



Research for Characteristics of Rotating Dipole Acoustic Source in Spatial Acoustic Field

Zhihong Liu

Center of Energy and Environment, The QingDao Technology University
11 FuShun Road, QingDao 266033, China
E-mail: lzhqingdao@163.com

Chuijie Yi

Center of Energy and Environment, The QingDao Technology University
11 FuShun Road, QingDao 266033, China
E-mail: Chuijieyi@vip.163.com

Qian Zhang

Center of Energy and Environment, The QingDao Technology University
E-mail: Qianzhang@163.com

Abstract

Formula for calculating the acoustic pressure of spin dipole acoustic source at any point in space was deduced on the base of frequency-domain solution of turning point acoustic source and acoustic field in free space. Which discussed the acoustic field characteristics during the harmonic dipole source rotating and studied the impact on the acoustic field and acoustic pressure at different source frequency, rotating frequency. Study shows that: dipole acoustic field is of an intense space directivity, the characteristics of acoustic field and acoustic source are closely related.

Keywords: Dipole Acoustic Source, Characteristics of acoustic field, Rotating acoustic source

1. Frequency-domain Solution of Spin Dipole Acoustic Source:

The acoustic pressure radiated by pulsating sphere acoustic source is described as:

$$P(r, t) = \frac{Q(a)}{4\pi r} \frac{-i\omega\rho}{1 - ika} e^{ik(r-a)} e^{-i\omega t} \quad (1)$$

Where: ρ is for air-space density, ω is for circular frequency, k is for wave number.

$Q(a)$ is for the intensity of spherical acoustic source, defined as the surface of the spherical acoustic source multiplied by the speed of the surface. Supposed if the radius of a point acoustic source tends to zero and $\lim_{a \rightarrow 0} Q(a) = Q_0$, Q_0 is for the intensity of the point acoustic source, then formula (1) is:

$$P(r, t) = -i\omega\rho Q_0 \frac{e^{ikr}}{4\pi r} e^{-i\omega t} \quad (2)$$

According to the literature [1,3,4,5], the radiation of acoustic pressure of a rotating Point Source in free space is expressed as:

$$P(X, t) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{Q(\tau)}{r_s} \delta(t - \tau - r_s/c_0) d\tau \quad (3)$$

(X, t) are for the location and time coordinates of observing. (ξ, τ) are for the location and time coordinates of point acoustic source. $Q(a)$ is for the intensity of acoustic source. $r_s = |X - \xi(\tau)|$ is for the distance between point acoustic source and observation point. $(t - \tau - r_s/c_0)$ is for time delay. c_0 is for the velocity of sound in medium. Taking the Fourier transform on both sides of the formula (3) can get frequency-domain solution of arbitrary movement of a

acoustic source and field:

$$P(X, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q(\tau) \frac{e^{ik_0 r_s}}{4\pi r_s} e^{i\omega \tau} d\tau \quad (4)$$

According to the literature [2], taking $\frac{e^{ik_0 r_s}}{4\pi r_s}$ expanded by Legendre and addition principle, supposed initial of point source is $(r_0, \theta_0, \varphi_0)$, the location of rotating is $(r_0, \theta_0, \varphi_0)$, observation location is (r, θ, φ) , $\varphi_0 = \varphi_b + m\Omega$, we can get the frequency domain solution of a rotating point source in free space and the acoustic field g_ω caused by unit intensity and harmonic point source.

$$g_\omega(r/r_0) = \sum_{n=0}^{\infty} ik_0 [(2n+1)/4\pi] j_n(k_0 r_0) h_n(k_0 r) \sum_{m=0}^n \varepsilon_m \frac{(n-m)!}{(n+m)!} P_n^m(0) P_n^m(\theta) \quad (5)$$

$$* \left(e^{im(\varphi_b - \varphi)} \delta(\omega + m\Omega - \omega_i) + e^{-im(\varphi_b - \varphi)} \delta(\omega - m\Omega - \omega_i) \right)$$

through the differential coordinates of the acoustic source, multi-pole field can be generated from the monopole field, so the dipole acoustic field is generated by unit intensity point source at the direction of $Z(\theta = 0)$:

$$g_z = -\partial g_\omega / \partial z = -(\partial g_\omega / \partial r) \cos \theta \quad (6)$$

Taking g_ω into formula (6):

$$g_z(r/r_0) = \sum_{n=0}^{\infty} (-ik_0^2) [(2n+1)/4\pi] \cos \theta H'_n \sum_{m=0}^n \varepsilon_m \frac{(n-m)!}{(n+m)!} P_n^m(0) P_n^m(\cos \theta) \quad (7)$$

$$* \left(e^{im(\varphi_b - \varphi)} \delta(\omega + m\Omega - \omega_i) + e^{-im(\varphi_b - \varphi)} \delta(\omega - m\Omega - \omega_i) \right)$$

Among the formula:

$$H'_n = \begin{cases} j_n(kr_0) h'_n(kr) & r > r_0 \\ j'_n(kr) h_n(kr_0) & r < r_0 \end{cases} \quad \varepsilon_m = \begin{cases} 1/2 & m=0 \\ 1 & m=1 \cdots n \end{cases}$$

$k_0 = \omega / c_0$ is acoustic wave number, ω is for acoustic circular frequency, Ω is for rotating circular frequency, ω_i is for source vibration frequency, Similarly, the dipole acoustic field is generated by unit point source at the direction of φ_0 dipole :

$$g_\varphi = -\frac{1}{r_0} \frac{\partial g_\omega}{\partial \varphi}$$

Taking g_ω into the formula above:

$$g_\varphi(r/r_0) = \sum_{n=0}^{\infty} (-ik_0 / r_0) [(2n+1)/4\pi] j_n(k_0 r_0) h_n(k_0 r) * \quad (8)$$

$$\left\{ P_n(0) P_n(\theta) \delta(\omega - \omega_i) - \sum_{m=1}^n \frac{(n-m)!}{(n+m)!} P_n^m(0) P_n^m(\theta) \right.$$

$$\left. * im \left[e^{im(\varphi_b - \varphi)} \delta(\omega + m\Omega - \omega_i) - e^{-im(\varphi_b - \varphi)} \delta(\omega - m\Omega - \omega_i) \right] \right\}$$

formula(7) and (8) set up the acoustic field frequency-domain solution of a rotating dipole source respectively. In order to discuss the directivity, the above directive function of the acoustic field $D(\theta, \varphi)$ was set up according to the definition of far-field directivity of the acoustic field:

$$D_z(\theta, \varphi) = re^{-ikr} g_z(r, \theta, \varphi), \quad D_\varphi(\theta, \varphi) = re^{-ikr} g_\varphi(r, \theta, \varphi) \quad (9)$$

2. The Acoustic Field Characteristics of Rotating Dipole Sound Source

In order to compare with the characteristics of the monopole rotating sound source, relevant parameters were selected according to the literature [2], namely

$$\varphi_b = 0, \varphi = -\pi/4, \omega_i = 6800 \text{ rad/s}, 2300 \text{ rad/s}, \Omega = 560 \text{ rad/s}, 112 \text{ rad/s}, \omega = \omega_i + k\Omega.$$

Here k is for arbitrary integer, set $k = -5, \dots, 5$. ω_0 for rotating circular frequency substitute for Ω , Y substitutes for θ , valued $0, \pi/18, \pi/6, \pi/3, \pi/2$.

2.1 The Characteristics of Far-Field Acoustic Pressure

For far field $r = 2$, $r_0 = 0.3$, we can calculate the value changes of acoustic pressure with the frequency and directivity. harmonic distribution of acoustic pressure as Fig1 and Fig. 3. Acoustic pressure amplitude distribution along the observation angle as Fig2 and Fig. 4. We can conclude from figures: ① Fig1(a)~Fig1(d), far-field acoustic pressure amplitude increases with the auto-oscillation frequency increasing in direction of Z and decreases with the observed angle increasing, the scope of harmonic expands. ② There is only fundamental frequency in the direction of $\theta = 0$, namely the direction of rotation axis and acoustic pressure reaches the maximum amplitude; There is not harmonic distribution in the direction of $\theta = \pi/2$ namely plane of rotation. ③ Fig1(e), with the increasing of rotation frequency, it shows Doppler shift apparently, and the changes of rotation frequency have a greater impact on the harmonic distribution at a larger observation angle. ④ Fig3 (a)~(d), far-field acoustic pressure amplitude is impacted greatly by the auto-oscillation frequency, the scope of whose harmonic expands with the observed angle increasing in direction of Y. ⑤ There is only fundamental frequency in the direction of $\theta = 0$, namely the direction of rotation axis; Harmonic distribution is the most abundant in the direction of $Y = \pi/2$ namely plane of rotation. ⑥ The frequency change greatly impacts on the harmonic distribution, consistent with it in the direction of Z. ⑦ Directivity aspect, there is an intensive space directivity in the direction of Z and Y. With the decreasing of auto-oscillation frequency and rotation frequency, there is more harmonic deviated from the fundamental frequency distributed near the plane of the observed angle $\theta = \pi/2$, range from 300 to 900 in the direction of Z. With the increasing value of k, it points to $\theta = 0$ gradually, and when $k=0$, it points to $\theta = 0$ intensively. The changes of auto-oscillation frequency and rotation frequency impacts on directivity mildly in the direction of Y, and there is more harmonic deviated from the fundamental frequency distributed near the plane of the observed angle $\theta = \pi/2$, With the increasing value of k, it points to $\theta = 0$ gradually, and when $k=0$, it points to $\theta = 0$ intensively.

2.2 The Characteristics of Near-Field Acoustic Pressure:

For near-field $r = 0.4$, $r_0 = 0.3$, the other parameters are the same as far-field, and discusses its acoustic characteristics in accordance with the above points similarly. It can be seen from Fig1.a to Fig1.d that near-field affects the harmonic distribution acoustic pressure amplitude mildly in the case of small θ , but it impacts intensively in the case of θ close to $\pi/2$ in the direction of Z. With the increasing value of θ , the acoustic pressure amplitude decreases gradually, but the scope of harmonic distribution expands. Reducing the rotation frequency affects the acoustic pressure amplitude and the scope of harmonic distribution mildly. Reduction of the auto-oscillation frequency decreases the acoustic pressure amplitude and the scope of harmonic distribution remarkable. It can be seen from Fig1e for near-field: ① When the rotation frequency is below 100Hz, the changes of it does not affect the distribution of harmonics, but there will be frequency shift when the value of θ is big. ② When the rotation frequency is above 1000Hz, the changes of it affects the distribution of harmonics intensively, and there will be Doppler frequency shift remarkable. ③ There is an intensive directivity in the direction of $\theta = 0$ and harmonic distribution is abundant near the plane of $\theta = \pi/2$.

It can be seen from Fig3.a to Fig3.d that near-field affects the harmonic distribution acoustic pressure amplitude mildly in the case of big θ , but it affects intensively in the case of θ close to $\pi/2$ in the direction of Y. With the increasing value of θ , the acoustic pressure amplitude increases gradually, but it affects the scope of harmonic distribution mildly. Reducing the rotation frequency affects the acoustic pressure amplitude and the scope of harmonic distribution mildly. Reduction of the auto-oscillation frequency decreases the acoustic pressure amplitude and the scope of harmonic distribution remarkable. It can be seen from Fig1e for near-field: ① When the rotation frequency is below 100Hz, the changes of it does not affect the distribution of harmonics. ② When the rotation frequency is higher, the changes of it affects the distribution of harmonics intensively, and there will be Doppler frequency shift remarkable. ③ There is an directivity in the direction of $\theta = \pi/2$ and harmonic distribution is abundant in the plane of $\theta = \pi/2$. when $k=0$, it points to $\theta = 0$.

3. Conclusion

From the results of the study we can get the key point of the rotating source acoustic traits.

- 1) The spatial dipole acoustic field is of an intense space directivity, specially in the direction of $\theta = 0$ and $\theta = \pi/2$.
- 2) The characteristics of acoustic field is closely depended on the traits of acoustic source.
- 3) The auto-oscillation frequency and rotation frequency of source impacts the directivity mildly in the direction of $\theta = 0$ and $\theta = \pi/2$.

References

- A. Zinoviev, D.A. Bies. (2004). On acoustic radiation by a rigid object in a fluid flow. *Journal of Sound and Vibration*. PP. 535–548.
- C. Testa, S. Ianniello, G. Bernardini, M. Gennaretti. (2007). Sound scattered by a helicopter fuselage in descent flight

condition, AIAA Paper13th AIAA/CEAS Aeroacoustics Conference.

Ju Hongbin. (1995). The Analysis of the Near-Field Acoustic Characteristics about the rotating sound source. *Journal of Aerospace Power*. PP. 131-134.

M. Gennaretti, C. Testa. (2008). A boundary integral formulation for sound scattered by elastic moving bodies. *Journal of Sound and Vibration* 314. PP. 712-737.

WuJiuhui, Chen Hualing, HuangXieqIng. (2000). Acoustic Solution of RotatingPoint Source in Frequency Domain. *JOURNAL OF XI AN JIAO'TONG UNIVERSITY*.PP.71-75.

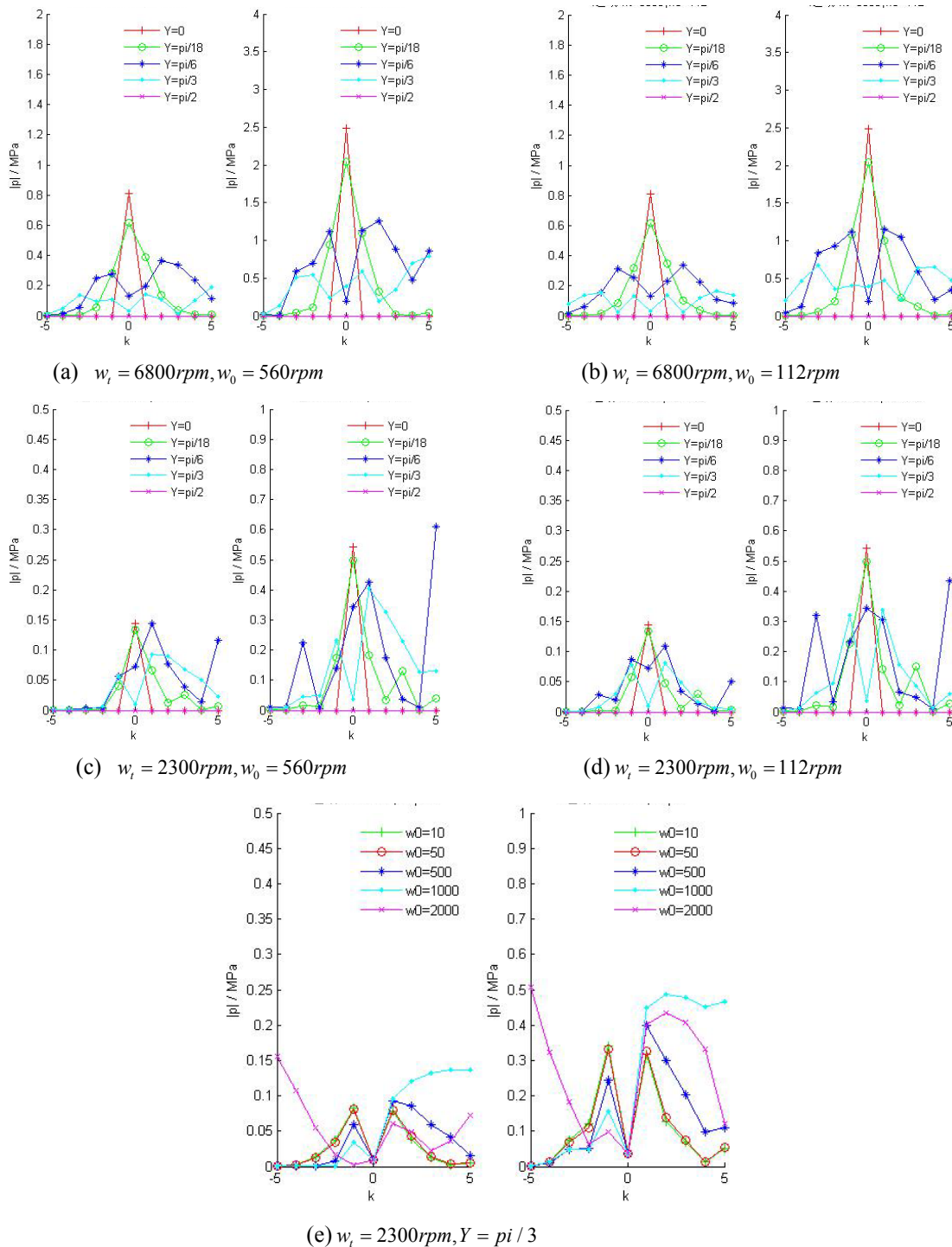


Figure 1. Acoustic pressure harmonic distribution of the far-field on the left and the near-field on the right in the direction of Z

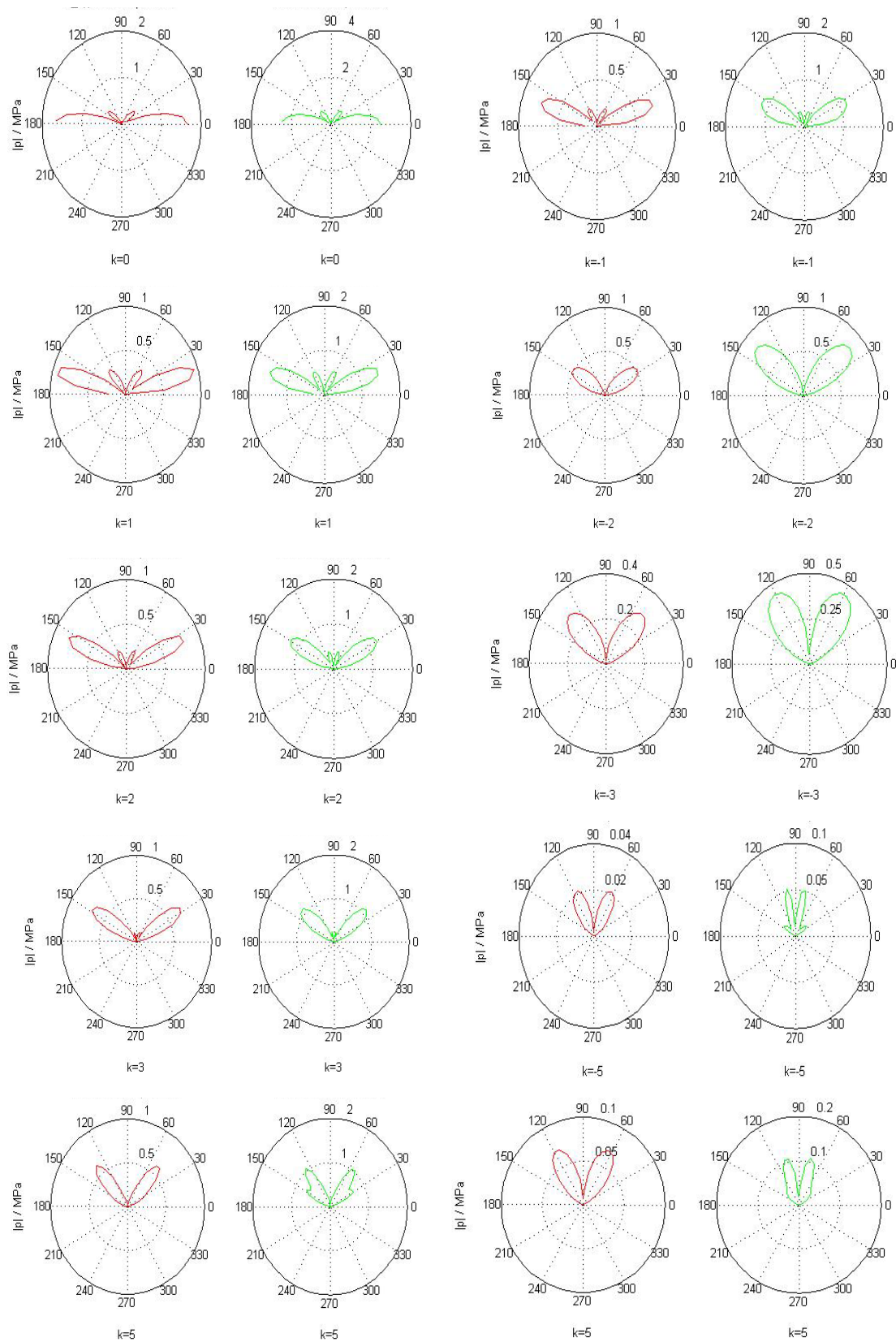


Figure 2. Acoustic pressure amplitude distribution along the observed angle of the far-field on the left and the near-field on the right in the direction of Z at the rotate speed $w_i = 6800rpm$, $w_0 = 560rpm$

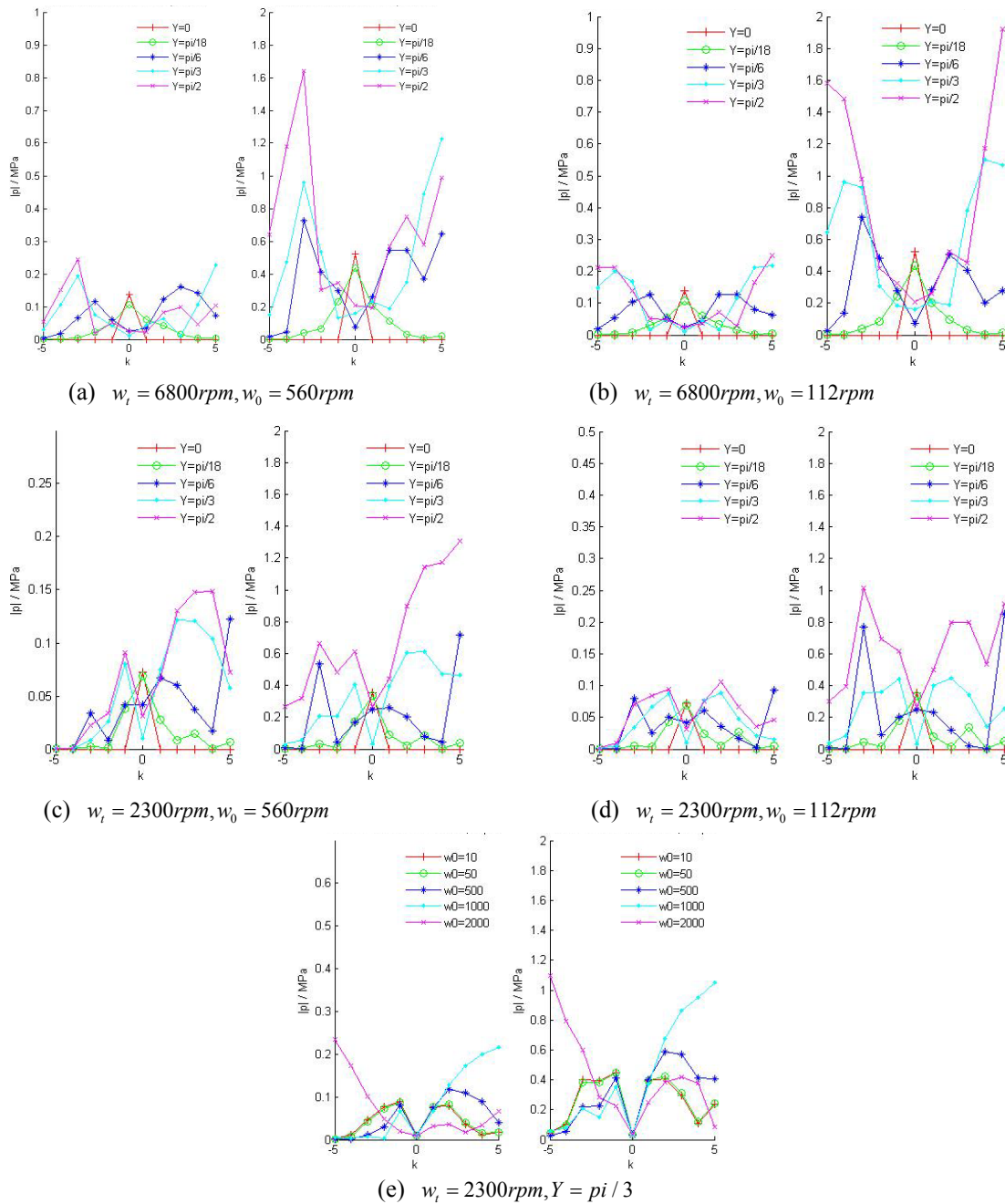
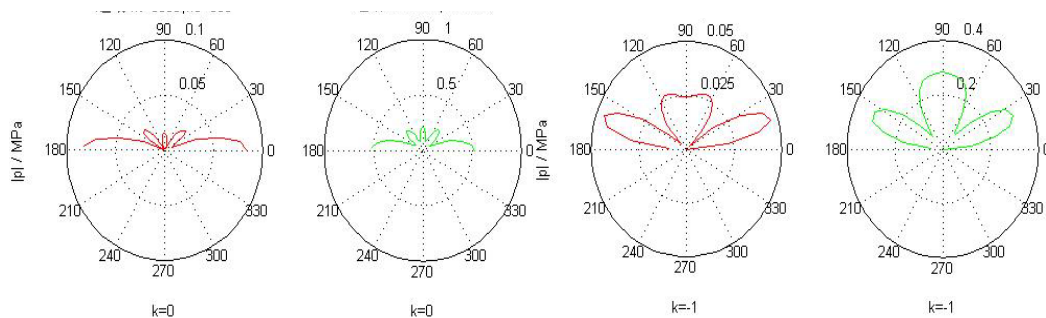


Figure 3. Acoustic pressure harmonic distribution of the far-field on the left and the near-field on the right in the direction of Y



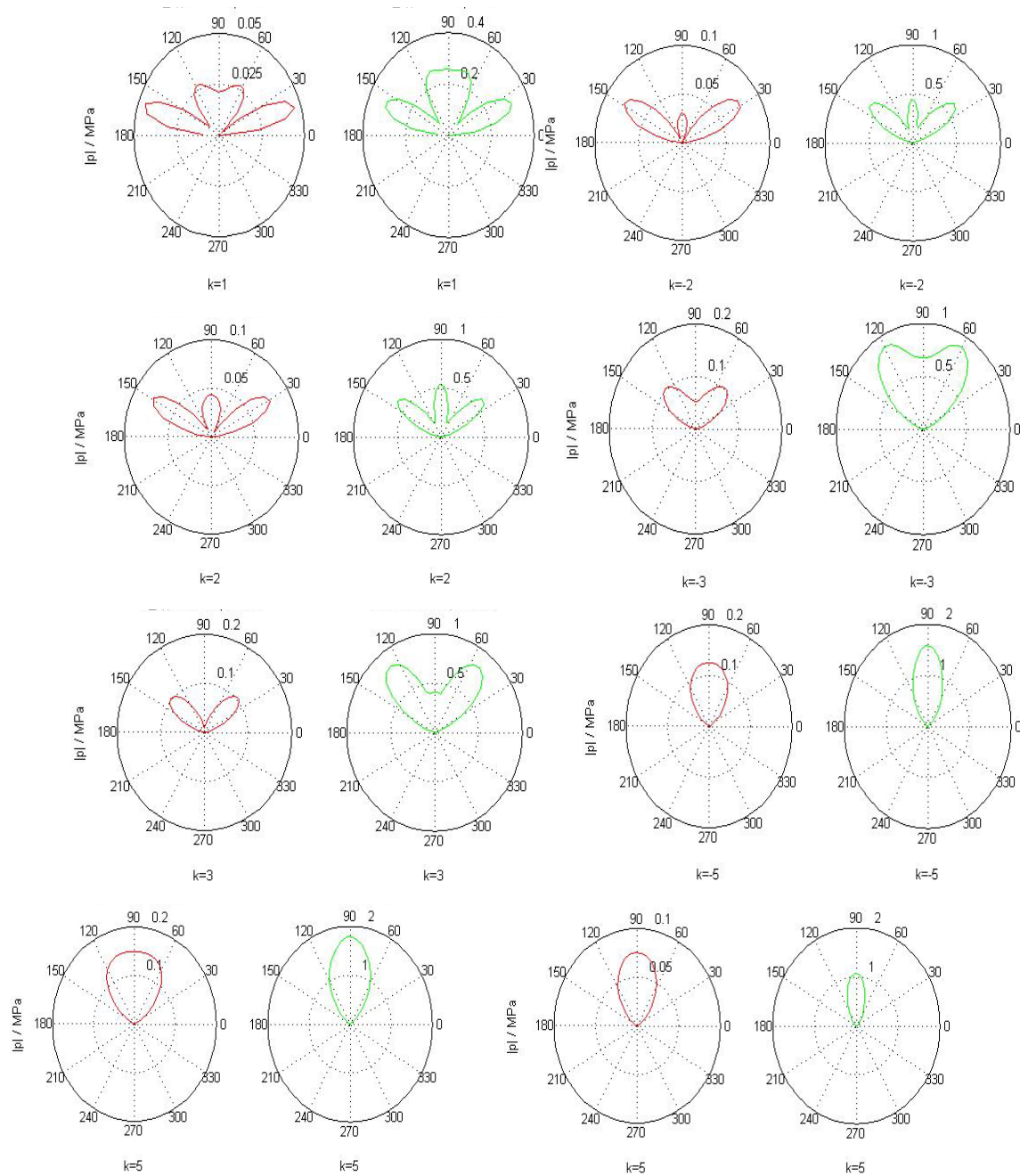


Figure 4. Acoustic pressure amplitude distribution along the observed angles of far-field on the left and the near-field on the right in the direction of Y at the rotate speed $w_i = 6800rpm$, $w_0 = 560rpm$