

Fuzzy Reliability of Two Units of the Cold Storing System

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Abstract

This paper which adopts Probability statistics fuzzy mathematical principles and methods gives the Fuzzy reliability index of the cold storing system with two different units when the switch is completely reliable and the switch is not completely reliable (Switch life 0-1 and Exponential distribution). And this paper gives a new kind of Failure mode, that is: system will be immediately failure if the switch is failure, meanwhile, it gives the new mode's Fuzzy reliability index.

Keywords: Cold storing system, Reliability, Fuzzy reliability

1. Prior knowledge

Knows C by literature [1] to express in the classical reliable definition "the product in…maintains its stipulation function" this clear event, C_1, C_2, \dots, C_n expressed separately each fuzzy function represent fuzzy event. Obviously C separately belongs to C_1, C_2, \dots, C_n in varying degrees. \overline{C} expresses the system breakdown, C_i expressed "the ist unit is working", C_i expressed the fuzzy function subset which we discussed.

By fuzzy conditional probability definition we obtain:

$$P(C\Delta C_{j}) = P(C_{j}/C) \cdot P(C)$$
⁽¹⁾

$$P(C_i \Delta C_i) = p(C_i / C_i) \cdot P(C_i)$$
⁽²⁾

According to the fuzzy reliable theory and the ordinary reliable theory we have:

$$P(C\Delta C_{j}) = R_{s} \qquad P(C_{i}\Delta C_{j}) = R_{i}$$

$$P(C_{j}/C) = uc_{j}(R_{s}) \qquad P(C_{j}/C_{i}) = uc_{j}(R_{i}) \qquad (3)$$

Substitutes (3) into (1) (2) we have:

$$R_s = u c_j(R_s) \cdot R_s \tag{4}$$

$$R_i = u c_j(R_i) \cdot R_i \tag{5}$$

Based on the literature [1], [4] knowledge, the relations between every unit fuzzy failure rate λ_i and the ordinary

failure rate λ_i is:

$$\lambda_{i} = \lambda_{i} - \frac{du c_{j}(R_{i}) \cdot dt}{u c_{j}(R_{i}) dt} = \lambda_{i} - \overline{u' c_{j}(R_{i})}$$

$$(6)$$

Where $\overline{u'c_j(R_i)}$ is the relative rate of $u'c_j(R_i)$.

2. Fuzzy Reliability analysis

Theorem 1 Suppose the system is the cold storing system with two different units and the switch is completely reliable, Its life respectively is x_1, x_2 , also obeys separately exponential distribution λ_1, λ_2 , mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

$$R_{s} = u c_{j}(R_{s}) \cdot \left\{ \frac{\lambda_{2} + \overline{u' c_{j}(R_{2})}}{\lambda_{2} - \lambda_{1} + \overline{u' c_{j}(R_{2})} - \overline{u' c_{j}(R_{1})}}{\sum_{i} - \overline{u' c_{j}(R_{1})}} \cdot \frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u' c_{j}(R_{1})}}{\lambda_{1} - \lambda_{2} + \overline{u' c_{j}(R_{1})} - \overline{u' c_{j}(R_{2})}} \cdot \frac{R_{2}}{\overline{u' c_{j}(R_{2})}} \right\}$$

$$MTTF = u c_{j}(R_{s})_{m} \cdot \frac{\lambda_{1} + \lambda_{2} + \overline{u' c_{j}(R_{1})} + \overline{u' c_{j}(R_{2})}}{\left\{\lambda_{1} + \overline{u' c_{j}(R_{1})}\right\} \cdot \left\{\lambda_{2} + \overline{u' c_{j}(R_{2})}\right\}}$$

Proof: Known two unit life distributions respectively are $F_1 = 1 - e^{-\lambda_1 t}$, $F_2 = 1 - e^{-\lambda_2 t}$, also knows the system by literature [2] the reliability is:

$$R_{s} = \frac{\lambda_{2}}{\lambda_{2} - \lambda_{1}} \cdot R_{1} + \frac{\lambda_{1}}{\lambda_{1} - \lambda_{2}} \cdot R_{2}$$
(7)

So substitutes (5)(6)(7) into (4) we obtain the fuzzy reliability

$$R_{s} = u c_{j}(R_{s}) \cdot \left\{ \frac{\lambda_{2} + \overline{u' c_{j}(R_{2})}}{\lambda_{2} - \lambda_{1} + \overline{u' c_{j}(R_{2})} - \overline{u' c_{j}(R_{1})}} \cdot \frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u' c_{j}(R_{1})}}{\lambda_{1} - \lambda_{2} + \overline{u' c_{j}(R_{1})} - \overline{u' c_{j}(R_{2})}} \cdot \frac{R_{2}}{\overline{u' c_{j}(R_{2})}} \right\}$$

Result of $R_s = \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot R_1 + \frac{\lambda_1}{\lambda_1 - \lambda_2} \cdot R_2$, obtain easily:

$$MTTF = \int_{0}^{\infty} R_{s} dt = \int_{0}^{\infty} u c_{j}(R_{s}) \cdot R_{s} \cdot dt = u c_{j}(R_{s})_{m} \cdot \int_{0}^{\infty} R_{s} \cdot dt$$
$$= u c_{j}(R_{s})_{m} \cdot \int_{0}^{\infty} \left[\frac{\lambda_{2}}{\lambda_{2} - \lambda_{1}} \cdot e^{-\lambda_{1}t} + \frac{\lambda_{1}}{\lambda_{1} - \lambda_{2}} \cdot e^{-\lambda_{2}t} \right] \cdot dt$$
$$= u c_{j}(R_{s})_{m} \cdot \left[\frac{1}{\lambda_{1}} + \frac{1}{\lambda_{2}} \right]$$
(8)

Therefore substitute (6) into equation (8), we have:

$$MTTF = u c_{j}(R_{s})_{m} \cdot \frac{\lambda_{1} + \lambda_{2} + \overline{u'c_{j}(R_{1})} + \overline{u'c_{j}(R_{2})}}{[\lambda_{1} + \overline{u'c_{j}(R_{1})}] \cdot [\lambda_{2} + \overline{u'c_{j}(R_{2})}]}$$
(9)

Where, $uc_j(R_s)_m - uc_j(R_s)$ is an average value which is in operating time sector

 $[0,\infty)$, and it is a constant.

Theorem 2 suppose the system is the cold storing system with two different units and the switch is not completely reliable. Its life respectively is x_1, x_2 , Also obeys separately exponential distribution λ_1, λ_2 , mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

$$R_{s} = uc_{j}(R_{s}) \cdot \left[\frac{R_{1}}{\underbrace{uc_{j}(R_{1})}_{\sim} + \frac{P(\lambda_{1} + \overline{u'c_{j}(R_{1})})}{\lambda_{1} - \lambda_{2} + \overline{u'c_{j}(R_{1})} - \overline{u'c_{j}(R_{2})}} \cdot \left[\frac{R_{2}}{uc_{j}(R_{2})} - \frac{R_{1}}{uc_{j}(R_{1})} \right] \right]$$

$$MTTF = uc_{j}(R_{s})_{m} \cdot \left[\frac{1}{\lambda_{1} + \overline{u'c_{j}(R_{1})}} + p \cdot \frac{1}{\lambda_{2} + \overline{u'c_{j}(R_{2})}} \right]$$

Proof: Introduces a random variable v, we have:

$$p\{v=j\} = \begin{cases} q(\underline{\exists} j=1) \\ p(\underline{\exists} j=2) \end{cases}$$

The reliability of system is

$$R = P\left\{\sum_{J=1}^{\nu} x_{j} > t\right\} = q \cdot p\{x_{1} > t\} + p \cdot p\{x_{1} + x_{2} > t\}$$
$$= e^{-\lambda_{1}t} + \frac{p\lambda_{1}}{\lambda_{1} - \lambda_{2}} (e^{-\lambda_{2}t} - e^{-\lambda_{1}t}) = R_{1} + \frac{p\lambda_{1}}{\lambda_{1} - \lambda_{2}} \cdot (R_{2} - R_{1})$$
(10)

Substituting (10) (5) (6) into (4) entails that:

$$R_{s} = u c_{j}(R_{s}) \cdot R_{s}$$

$$= u c_{j}(R_{s}) \cdot \left[\frac{R_{1}}{\frac{u}{c_{j}(R_{1})}} + \frac{P(\lambda_{1} + \overline{u'c_{j}(R_{1})})}{\lambda_{1} - \lambda_{2} + \overline{u'c_{j}(R_{1})} - \overline{u'c_{j}(R_{2})}} \cdot \left[\frac{R_{2}}{\frac{u}{c_{j}(R_{2})}} - \frac{R_{1}}{uc_{j}(R_{1})} \right] \right]$$

Because the mean life of system is:

$$MTTF = \int_{0}^{\infty} R_{s} dt = \int_{0}^{\infty} e^{-\lambda_{1}t} + \frac{p\lambda_{1}}{\lambda_{1} - \lambda_{2}} (e^{-\lambda_{2}t} - e^{-\lambda_{1}t}) dt = \frac{1}{\lambda_{1}} + p\frac{1}{\lambda_{2}}$$

So the MTTF is:

$$MTTF = \int_{0}^{\infty} R_{s} dt = \int_{0}^{\infty} u c_{j}(R_{s}) \cdot R_{s} \cdot dt = u c_{j}(R_{s})_{m} \cdot \left[\frac{1}{\lambda_{1}} + p \frac{1}{\lambda_{2}}\right]$$
$$= u c_{j}(R_{s})_{m} \cdot \left[\frac{1}{\lambda_{1} + u'c_{j}(R_{1})} + p \cdot \frac{1}{\lambda_{2} + u'c_{j}(R_{2})}\right]$$
(11)

Theorem 3 The system not immediately expires when the switch is not working, the life of two different units is x_1, x_2 , he life of switch is x_K , Obeys the exponential distribution separately and the parameter is λ_1, λ_2 and λ_K , mutually independent, so the fuzzy reliability and fuzzy mean lifetime are:

$$\begin{split} R_{s} &= u c_{j}(R_{s}) \Biggl[\frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u' c_{j}(R_{1})}}{\lambda_{K} + \lambda_{1} - \lambda_{2} + \overline{u' c_{j}(R_{K})} + \overline{u' c_{j}(R_{1})} - \overline{u' c_{j}(R_{2})}} \\ \cdot \Biggl[\frac{R_{2}}{u c_{j}(R_{2})} - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{K}}{u c_{j}(R_{K})} \Biggr] \Biggr] \\ MTTF &= u c_{j}(R_{s})_{m} \end{split}$$

$$\cdot \left[\frac{1}{\lambda_1 + \overline{u'c_j(R_1)}} + \frac{\lambda_1 + \overline{u'c_j(R_1)}}{[\lambda_2 + \overline{u'c_j(R_2)}] \cdot [\lambda_1 + \overline{u'c_j(R_1)} + \lambda_K + \overline{u'c_j(R_K)}]} \right]$$

Proof: From literature [2] we know

$$R = e^{-\lambda_{1}t} + \frac{\lambda_{1}}{\lambda_{K} + \lambda_{1} - \lambda_{2}} \left[e^{-\lambda_{2}t} - e^{-(\lambda_{1} + \lambda_{K})t} \right]$$
(12)
$$MTTF = \frac{1}{\lambda_{1}} + \frac{\lambda_{1}}{\lambda_{2}(\lambda_{1} + \lambda_{K})}$$
(13)

Substituting (12)(5)(6) into (4) entails that

$$R_{s} = u c_{j}(R_{s}) \cdot R_{s}$$

$$= u c_{j}(R_{s}) \left[\frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u'c_{j}(R_{1})}}{\lambda_{K} + \lambda_{1} - \lambda_{2} + \overline{u'c_{j}(R_{K})} + \overline{u'c_{j}(R_{1})} - \overline{u'c_{j}(R_{2})}} \right]$$

$$\cdot \left[\frac{R_{2}}{u c_{j}(R_{2})} - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{K}}{u c_{j}(R_{K})} \right]$$

Because also

$$MTTF = \int_{0}^{\infty} R_{s} dt = \int_{0}^{\infty} u c_{j}(R_{s}) \cdot R_{s} \cdot dt$$
$$= u c_{j}(R_{s})_{m} \cdot \int_{0}^{\infty} R_{s} \cdot dt = u c_{j}(R_{s})_{m} \cdot \left[\frac{1}{\lambda_{1}} + \frac{\lambda_{1}}{\lambda_{2}(\lambda_{1} + \lambda_{K})}\right]$$
(14)

Substituting (6) into (14) entails that

$$MTTF = u c_j (R_s)_m$$

$$\cdot \left[\frac{1}{\lambda_1 + \overline{u'c_j(R_1)}} + \frac{\lambda_1 + \overline{u'c_j(R_1)}}{[\lambda_2 + \overline{u'c_j(R_2)}] \cdot [\lambda_1 + \overline{u'c_j(R_1)}] + \lambda_K + \overline{u'c_j(R_K)}]} \right]$$

Theorem 4 The system immediately expires when the switch is not working, the life of two different units is x_1, x_2 , the life of switch is x_K , Obeys the exponential distribution separately and the parameter is λ_1, λ_2 and λ_K , mutually independent, so the fuzzy reliability is:

$$\begin{split} R_{s} &= u c_{j}(R_{s}) \cdot \left[\frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u' c_{j}(R_{1})}}{\lambda_{K} + \lambda_{1} - \lambda_{2} + \overline{u' c_{j}(R_{K})} + \overline{u' c_{j}(R_{1})} - \overline{u' c_{j}(R_{2})}} \right] \\ & \cdot \left[\frac{R_{2}}{u c_{j}(R_{2})} - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{K}}{u c_{j}(R_{K})} \right] \\ & - \frac{[\lambda_{K} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{1} + \overline{u' c_{j}(R_{1})}]}{[\lambda_{1} + \overline{u' c_{j}(R_{1})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{1})}]} \\ & - \frac{[\lambda_{K} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{2})} + \lambda_{1} + \overline{u' c_{j}(R_{1})}]}{[\lambda_{1} + \overline{u' c_{j}(R_{1})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{2})} + \lambda_{1} + \overline{u' c_{j}(R_{1})}]} \\ & \cdot \left[1 - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{K}}{u c_{j}(R_{K})} \right] - \frac{[\lambda_{K} + \overline{u' c_{j}(R_{2})} + \lambda_{K} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{2})}]}{[\lambda_{2} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{K})}] \cdot [\lambda_{2} + \overline{u' c_{j}(R_{2})} - \lambda_{1} - \overline{u' c_{j}(R_{1})}]} \right] \end{split}$$

$$\cdot \left[1 - \frac{R_2}{uc_j(R_2)} \cdot \frac{R_K}{uc_j(R_K)}\right]$$

Proof: the switch is not expire when the unit x_1 is not working, $x_K > x_1$, unit x_1 is replaced by storing unit x_2 , the life of system is x_K when unit x_2 is not expire.

Because the life distribution of system is:

$$\begin{split} 1 - R_{s} &= \iiint_{t_{1} \leq t, t_{K} \leq t_{1}} \lambda_{1} \lambda_{K} e^{-\lambda_{1} t_{1}} e^{-\lambda_{K} t_{K}} dt_{1} dt_{K} + \\ & \iiint_{t_{1} + t_{2} \leq t, t_{K} > t_{1}} \lambda_{2} \lambda_{K} e^{-\lambda_{1} t_{1}} e^{-\lambda_{2} t_{2}} e^{-\lambda_{K} t_{K}} dt_{1} dt_{2} dt_{K} + \\ & \iiint_{t_{1} + t_{2} \geq t_{K}, t_{1} \leq t_{K} \leq t} \lambda_{K} e^{-\lambda_{1} t_{1}} e^{-\lambda_{2} t_{2}} e^{-\lambda_{K} t_{K}} dt_{1} dt_{2} dt_{K} \\ &= 1 - e^{-\lambda_{1} t} - \frac{\lambda_{1}}{\lambda_{K} + \lambda_{1} - \lambda_{2}} \Big[e^{-\lambda_{2} t} - e^{-(\lambda_{1} + \lambda_{K}) t} \Big] + \\ & \frac{\lambda_{K} \lambda_{1}}{(\lambda_{K} + \lambda_{1})(\lambda_{2} - \lambda_{1})} \Big[1 - e^{-(\lambda_{K} + \lambda_{1}) t} \Big] + \frac{\lambda_{K} \lambda_{2}}{(\lambda_{K} + \lambda_{2})(\lambda_{2} - \lambda_{1})} \Big[1 - e^{-(\lambda_{K} + \lambda_{2}) t} \Big] \end{split}$$

So the reliability of system is:

$$R_{s} = e^{-\lambda_{1}t} + \frac{\lambda_{1}}{\lambda_{K} + \lambda_{1} - \lambda_{2}} \left[e^{-\lambda_{2}t} - e^{-(\lambda_{1} + \lambda_{K})t} \right] - \frac{\lambda_{K}\lambda_{2}}{(\lambda_{K} + \lambda_{1})(\lambda_{2} - \lambda_{1})} \left[1 - e^{-(\lambda_{K} + \lambda_{1})t} \right] - \frac{\lambda_{K}\lambda_{2}}{(\lambda_{K} + \lambda_{2})(\lambda_{2} - \lambda_{1})} \left[1 - e^{-(\lambda_{K} + \lambda_{2})t} \right]$$
(15)

Substituting (15)(5)(6) into (4) we obtain:

$$\begin{split} R_{s} &= u c_{j}(R_{s}) \cdot R_{s} \\ &= u c_{j}(R_{s}) \cdot \left[\frac{R_{1}}{u c_{j}(R_{1})} + \frac{\lambda_{1} + \overline{u' c_{j}(R_{1})}}{\lambda_{k} + \lambda_{1} - \lambda_{2} + \overline{u' c_{j}(R_{k})} + \overline{u' c_{j}(R_{1})} - \overline{u' c_{j}(R_{2})}} \right] \\ &\cdot \left[\frac{R_{2}}{u c_{j}(R_{2})} - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{k}}{u c_{j}(R_{k})} \right] - \frac{\left[\lambda_{k} + \overline{u' c_{j}(R_{k})}\right] \cdot \left[\lambda_{2} + \overline{u' c_{j}(R_{2})} + \lambda_{1} + \overline{u' c_{j}(R_{1})}\right]}{\left[\lambda_{2} + \overline{u' c_{j}(R_{k})}\right] \cdot \left[\lambda_{2} + \overline{u' c_{j}(R_{2})} + \lambda_{1} + \overline{u' c_{j}(R_{1})}\right]} \right] \\ &\cdot \left[1 - \frac{R_{1}}{u c_{j}(R_{1})} \cdot \frac{R_{k}}{u c_{j}(R_{k})} \right] - \frac{\left[\lambda_{k} + \overline{u' c_{j}(R_{k})}\right] \cdot \left[\lambda_{2} + \overline{u' c_{j}(R_{2})}\right]}{\left[\lambda_{2} + \overline{u' c_{j}(R_{2})}\right] \cdot \left[\lambda_{2} + \overline{u' c_{j}(R_{2})}\right]} \right] \\ &\cdot \left[1 - \frac{R_{2}}{u c_{j}(R_{2})} \cdot \frac{R_{k}}{u c_{j}(R_{k})} \right] \right] \end{split}$$

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