

# Two Vehicular Headways Time Dichotomic Models

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## Abstract

Vehicular time headway is an important parameter which characterises traffic flows and a great number of Road Engineering applications utilise time headway statistical models (for example the distribution of time headways contributes to the determination of the capacity, delay and queue at intersection). In this paper two dichotomic laws are presented: the log-normal shifted negative exponential distribution (ENTLN) and the Pearson's type III shifted negative exponential distribution (ENTPIII). These distributions can be useful in studying partially conditioned flows. A complete method for the calibration of these models is then given and is an alternative to the method of moments. Finally, a comparative analysis is made between the two models proposed and others available in literature on the basis of a common significant set of measured time headways.

**Keywords:** dichotomic vehicular time headway, statistical models, stationary traffic flow conditions

## 1. Introduction

The fundamental, practical utility of the analyses of time headway in road engineering is well known (e.g. Griffiths & Hunt, 1991; Sullivan & Troutbeck, 1994; Luttinen, 2003; Rossi, 2012): the distribution of time headways contributes to the determination, for instance, of the characteristics of overtaking manoeuvres, of merging and intersection, whereas the capacity of many elements of the infrastructural system is generally ruled by minimum vehicular time headway and by the distribution of time headways under saturation conditions. These circumstances have always motivated the theoretical knowledge of the vehicular time headways and in particular, the research of realistic models in conditions of uncertainty to use in simulation procedures, in traffic control and in functional road planning. This explains the attention towards the issue ever since the beginning of traffic studies, from the early thirties until the eighties (for a first review of the model approaches suggested see Gerlough & Huber, 1975; VV. AA., 2001; for a calibration technique see Luttinen, 1996). Nowadays, the vehicular time headways is back again of great interest in the field of road engineering applications (Ha et al., 2010, 2012; Rossi, 2012). The present work is thus contributing to this renewed interest about vehicular headways.

In this paper, after a brief introduction to the main laws used at present in the field of headways, we bring forward a proposal regarding the identification and calibration of dichotomic probability density functions, useful when vehicular flows are a casual succession of free-moving and constrained vehicles. The technique used to estimate parameters of the models elaborated and discussed here resorts to a different procedure from the one comparing theoretical and statistical moments. With this method we identify laws in a more efficient and rapid way, obtaining the best fit through preliminary hypotheses with regard to some parameters (carried out for instance according to information available in literature) and through a correct combination of the method of moments with a procedure aiming at maximizing conformity. The models here proposed and the techniques used for the evaluation of the parameters seem to be original and of a certain practical interest in the study of vehicular headways and in the application of the traffic flow theory.

## 2. Dichotomic Models of Vehicular Time Headway

In stationary traffic flow conditions, the description of vehicular headway  $\{\tau\}$  as a random variable associated to the process section can derive from different laws, according to traffic characteristics. We discuss the probability density functions that are mostly used in applications: negative exponential, shifted negative exponential, Erlang's distribution and the Log-normal distribution (Luttinen, 1996). The models mentioned

above are known as non-dichotomic (or composite) because they do not allow a characterization in probability of headways that compete to free-moving and constrained vehicles. Further non-dichotomic models are available in literature along with the ones mentioned at the beginning of this paper: among these the most common is the Gamma probability density function, a more specific aspect of Pearson's type III distribution (May, 1990) (as are the negative exponential, the shifted negative exponential and Erlang's distribution). As regards the prevailing field of application of the probability density functions mentioned up to here, we would like to mention that (Gerlough & Huber, 1975):

- the negative exponential and shifted negative exponential probability density functions are used for flows with a volume that does not exceed 400-500 vehicles per hour;
- Erlang's law is suitable for short headways, characterized by low frequency as it is for flows with a volume above 400-500 vehicles per hour;
- the Log-normal probability density function is suitable for the interpretation of the traffic flow conditions that narrow down most of the determinations of  $\{\tau\}$  to a small number of classes concerning a few seconds.

Referring to dichotomic laws (or composite), in the following chapter we provide the initial elements with the specification of the hyper-exponential probability density function. In general, composite functions are linear combinations of probability density functions (otherwise called "distribution mixture models" in statistics); the flow is considered to form, in casual succession, with vehicles in platoon and vehicles that do not interfere. Indicating the percentages with  $P_f$  and  $P_c$ , respectively free-moving and constrained vehicles, since there are no vehicles that do not belong to one of the two, we have that

$$P_f + P_c = 1 \quad (1)$$

Furthermore, if  $f_{T_f}$  and  $f_{T_c}$  are the probability density functions of  $\{\tau\}$ , respectively for free-moving and constrained vehicles, the dichotomic law is written as follows:

$$f_T(\tau) = P_f f_{T_f}(\tau) + P_c f_{T_c}(\tau) \quad (2)$$

Therefore as regards the  $F_T$  distribution function we have

$$F_T(\tau) = P_f F_{T_f}(\tau) + P_c F_{T_c}(\tau) = P_f \int_{-\infty}^{\tau} f_{T_f}(\xi) d\xi + P_c \int_{-\infty}^{\tau} f_{T_c}(\xi) d\xi \quad (3)$$

Distributions  $f_{T_f}$  and  $f_{T_c}$  are called components of the dichotomic law. The mathematical expectation and the variance of such a distribution are:

$$\mu_T = P_f \mu_{T_f} + P_c \mu_{T_c} \quad (4)$$

$$\sigma_T^2 = P_f \sigma_{T_f}^2 + P_c \sigma_{T_c}^2 + P_f P_c (\mu_{T_f} - \mu_{T_c})^2 \quad (5)$$

having indicated with  $\mu_{T_f}$ ,  $\mu_{T_c}$ ,  $\sigma_{T_f}^2$  and  $\sigma_{T_c}^2$ , the mathematical expectations and variances of the distributions  $f_{T_f}$  and  $f_{T_c}$ .

$k$  centered moments are expressed as

$$M_T^{(k)} = P_f M_{T_f}^{(k)} + P_c M_{T_c}^{(k)} \quad (6)$$

where  $M_{T_f}^{(k)}$  and  $M_{T_c}^{(k)}$  are  $k$  centred moments of distributions  $f_{T_f}$  and  $f_{T_c}$ . Along with the hyper-exponential distribution previously mentioned, we also refer to May (May, 1990) and Schuhl's (Gerlough et al., 1971) composite models in the applications. May's probability density function proved to be able to interpret flows with constraint between vehicles such that the percentage of vehicles in platoon is strongly predominant over free-moving vehicles. Schuhl's distribution is an evolution of the hyper-exponential model and does not seem to offer particular advantages in processing empirical data and in applications. As regards expressions, calibration and further details regarding May and Schuhl's probability density functions refer to Gerlough and Huber (1975).

### 3. The Log-normal Shifted Negative Exponential Distribution (ENTLN)

In this and in the following paragraphs, we examined 14570 measurements of vehicular time headway dealt with in (May, 1990), where they are gathered and presented as functions of  $Q$  traffic volumes of the flows observed.

According to the descriptive and predictive abilities demonstrated by shifted negative exponential and log-normal distributions, we suggest that the vehicular time headway of free-moving vehicles is distributed according to the first probability density function and that the vehicular headway of constrained vehicles is concordant with the second function. For this reason the dichotomic function expressed by (2) is, a composite Log-normal shifted negative exponential distribution. Therefore,

$$f_{\tau_1}(\tau) = 0 \quad \text{for } \tau < \alpha$$

$$f_{\tau_1}(\tau) = \lambda e^{-\lambda(\tau-\alpha)} = \frac{1}{\mu_{\tau_1} - \alpha} \exp\left(-\frac{\tau - \alpha}{\mu_{\tau_1} - \alpha}\right) \quad \text{for } \tau \geq \alpha \quad (7)$$

$$f_{\tau_c}(\tau) = \frac{1}{\tau \zeta \sqrt{2\pi}} \exp\left[-\frac{(\ln(\tau) - \nu)^2}{2\zeta^2}\right] \quad (8)$$

where with  $\nu$  and  $\zeta$ , we have the mathematical expectation and standard deviation of the random variable  $\ln(\tau)$ . From (3) we have that

$$F_{\tau_1}(\tau) = 0 \quad \text{for } \tau < \alpha$$

$$F_{\tau_1}(\tau) = 1 - e^{-\lambda(\tau-\alpha)} \quad \text{for } \tau \geq \alpha \quad (9)$$

$$F_{\tau_c}(\tau) = \Phi^*\left(\frac{\ln(\tau) - \nu}{\zeta}\right) \quad (10)$$

where  $\Phi^*$  is the distribution function of the normal reduced variable. As regards mathematical expectations and variations of the component probability density functions we have

$$\mu_{\tau_1} = \frac{1}{\lambda} + \alpha \quad (11)$$

$$\mu_{\tau_c} = e^{\nu + \frac{\zeta^2}{2}} \quad (12)$$

$$\sigma_{\tau_1}^2 = \frac{1}{\lambda^2} \quad (13)$$

$$\sigma_{\tau_c}^2 = e^{2\nu + \zeta^2} (e^{\zeta^2} - 1) \quad (14)$$

In this case the parameters cannot be immediately expressed as a function of  $\bar{\tau}$  and  $s$ , that is to say of the sampling mean value and standard deviation. There are six unknown quantities from measured time headways:  $\nu$ ,  $\lambda$ ,  $\alpha$ ,  $\zeta$ ,  $P_l$ ,  $P_c$ . Following the consolidated procedure for the estimate, the equations we must refer to are estimation method of moments obtained by the equality of moments of theoretical distributions and those determined by traffic measurements. In this way, due to the structure of the relations involved, the research of solutions can be burdensome, also resorting to a numerical procedure and not always efficient in identifying the most suitable probability density function for the data. We here present a different calculation approach. We impose equality of the mean value and of the variation of the theoretical and experimental distributions, as well as conserve the complementarity to one of the two percentages of free-moving and constrained vehicles ( $P_c = 1 - P_l$ ). Indicated for the sake of brevity,  $\mu_{\tau_1}$ ,  $\mu_{\tau_c}$ ,  $\sigma_{\tau_1}$ ,  $\sigma_{\tau_c}$ , with  $\tau_1$ ,  $\tau_c$ ,  $\sigma_1$ ,  $\sigma_c$ , we express  $P_l$ ,  $P_c$ ,  $\sigma_1$ ,  $\sigma_c$ , as a function of  $\tau_1$ ,  $\tau_c$ . Therefore:

$$P_l = \frac{\bar{\tau} - \tau_c}{\tau_1 - \tau_c} \quad (15)$$

$$P_c = 1 - P_l = \frac{\tau_1 - \bar{\tau}}{\tau_1 - \tau_c} \quad (16)$$

$$\sigma_1^2 = \tau_1^2 \quad (17)$$

$$\sigma_c^2 = \frac{\tau_l - \tau_c}{\tau_l - \bar{\tau}} \left[ s^2 - \left( \frac{\bar{\tau} - \tau_c}{\tau_l - \tau_c} \right) \tau_l^2 - \left( \frac{\tau_l - \bar{\tau}}{\tau_l - \tau_c} \right) \left( \frac{\bar{\tau} - \tau_c}{\tau_l - \tau_c} \right) (\tau_l - \tau_c)^2 \right] \tag{18}$$

The calibration of a Log-normal shifted negative exponential distribution follows the estimate of parameters  $\tau_l$ ,  $\tau_c$  and  $\alpha$ . The value of  $\alpha$  can be assigned considering that it represents the shifting time headway of the negative exponential probability density function and can be chosen according to information already available. The estimate of  $\tau_l$  and  $\tau_c$  results after verifications through opportune criterion of the fit of the law, carried out while the same two unknown residual quantities  $\tau_l$  and  $\tau_c$  change. Further in detail:

- $\tau_l$ ,  $\tau_c$  are varied with constant increments  $\Delta\tau_l$ ,  $\Delta\tau_c$  respectively between  $\tau_{lmin}$  and  $\tau_{lmax}$  and between  $\tau_{cmin}$  and  $\tau_{cmax}$ , obtaining  $i=1,2,\dots,n$  determinations of  $\tau_{li}$  of  $\tau_l$  and  $j=1,2,\dots,m$  determinations of  $\tau_{cj}$  of  $\tau_c$ ;
- for each possible couple  $(\tau_{li}, \tau_{cj})$  that can be obtained with the values of  $\tau_l$ , and  $\tau_c$  we calculate with (15) and (16) the percentiles  $P_{li,j}, P_{ci,j}$  and the calibration vector  $(P_{li,j}, P_{ci,j}, \tau_{li}, \tau_{cj}, \alpha)$  with which we completely specify (as a function of  $\tau_{li}, \tau_{cj}$ ) an expression  $(ENTLN)_{ij}$  of the Log-normal shifted negative exponential distribution model;
- if a vector  $(P_{li,j}, P_{ci,j}, \tau_{li}, \tau_{cj}, \alpha)$  presents values of  $P_{li,j}, P_{ci,j}$  that do not respect the constraints (instantly comprehended)  $P_l \in [0,1]$ ;  $P_c \in [0,1]$  it cannot be accepted; in the same way, it is rejected if, as regards the times that appear, it results  $\tau_{li} \leq \tau_{cj}$ , that is to say, if free-moving vehicle headways are not higher than those of constrained vehicles.

Now that calibration solutions have been limited, (that is to say the couples  $(\tau_{li}, \tau_{cj})$  on which these solutions depend), for each possible specification of the Log-normal shifted negative exponential model  $(ENTLN)_{p,q}$  (with  $p=1,2,\dots,h \leq m$ ;  $q=1,2,\dots,k \leq n$ ) we carry out a statistical test to the same group of data that must be interpreted, determining the relative value  $m_{p,q}$  of the statistics of the *test*, therefore, an adaptation matrix  $M = \{m_{p,q}\}$ . To summarize, with the shifted negative exponential procedure articulated in such a way, we can consider the elements of matrix  $M$  as measurements of conformity only of admissible distributions, whatever their grade of fit to the data processed. From the  $m_{p,q}$  statistics of  $M$  that indicates the highest fit degree, we individuate the vector of the solution  $(P_{lp,q}, P_{cp,q}, \tau_{lp}, \tau_{cq}, \alpha)$  that resolves the Log-normal as regards the information considered. Log-normal shifted negative exponential distributions in Figures 1 to 4 have been determined according to the procedure described starting from statistical distributions  $\{\tau\}$  of an experimental nature, regarding samples by no means numerous, in May (1990).

For free-moving and constrained vehicles percentages and free-moving and constrained mean vehicular headway for log-normal shifted negative exponential distribution (ENTLN) see Figure 5 and Figure 6.

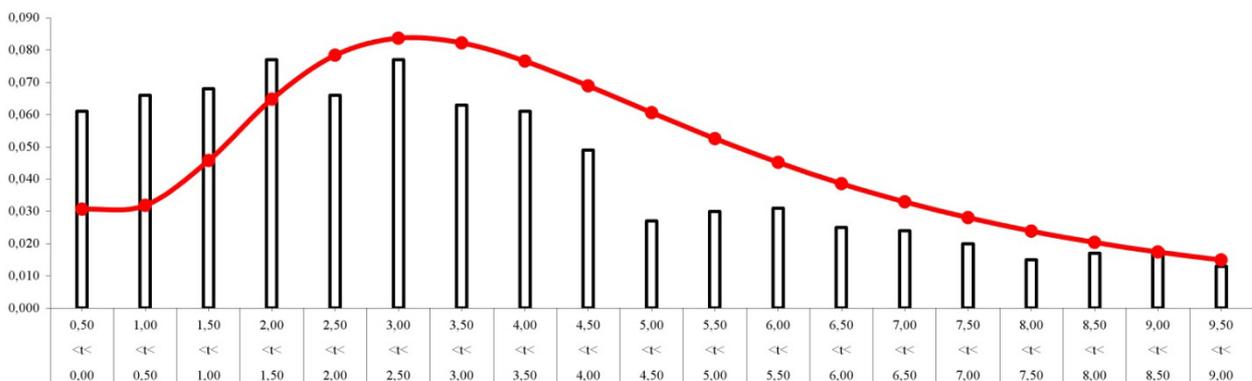


Figure 1. Probabilities related to headway classes: measured distribution (1320 observations) and ENTNLN distribution for traffic volume  $Q=10-14$  veh/min= $600-840$  veh/h

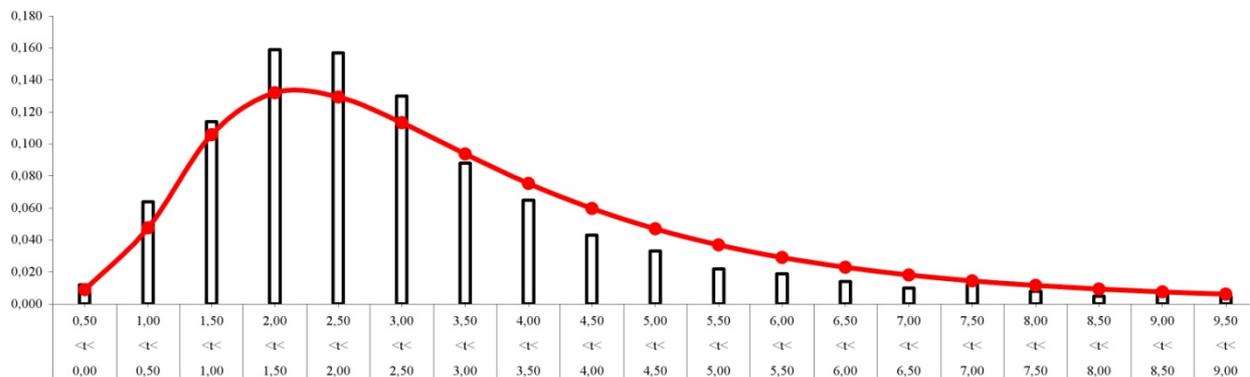


Figure 2. Probabilities related to headway classes: measured distribution (3432 observations) and ENTLN distribution for traffic volume  $Q=15-19$  veh/min= $900-1140$  veh/h

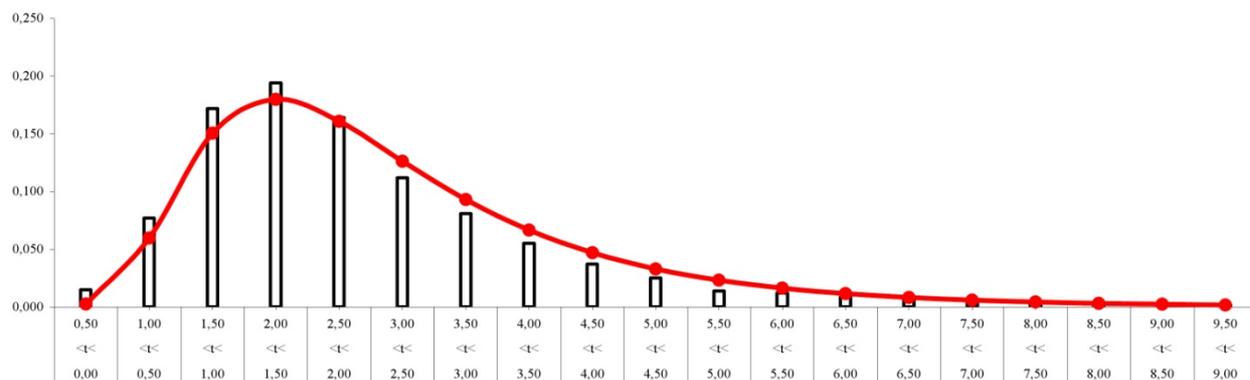


Figure 3. Probabilities related to headway classes: measured distribution (6327 observations) and ENTLN distribution for traffic volume  $Q=20-24$  veh/min= $1200-1440$  veh/h

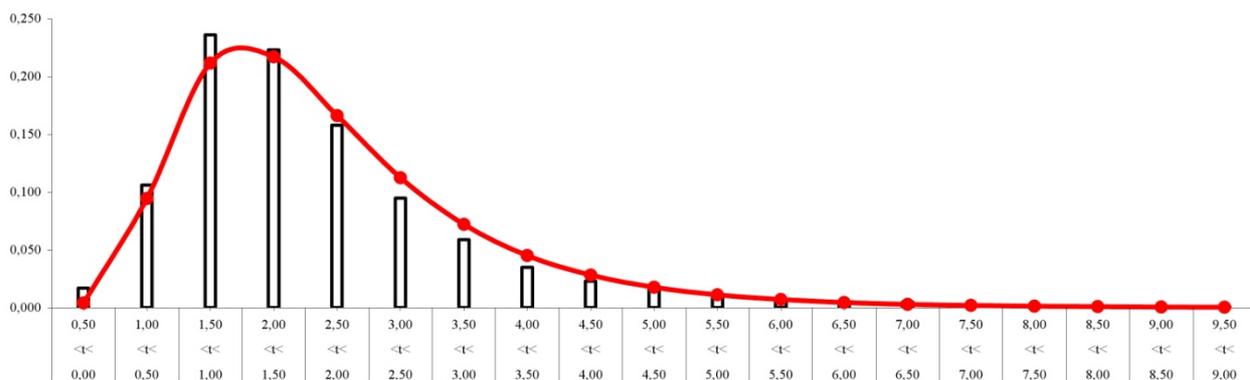


Figure 4. Probabilities related to headway classes: measured distribution (3491 observations) and ENTLN distribution for traffic volume  $Q=25-29$  veh/min= $1500-1740$  veh/h

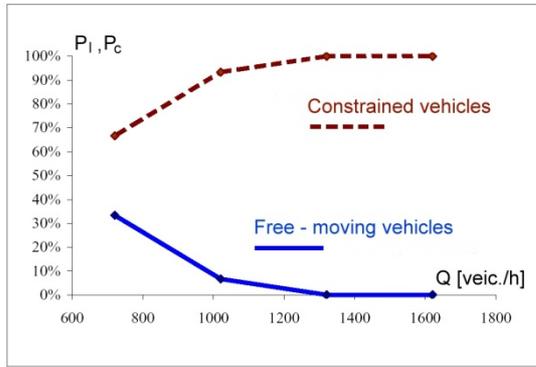


Figure 5. Free-moving and constrained vehicles percentages (ENTLN model)

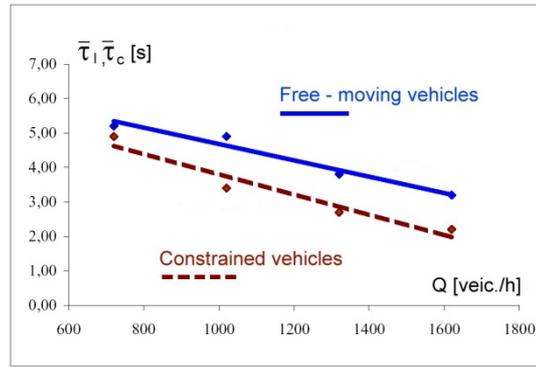


Figure 6. Free-moving and constrained mean vehicular headway (ENTLN model)

As regards calculations we adopted  $\tau_{1min} = 1.0$  sec,  $\tau_{1max} = 10.0$  sec,  $\tau_{cmin} = 0.1$  sec,  $\tau_{cmax} = 5.0$  sec,  $\Delta\tau_1 = \Delta\tau_c = 0.1$  sec,  $\alpha = 0.0$  sec. We point out that the limits inferior of  $\tau_1$  and  $\tau_c$  are assumed, even differently from real numbers, in order to rebuild the trend of the functional that appear in the model. In the application carried out we used the Chi-square test as a statistical test and therefore, as elements of matrix  $M$  (Benjamin & Cornell, 1970), we calculated the relations

$$m_{p,q} = \frac{\chi^2(\tau_{1p}, \tau_{cq})}{\chi^2_{threshold}} = \frac{\chi^2_{p,q}}{\chi^2_{threshold}} \tag{19}$$

where  $\chi^2_{threshold}$  was chosen equal to  $\chi^2(0.05)$  (obviously depending on the number of degrees of freedom of  $\chi^2$ ).

#### 4. Pearson's Type III Shifted Negative Exponential Distribution (ENTPIII)

The second dichotomic model we discuss here is an evolution of Pearson's type III model. We assume that the vehicular time headway of constrained vehicles is distributed according to a Pearson's type III model, whereas we suppose that the vehicular time headway of free-moving vehicles is distributed according to a shifted negative exponential:

$$f_{T_1}(\tau) = \begin{cases} 0 & \text{if } \tau < \alpha_1 \\ \lambda_1 e^{-\lambda_1(\tau-\alpha_1)} = \frac{1}{\mu_{T_1} - \alpha_1} \exp\left(-\frac{\tau - \alpha_1}{\mu_{T_1} - \alpha_1}\right) & \text{if } \tau \geq \alpha_1 \end{cases} \tag{20}$$

$$f_{T_c}(\tau) = \begin{cases} 0 & \text{if } \tau < \alpha_c \\ \frac{\lambda_c e^{-\lambda_c(\tau-\alpha_c)}}{\Gamma(k)} [\lambda_c(\tau - \alpha_c)]^{k-1} & \text{if } \tau \geq \alpha_c \end{cases} \tag{21}$$

where with  $\alpha_1$  and  $\alpha_c$  we indicate minimum vehicular time headways put into effects by drivers of free-moving and constrained vehicles. Equations from (2) to (6) remain valid. As regards mathematical expectation and variations of the probability density functions we have that:

$$\mu_{T_1} = \frac{1}{\lambda_1} + \alpha_1 \tag{22}$$

$$\mu_{T_c} = \frac{k}{\lambda_c} + \alpha_c \tag{23}$$

$$\sigma^2_{T_1} = \frac{1}{\lambda_1^2} \tag{24}$$

$$\sigma^2_{T_c} = \frac{k}{\lambda_c^2} \tag{25}$$

It is impossible to immediately express Pearson's type III shifted negative exponential distribution parameters as

functions of  $\bar{\tau}$  and  $s$ . There are seven unknown quantities that must be determined in order to identify Pearson's type III shifted negative exponential distribution that better fits to a distribution of measured time headways  $\lambda_1, \alpha_1, \lambda_c, \alpha_c, k, P_b, P_c$ . Thanks to the method of moments the calibration of the model can derive from Equation (1) and from equality between theoretical statistical moments as far as the sixth order. The procedure set out in the previous section for the calibration of the Log-normal shifted negative exponential distribution can be applied to Pearson's type III shifted negative exponential distribution, bearing in mind that in this case, a further unknown quantity occurs.

With reference to Equations (15), (16), (17), (18), four parameters must be determined:  $\tau_1, \alpha_1, \tau_c, \alpha_c$ . As for the Log-normal shifted negative exponential probability density function, the decision to choose the value of  $\alpha_1$  can be made for instance, according to information already available. The  $\alpha_c$  measurement represents the minimum vehicular time headway in platoon. It is a function of drivers' behaviour and of vehicle dimensions.

In the procedure we here discuss, we assign  $\alpha_c$  values determined within a previously set variation field  $[\alpha_{c_{min}}, \alpha_{c_{max}}]$  narrowing down parameters of interest to  $\tau_1, \tau_c$ . Values  $\alpha_{c_k}$  and  $\alpha_c$  are determined by  $\alpha_{c_{min}}$  with constant increments of  $\Delta\alpha_c$  variation as far as  $\alpha_{c_{max}}$ . For each determination  $\alpha_{c_k}$  and  $\alpha_c$  with the same criteria used for the Log-normal shifted negative exponential probability density function, we calculate the matrix  $M(\alpha_{c_k}) = \{m(\alpha_{c_k})_{p,q}\}$  and the couples  $(\tau_i, \tau_{c_j})$  that determine the "best"  $f_T$ , given  $\alpha_c = \alpha_{c_k}$ .

In this way we have a vector of solutions  $(P_{1p,q,k}, P_{cp,q,k}, \tau_{1p,k}, \tau_{cq,k}, \alpha_1, \alpha_{c_k})$  as determined by the vector  $(\tau_{1p,k}, \tau_{cq,k}, \alpha_{c_k})$  that supplies the best fit to the data considered.

Pearson's type III shifted negative exponential distributions in Figures 7 to 10 were determined according to the procedure mentioned above dealing with the same measurements and same input parameters as for the application in the previous section, to which the specifications  $\alpha_{c_{min}} = 0.1$  sec,  $\alpha_{c_{max}} = 1.0$  sec,  $\Delta\alpha_c = 0.1$  sec were added, as regards  $\alpha_c$ .

For free-moving and constrained vehicles percentages and free-moving and constrained mean vehicular headway for Pearson's type III shifted negative exponential distribution (ENTPIII) see Figure 11 and Figure 12.

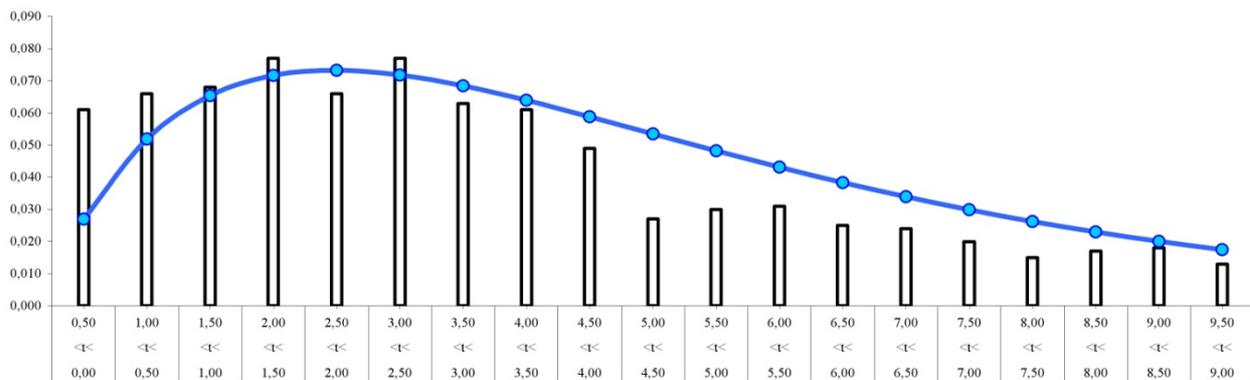


Figure 7. Probabilities related to headway classes: measured distribution (1320 observations) and ENTPIII distribution for traffic volume  $Q=10-14$  veh/min= $600-840$  veh/h

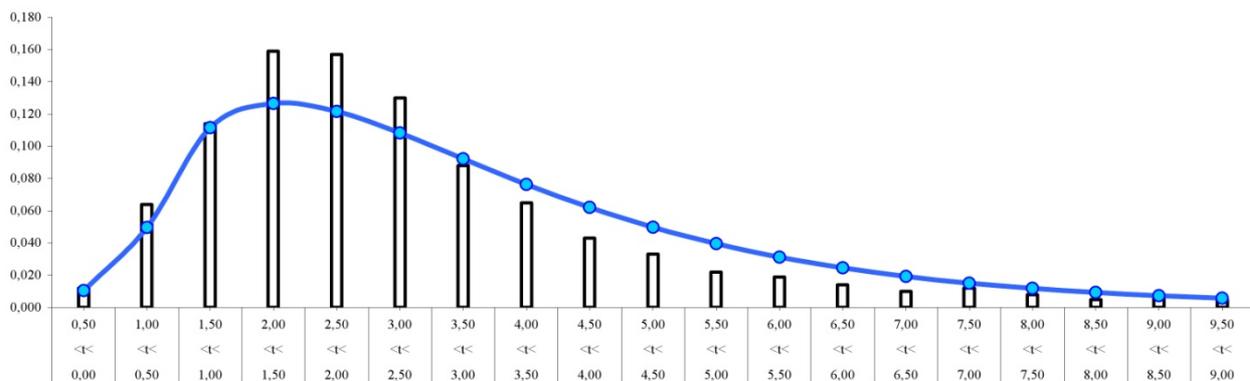


Figure 8. Probabilities related to headway classes: measured distribution (3432 observations) and ENTPIII distribution for traffic volume  $Q=15-19$  veh/min= $900-1140$  veh/h

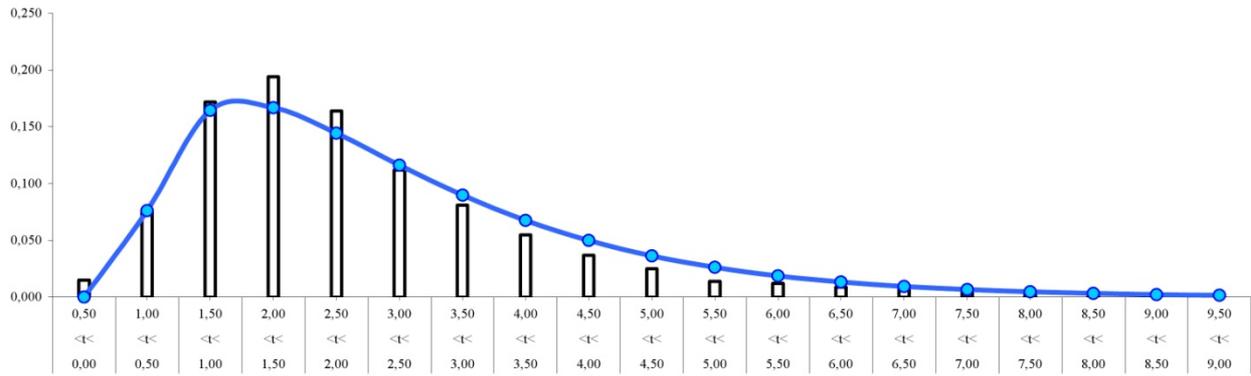


Figure 9. Probabilities related to headway classes: measured distribution (6327 observations) and ENTPIII distribution for traffic volume  $Q=20-24$  veh/min= $1200-1440$  veh/h

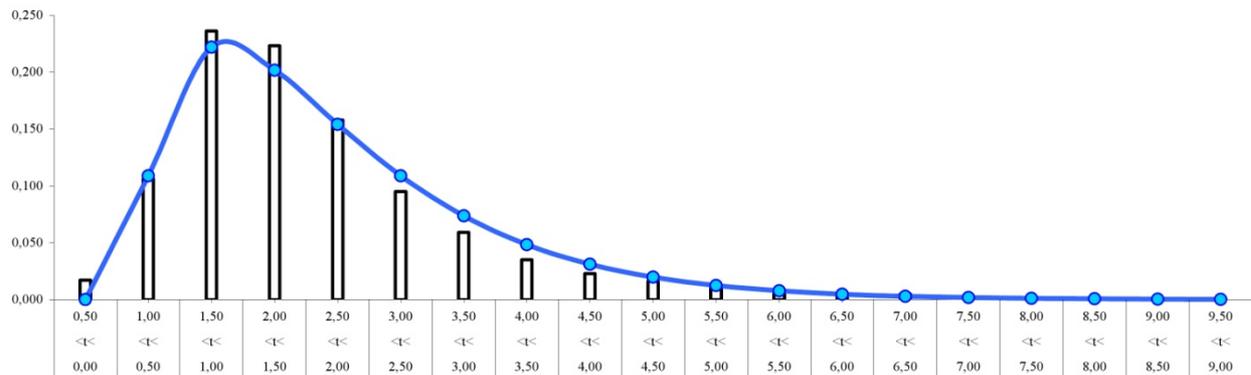


Figure 10. Probabilities related to headway classes: measured distribution (3491 observations) and ENTPIII distribution for traffic volume  $Q=25-29$  veh/min= $1500-1740$  veh/h

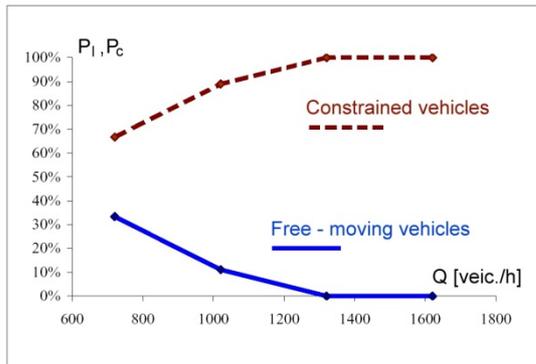


Figure 11. Free-moving and constrained vehicles percentages (ENTPIII model)

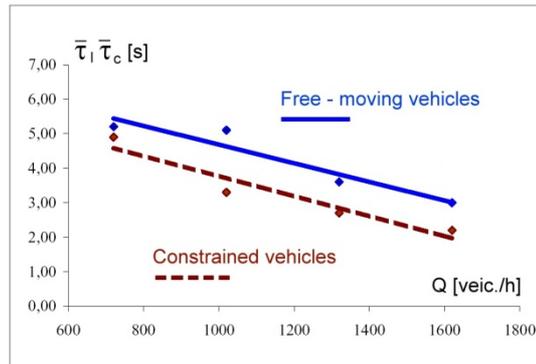


Figure 12. Free-moving and constrained mean vehicular headway (ENTPIII model)

### 5. Results and Comparisons with Other Models of Time Headway

In writing this paper we examined 14570 measurements of vehicular time headway dealt with in (May, 1990), where they are gathered and presented as functions of  $Q$  traffic volumes of the flows observed. The characteristics of the samples are the following ( $N$  indicates the number in terms of observations):  $Q = 600-840$  veh/h,  $N = 1320$ ;  $Q = 900-1140$  veh/h,  $N = 3432$ ;  $Q = 1200-1440$  veh/h,  $N = 6327$ ;  $Q = 1500-1740$  veh/h,  $N = 3491$ . The bar charts that refer to this data are in fig. 1 to 4 and also in Figures 7 to 10. The adaptation of Log-normal shifted negative exponential and Pearson's type III shifted negative exponential models is ever more increasing (in terms of response to the Chi-square test) with the progress of intensity of the mean value of flow rates from low/medium (600-1440 veh/h) to high (more than 1200 veh/h) (see Figures 1 to 4 and 7 to 10). These

results are coherent with the limits of the models developed in this work that derive from their genesis and structures: in fact, as regards high traffic flow values (see Figures 4 and 10) the two distributions are reduced to probability density functions of flows with completely constrained vehicles and for this reason, the Log-normal shifted negative exponential distribution and Pearson's type III shifted negative exponential distribution laws coincide, respectively with a Log-normal and a Pearson type III distribution. The latter are the models that generally describe headways  $\{\tau\}$  when traffic flow conditions limit most of sensed values of  $\tau$  to a small number of classes related to a few seconds. This circumstance occurs in the flow all the more when the flow rate is high. In Figures 13 to 16 we can see comparisons between the adaptation achieved in relation to the sample of reference, by means of different theoretical distributions of time headways: a part from Log-normal shifted negative exponential distributions and Pearson's type III shifted negative exponential distributions we also report the negative exponential distribution, a shifted negative exponential, Erlang's distribution, Pearson's type III distribution, the log-normal distribution, the normal distribution and May and Schuhl's composite distributions.

In Figure 17 we offer the trend of the ratio  $\chi^2_{\text{calculated}} / \chi^2_{(0,05)}$  when the mean flow rate varies for the different statistical models mentioned above.

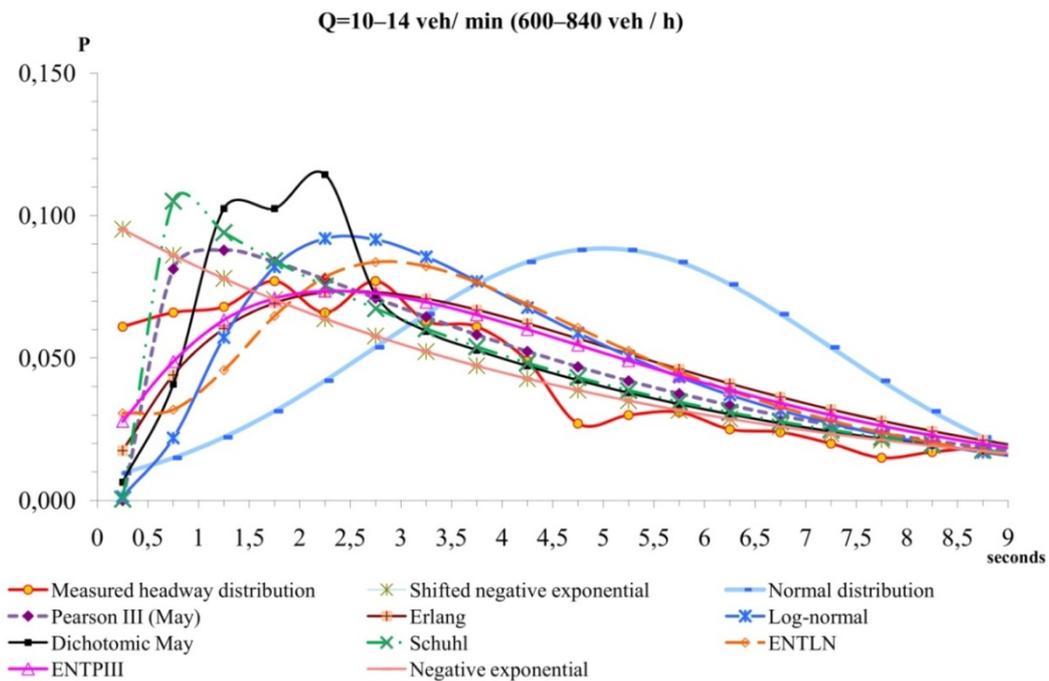


Figure 13. Probabilities related to headways classes: measured distribution (1320 observations) and theoretical distributions for traffic volume  $Q=10-14 \text{ veh/min}=600-840 \text{ veh/h}$

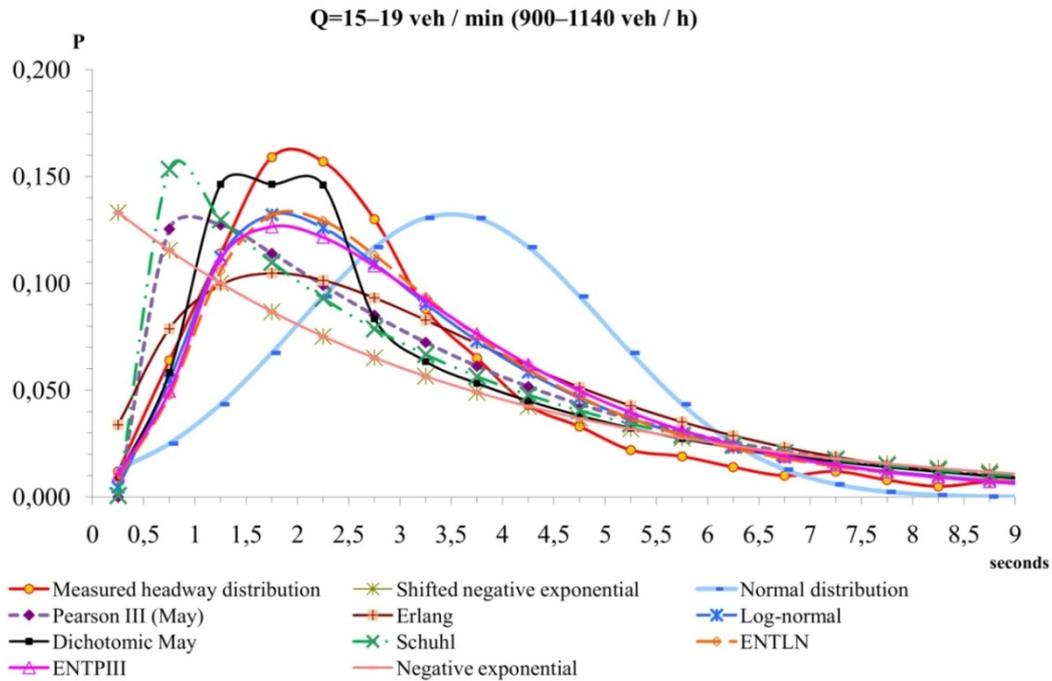


Figure 14. Probabilities related to headways classes: measured distribution (3432 observations) and theoretical distributions for traffic volume  $Q=15-19 \text{ veh/min}=900-1140 \text{ veh/h}$

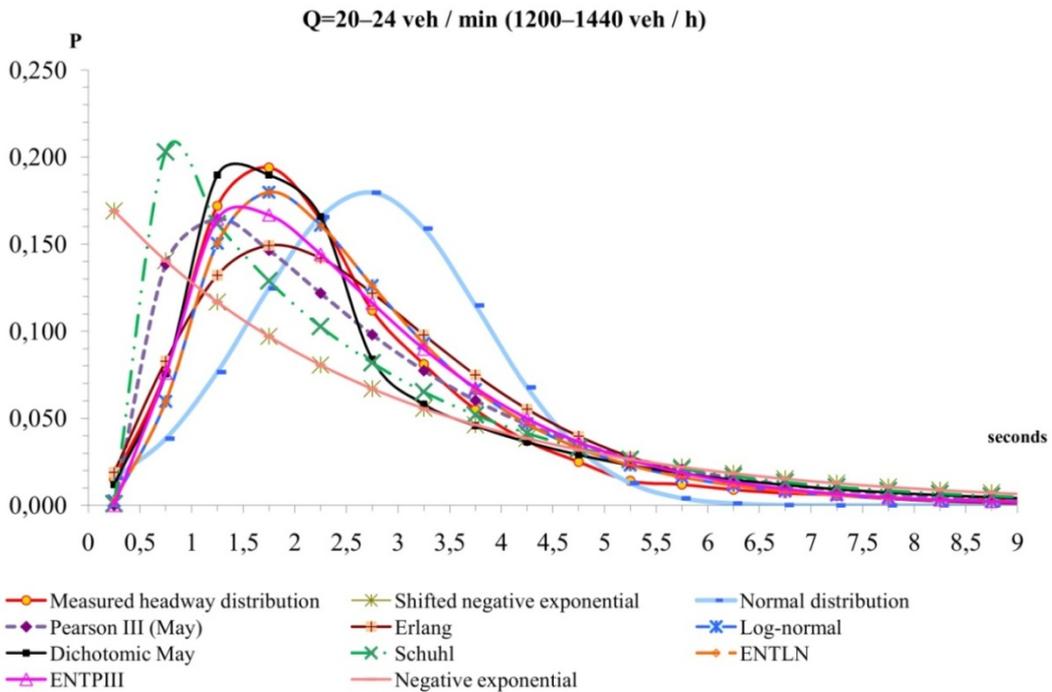


Figure 15. Probabilities related to headways classes: measured distribution (6327 observations) and theoretical distributions for traffic volume  $Q=20-24 \text{ veh/min}=1200-1440 \text{ veh/h}$

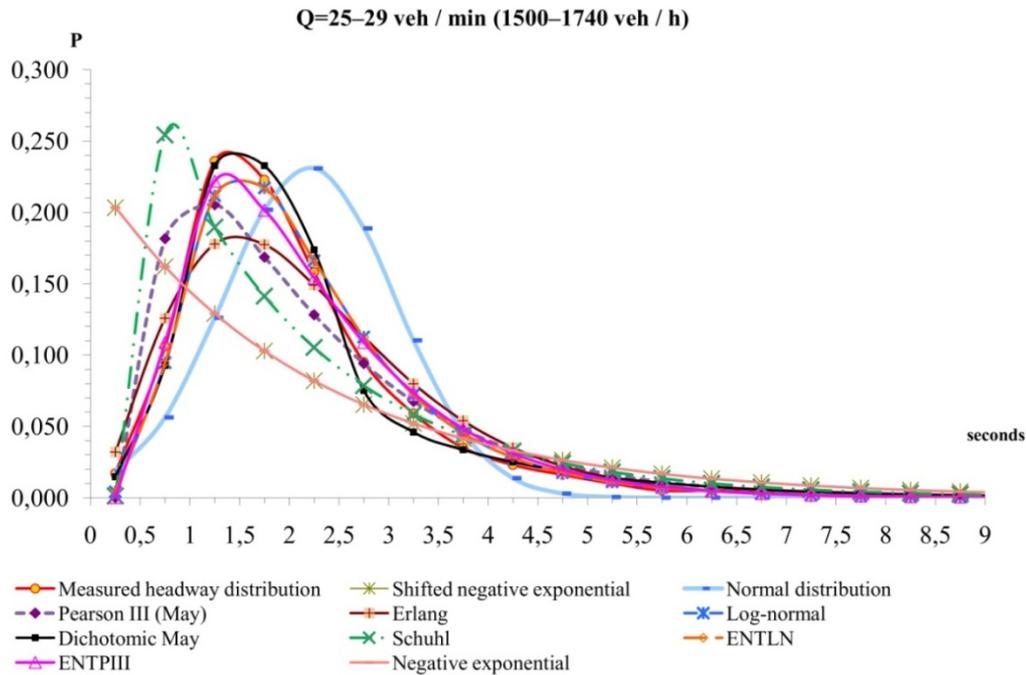


Figure 16. Probabilities related to headways classes: measured distribution (3491 observations) and theoretical distributions for traffic volume  $Q=25-29 \text{ veh/min}=1500-1740 \text{ veh/h}$

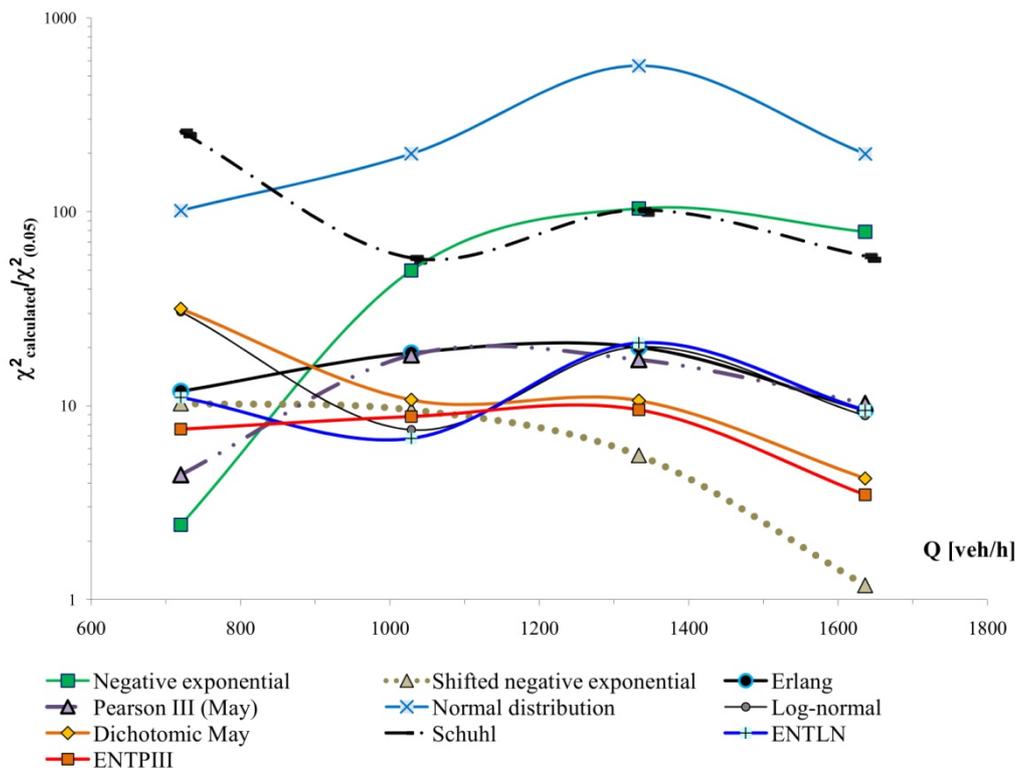


Figure 17. Ratio  $\chi^2_{calculated} / \chi^2_{(0.05)}$  in function of average flow rate, for the theoretical distributions (lower values mean better adjustment)

### 6. Conclusions

From the observation of the graphs called to attention and of the statistical measurements obtained from the Chi-square test we draw the conclusion that Pearson's type III shifted negative exponential probability density

function proves to have higher descriptive capacity, slightly above the Log-normal shifted negative exponential probability density function. Furthermore, the efficiency in interpreting of the two models results to be, as regards the samples considered, better than that of theoretical laws already available in literature, as confirmation of the suitability of composite functions and of the articulation of the estimation procedure elaborated (see fig. 17). Finally, we point out how a characteristic of the models analysed, being dichotomic, is that through them it is possible to obtain the percentages of free-moving and constrained vehicles within a circulating flow. The distinguishing element between the two derives from the fundamental hypothesis that free-moving vehicles, that do not interfere as a definition, can submit to a distribution of time headways with a negative exponential trend. Such trend corresponds to the perfectly casual collocation of events in time, where, for events we mean vehicular passages observed. It follows that vehicles of which headways are not distributed as a negative exponential, are considered to be constrained vehicles, for which the time spacing is to be interpreted with the second component of the dichotomic model.

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