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A New Way to Determine the Multinomial Divisibility in the Rational Coefficient Field

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Abstract

In this article, we utilize the corresponding parallelisms between rational coefficient multinomial and integral coefficient multinomial and between the integral coefficient multinomial and positive rational number to translate the problem of divisibility of the rational coefficient multinomial into the problem of multiplication cross operation among positive rational numbers. This method can determine the divisibility and obtain the quotient.

Keywords: Rational coefficient multinomial, Integral coefficient multinomial, Divisibility, Prime factorization

1. Introduction

As we know, any one positive integer m can be implemented by prime factorization and its formula of prime factorization is

$$m = 2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots p_n^{\alpha_n} = \prod_{k=1}^{n} p_k^{\alpha_k}.$$

Where, $\alpha_k \in N(k=1,2,\cdots,n)$ and $p_k(k=1,2,\cdots,n)$ are n prime numbers which are different each other and $p_i < p_{i+1}(i=1,2,\cdots,n-1)$.

In the same way, to any positive rational number q, we can obtain same conclusion.

Lemma 1: Any one positive rational number q can be denoted as

$$q = 2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots p_n^{\alpha_n} = \prod_{k=1}^{n} p_k^{\alpha_k}.$$

Where, $\alpha_k \in Z(k=1,2,\cdots,n)$ and $p_k(k=1,2,\cdots,n)$ are n prime numbers which are different each other and $p_i < p_{i+1}(i=1,2,\cdots,n-1)$.

Lemma 2: Suppose $Z[x]$ is the set of integer coefficient multinomial, $Q[x]$ is the set of rational coefficient multinomial, and to any $f(x) \in Q[x]$, the integer k must exist to make $kf(x) \in Z[x]$, so the sufficient and necessary condition that $f(x)$ is reducible on Q is that $kf(x)$ can be reducible on Z.

2. The multiplication cross operation of positive rational numbers

Definition 1: Suppose the positive rational numbers $q_1 = 2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots p_n^{\alpha_n} = \prod_{k=1}^{n} p_k^{\alpha_k}$ and

$$q_2 = 2^{\beta_1}3^{\beta_2}5^{\beta_3}\cdots p_n^{\beta_n} = \prod_{k=1}^{n} p_k^{\beta_k}.$$

Where, $\alpha_k, \beta_k \in N(k=1,2,\cdots,n)$ and $p_k(k=1,2,\cdots,n)$ are n prime numbers which are different each other and $p_i < p_{i+1}(i=1,2,\cdots,n-1)$.

So the multiplication cross operation between q_1 and q_2 is

$$q_1 \otimes q_2 = (2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots p_n^{\alpha_n}) \otimes (2^{\beta_1}3^{\beta_2}5^{\beta_3}\cdots p_n^{\beta_n}) = \prod_{k=1}^{n} p_k^{\sum_{i+j=k}\alpha_i\beta_j}.$$

The summation cross operation between q_1 and q_2 is

$$q_1 \oplus q_2 = (2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots p_n^{\alpha_n}) \oplus (2^{\beta_1}3^{\beta_2}5^{\beta_3}\cdots p_n^{\beta_n}) = \prod_{k=1}^{n} p_k^{\alpha_k-\beta_k}.$$

3. Main conclusions

To the integer coefficient multinomial $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n(a_n\neq 0)$, we can adopt the method $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n(a_n\neq 0)$ to correspond with $q=2^{a_0}3^{a_1}5^{a_2}\cdots p_n^{a_n}$.

Where, $\alpha_k \in Z(k=0,1,2,\cdots,n)$ and $p_k(k=0,1,2,\cdots,n)$ are n+1 prime numbers which are different each other and $p_i < p_{i+1}(i=1,2,\cdots,n-1)$ to make any integer coefficient multinomial $f(x)$ correspond with the positive rational number q , and obviously, this parallelism is the parallelism one by one. So we have:

Theorem 3: Suppose $Z[x]$ is the set of integer coefficient multinomial and Q^+ is the collectivity of positive rational numbers, so $\Phi: Z[x]\to Q^+$ exists to make any $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n(a_n\neq 0)\in Z[x]$, and $\Phi(f(x))\in Q^+$. So Φ is the isomorphic mapping from $<Z[x],+,\times>$ to $<Q^+,\oplus,\otimes>$, i.e. $<Z[x],+,\times>\cong<Q^+,\oplus,\otimes>$.

Deduction 4: Suppose $Z[x]$ is the set of integer coefficient multinomial, Q^+ is the collectivity of positive rational numbers, and Φ is the isomorphic mapping from $<Z[x],+,\times>$ to $<Q^+,\oplus,\otimes>$,

(1) If to any $f(x),g(x)(g(x)\neq 0)\in Z[x]$, $h(x),r(x)\in Z[x]$, exists and makes $f(x)=g(x)h(x)+r(x)$, where, $r(x)=0$ or $\partial^0(r(x))\le\partial^0(g(x))$ (mark $\Phi(f(x))=q_f,\Phi(g(x))=q_g,\Phi(h(x))=q_h,\Phi(r(x))=q_r,$) so $q_f=(q_g\otimes q_h)\oplus q_r$.

(2) In (1), the sufficient and necessary condition of $g(x)\mid f(x)$ is $q_r=1$.

(3) In (1), the sufficient and necessary condition of $g(x)\mid f(x)$ is $q_f=q_g\otimes q_h$.

4. Example

Example: Determine whether $g(x)=x^2-3x+2$ can divide $f(x)=x^3-4x^2+5x-2$ exactly?

Solution: According to Theorem 3,

The corresponding positive rational number of $f(x)=x^3-4x^2+5x-2$ is $2^{-2}3^5 5^{-4}7^1$, and the corresponding positive rational number of $g(x)=x^2-3x+2$ is $2^2 3^{-3}5^1 7^0$.

Suppose $g(x)\mid f(x)$, so $h(x)\in Z[x]$ exists and makes $f(x)=g(x)h(x)$.

Suppose the corresponding positive rational number of $h(x)$ is $2^{\beta_0}3^{\beta_1}5^{\beta_2}7^{\beta_3}$, so according to Deduction 4, $2^2 3^{-3}5^1\otimes 2^{\beta_0}2^{\beta_1}2^{\beta_2}2^{\beta_3}=2^{-2}3^5 5^{-4}7^1$.

According to the definition of multiplication cross, we can get

$$\begin{cases}\beta_0=-1\\ \beta_1=1\\ \beta_2=0\\ \beta_3=0\end{cases} \text{ from } \begin{cases}2\beta_0=-2\\ 2\beta_1-3\beta_0=5\\ \beta_0+2\beta_2-3\beta_1=-4\\ 2\beta_3-3\beta_2+\beta_0=1\end{cases},$$

so $h(x)=x-1$, i.e. $g(x)\mid f(x)$.

About the divisibility of rational coefficient multinomial, we only need translate rational coefficient multinomial into integer coefficient multinomial, i.e. the reducible problem of rational coefficient multinomial in the rational number field is coincident with the reducible problem of integer coefficient multinomial in the integer loop.

5. Conclusions

The method in the article is only a sort of theoretic method, and it is very difficult to use it to determine the divisibility of multinomial in practice, and this new way can also be used to discuss other problems about multinomial.

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