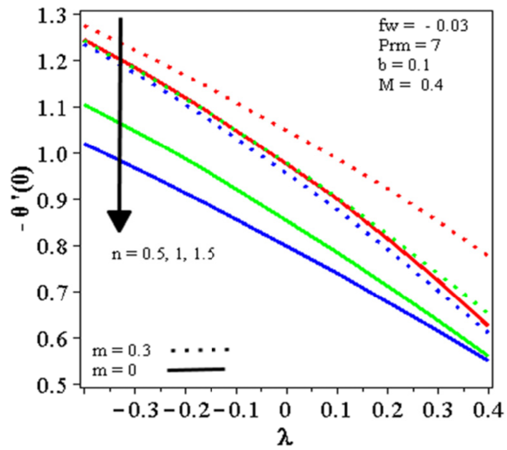

Figure 5. Variation of Nusselt number with λ , n and m

Figures 6-7 present the effects of various values of λ on the dimensionless velocity and the dimensionless temperature field. When $\mathrm{b} = 0.7, f_w = 0.03, M = -0.5, \mathrm{m} = 0.05$, $\Pr_m = 0.7$, and $\mathrm{n} = 0.4$, the velocity and the temperature profiles are determined. Velocity decreases with increasing λ , the temperature increases.

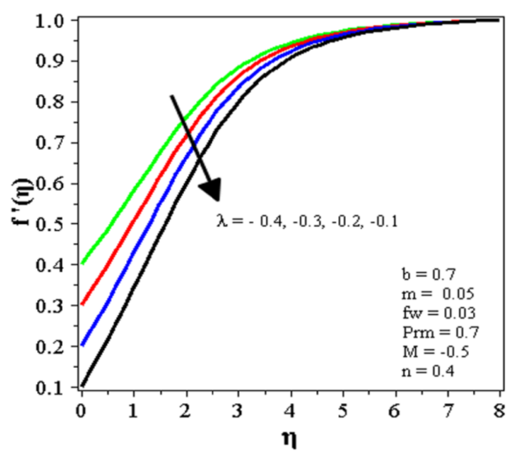

Figure 6. Effects of the wedge velocity parameter (λ) on the dimensionless velocity

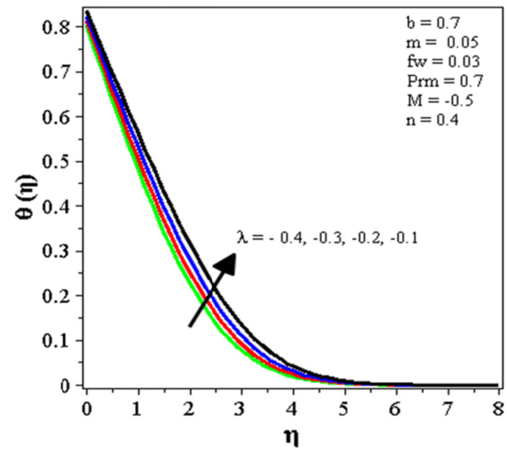

Figure **7**. Effects of the wedge velocity parameter (λ) on the dimensionless temperature

The effects of the suction/injection parameter on the dimensionless velocity and temperature are shown in Figures 8-9, when $\mathrm{b}=0.3, \mathrm{M}=-0.6, m=0.3, \lambda=-0.1, \mathrm{Pr}_m=0.7$ and $n=0.4$. It can be seen that the velocity and temperature are affected by the suction/injection parameter f_w, wherein velocity decreases with increasing f_w but temperature increases.

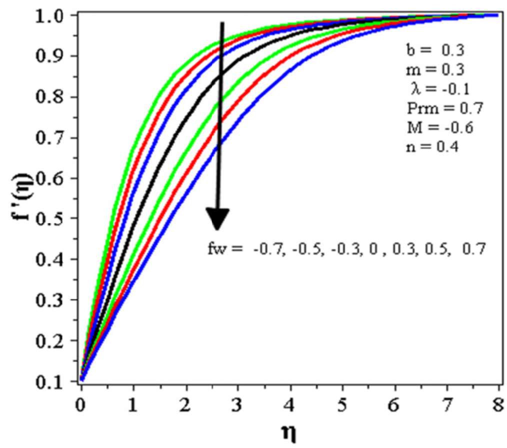

Figure 8. Effects of suction/injection parameter (f_w) on the dimensionless velocity

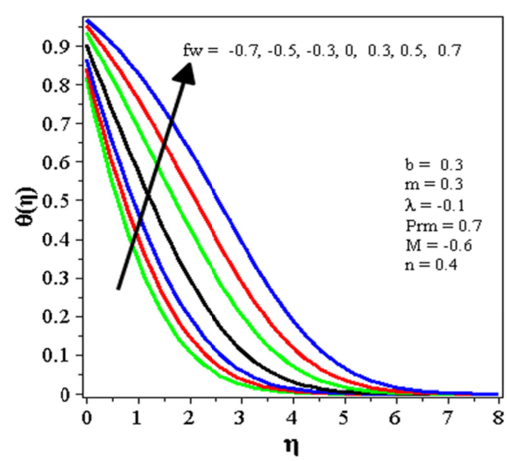

Figure 9. Effects of suction/injection parameter (f_w) on the dimensionless temperature

Figures 10-11 illustrate the variations of the power law-index n on the dimensionless velocity and temperature profiles. The velocity and the temperature are found to be increased with an increase in power law index n.

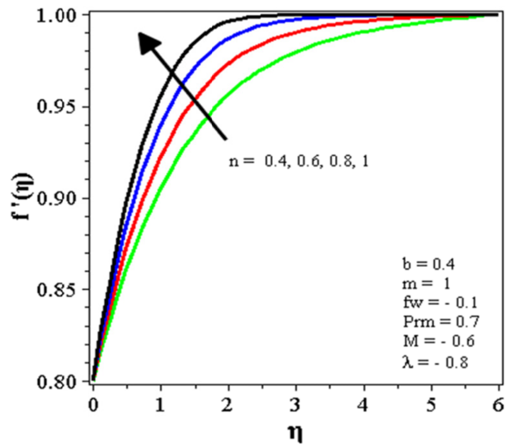

Figure 10. Effects of power-law index (n) on the dimensionless velocity

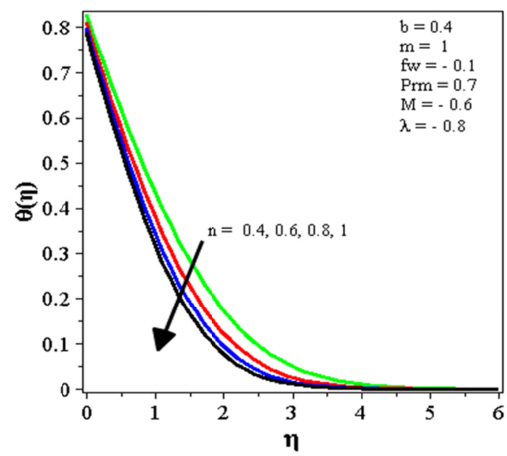

Figure 11. Effects of power-law index (n) on the dimensionless temperature

Figures 12-13 show the effects of the thermal slip parameter on the dimensionless velocity and temperature when $n = 0.1, M = -0.6, m = 0.05,\ f_w = -0.2,\ \Pr_m = 0.7\ , f_w = -0.2$ and $\lambda = -0.3$. With these values, the velocity is increased with increase in b and temperature is decreased with increase in b.

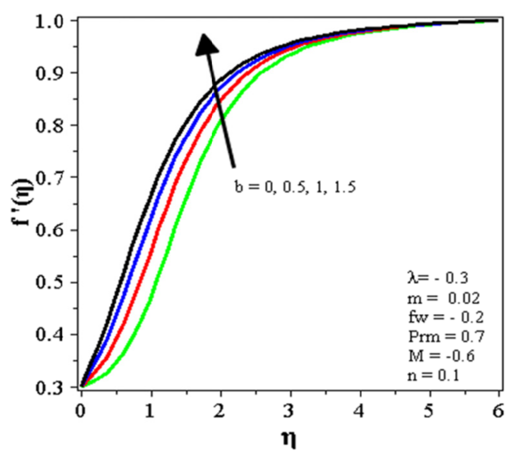

Figure 12. Effects of thermal slip parameter (b) on the dimensionless velocity

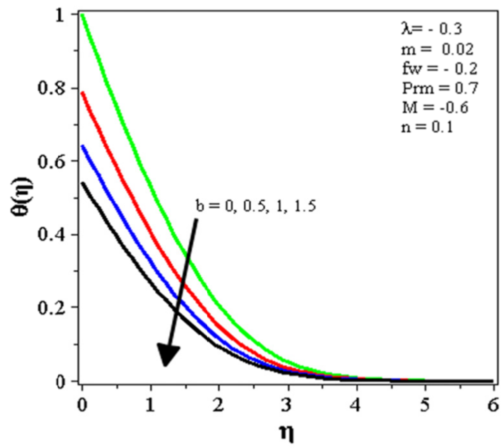

Figure 13. Effects of thermal slip parameter (b) on the dimensionless temperature

Figures 14-15 show the effects of the consistency variation parameter on dimensionless velocity and the temperature when $n = 0.5, f_w = 0.2, m = -0.1, b = 0.3, \Pr_m = 3$, and $\lambda = -0.07$. The velocity is increased with an increase in M and temperature is decreased with increase in M.

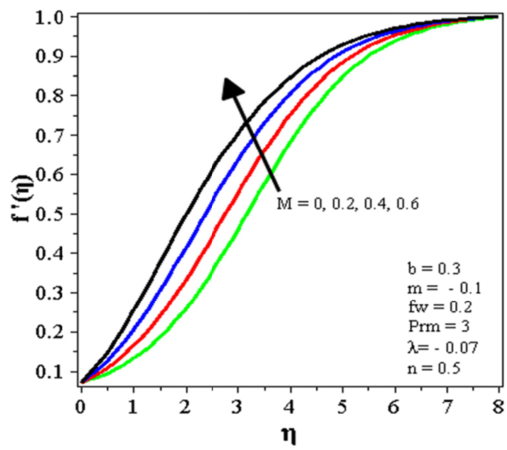

Figure 14. Effects ofconsistancy variation parameter (M) on the dimensionless velocity

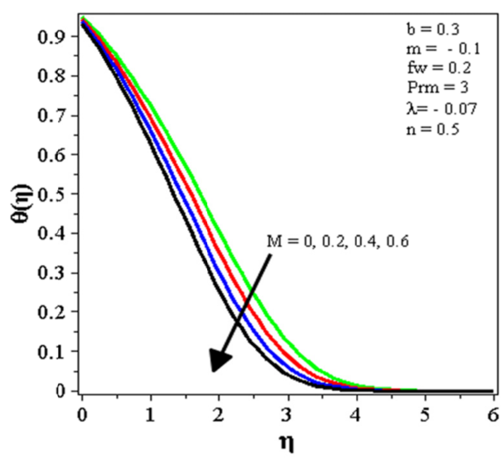

Figure 15. Effects ofconsistancy variation parameter (M) on the dimensionless temperature

The effects of the pressure gradient parameter m on the dimensionless velocity and temperature are shown in Figures 16-17, when $b = 0.2, f_w = 0.5, M = -0.6, \lambda = -0.6, \Pr_m = 0.7$ and $n = 0.1$. It is found that the dimensionless velocity and temperature are influenced by *m*. The dimensionless velocity and temperature increase with increasing m. White (2006) also draws a similar conclusion.

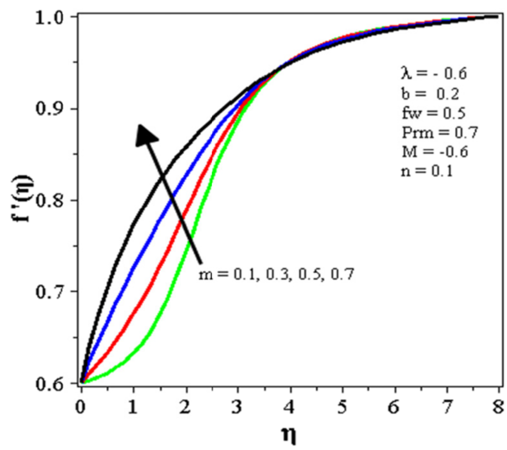

Figure 16. Effects of Falkner-Skan flow parameter (*m*) on the dimensionless velocity

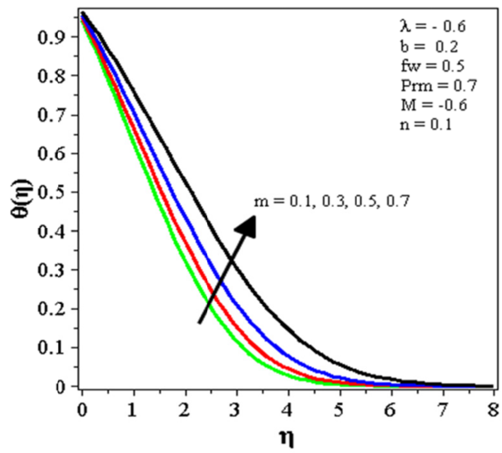

Figure 17. Effects of Falkner-Skan flow parameter (*m*) on the dimensionless temperature

4. Conclusions

The present study investigated the influence of thermal slip and variable consistency on the heat transfer Falkner–Skan stretching wedges flow of a power-law fluid. The main points can be summarized as follows:

The wall shear stress increased with an increase in consistency variation parameter, decreases with increasing *n*, *b*, Nusselt number increased with an increase in *M* and *b* decreases with increasing *n*.

The shear stress coefficient increases as λ and *m* increase while the Nusselt number increases as *m* increases, decreases with increasing values of *n*, λ.

Velocity decreases with increasing λ, whereas temperature increases.

The velocity is increased, whereas the temperature is decreased with an increase in the value of the power law index *n*.

The velocity is increased with an increase in the values of the *b* and temperature is decreased with increase in the values of *b*.

The velocity is increased with an increase in *M* and temperature is decreased with increase in *M*. The velocity and temperature increases with increasing the Falkner-Skan flow parameter *m*.

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