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Planning the Expansion of Long-Term Transmission Networks Using a Cycle-Based Formulation

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Abstract

This paper presents a methodology to solve the long-term transmission expansion planning problem, using a formulation that uses mathematical expressions that are alternatives to the second Kirchhoff's law and that are applied to the cycles critical of the system graph. The network transmission expansion planning problem of power systems is part of the socalled NP-complete problems, which belong to a category of problems that are dfficult to solve, for which polynomial solution algorithms are not known. The proposed methodology is applied to two test systems of the specialized literature with very good results.

Keywords: optimization, transmission planning, critical cycles, Kirchhoff's second law

1. Introduction

The problem of planning the expansion of the electricity transmission network determines the new investments in electrical infrastructure: transmission lines and substations, required to allow the adequate transfer of active power between the generation centers and the energy consumption centers, in a planning horizon of 10 or more years (Escobar et al, 2010).

The investment options associated with this problem are characterized by high costs, by the long periods of time required for its construction, by the long periods of recovery of the investment and by the need to keep the built infrastructure in operation until such investment is recovered. This problem takes as a reference the current network and considers the future increase of the demand in the nodes of the system, the existing generation, the alternatives of new generation, the repowering of the existing generation, the new demand nodes, the requirements of new substations and the repowering of existing substations, in the analyzed time horizon.

Static planning determines the minimum cost solution considering a single period of time in the planning horizon, this is denominated static planning, and assumes that the existing network is part of the optimal future solution, which means that the possibility of withdrawing, moving or leaving elements that are operating in the current network, is not considered. This is a problem of optimization of integer-mixed nonlinear programming (IMNLP). It is a multimodal, non-convex problem that can not be solved successfully using exact optimization techniques when the system is large and has a large number of isolated nodes. In systems of smaller dimensions, the optimal solution is found using methods such as Branch and Cut or Branch and Bound (Escobar et al, 2010; Dom´ınguez, 2014). In some of these cases, it is found that computer systems require large calculation times compared to those required by metaheuristic techniques such as (Tabu Search (TS) or the genetic algorithm proposed by Chu and Beasley (CBGA) (Escobar et al, 2016), which do not guarantee obtaining the optimal global solution. The problems that are simultaneously IMNLP and NP-complete are characterized as being the most difficult solution. To solve the problem of expansion planning, various methodologies have been used from heuristic and metaheuristic techniques, to exact methods (Dominguez, 2014).

In this paper a novel formulation of the original problem is proposed, replacing the Kirchhoff's second law by restrictions associated with the critical cycles of the system, which produce the same optimal solution with less computational effort (Escobar L, 2018). The problem is solved initially using the transport model. Then, the existing circuits at their upper limit and the investment in new corridors are identified in the relaxed solution. These corridors are identified and included in a cycle restriction or closed trajectory that must meet two conditions: it must contain the identified circuit and must associate this circuits with the paths that form a cycle with the least amount of products power-reactance, in which the relative direction of the active power flows is taken into account. The identified cycles are added to the transport model, and the problem is solved again, repeating the previous process until no overloaded circuits or investment in new corridors appear in the solution. This procedure allows to identify critical cycles, and allows to find the optimal solution obtained with the DC model, for this problem, without including Kirchhoff's second law in the traditional way.

2. Method

2.1 Problem Formulation

The mathematical model used to solve the problem of transmission planning is the following:

$$min = \sum_{(i,j)\in\Omega_1} C_{ij} \sum_{k\in\Omega_2} y_{ij,k} \tag{1}$$

$$s.a. \quad \sum_{(p,i)\in\Omega_1} \left(f^0_{pi} + \sum_{k\in\Omega_2} f_{pi,k} \right) - \sum_{(i,j)\in\Omega_1} \left(f^0_{ij} + \sum_{k\in\Omega_2} f_{ij,k} \right) + g_i = d_i \tag{2}$$

$$\left| f^0_{ij} \right| \leq n^0_{ij} \bar{f}_{ij}, \quad \forall\ (i,j) \in \Omega_1 \tag{3}$$

$$\left| f_{ij,k} \right| \leq y_{ij,k} \bar{f}_{ij}, \quad \forall\ (i,j) \in \Omega_1,\ k \in \Omega_2 \tag{4}$$

$$0 \leq g_i \leq \bar{g}_i \tag{5}$$

$$\sum_{k\in\Omega_2} y_{ij,k} \leq \bar{n}_{ij}, \quad \forall\ (i,j) \in \Omega_1 \tag{6}$$

$$y_{ij,k-1} \geq y_{ij,k}, \quad \forall\ (i,j) \in \Omega_1,\ k \in \Omega_2,\ k > 1 \tag{7}$$

$$\left| f_{ij,k} - f_{ij,k-1} \right| \leq M\left(1 - y_{ij,k}\right), \quad \forall\ (i,j) \in \Omega_1,\ k \in \Omega_2,\ k > 1 \tag{8}$$

$$\left| \left(f^0_{ij}/n^0_{ij} - f_{ij,1} \right) \right| \leq M(1 - y_{ij,1}), \quad \forall\ (i,j) \in \Omega_1,\ k \in \Omega_2,\ n^0_{ij} > 0 \tag{9}$$

In the previous model, C_{ij} is the cost of adding a circuit in the ij corridor. Ω_1 is the set of existing and expansion corridors. $y_{ij,k}$ is the binary variable associated with the investment option k of the corridor ij, and $k \in \Omega_2$. f^0_{ij} is the active power flow in the base network or existing circuits. $f_{ij,k}$ is the active power flow in the k investment option of the expansion circuits. $\bar{f}_{ij}$ is the maximum flow allowed for a circuit in the ij corridor. $\bar{g}$ is the maximum nodal generation vector. n_{ij} is the number of added circuits in the ij corridor. $\bar{n}_{ij}$ is the maximum number of circuits that can be added in the ij corridor. M is a parameter defined a priori of great size, which makes the disjunctive restrictions irrelevant when the variable $y_{ij,k} = 0$ (Domínguez, 2017).

In this model, (1) represents the objective function and characterizes traditional planning as a minimum cost problem. The restriction (2) represents the first *Kirchhoff* law. The restriction (3) allows to establish the limits of capacity in the existing circuits and the (4) in the expansion circuits. Through (5) the generation limits are established and (6) establishes the investment limit. (7) establishes an order of priority among the investment options and eliminates the existing symmetry between these options in the traditional disjunctive transport model. (8) and (9) guarantee that the active power flows are equal in circuits connected in parallel in the same corridor (Garver, 1970; Escobar L, 2018).

2.2 Cycle formulation

We use below the basic terminology of graph theory, which will guide us to the definition of the cycle. An unmanaged graph G is a pair (V, E), where V is a finite set and E is a family of (non-ordered) pairs of V elements. The elements of V are called *nodes* or *vertices* and the elements of E are called *paths* or *corridors* of G. Given a corridor between two vertices $i, j \in V$, with $i \neq j$, we denote this corridor for (i, j). Therefore, for a corridor $e = (i, j) \in E$, i and j are called their final points or vertices. In the same way we say that the e corridor is *incident* to the vertices i and j. Similarly, we say that the vertex i is *adjacent* to the vertex j. The degree of a vertex in a non-directed graph is the number of links or corridors incident to it, which we will denote as $deg(i_q)$. A path p of length k, which joins a vertex i to a vertex j, in a graph $G(V, E)$, is a sequence $\langle r_0, r_1, \ldots, r_k \rangle$ of vertices such that:

$$i = r_0,\ \ j = r_k,$$

with $(r_{m-1}, r_m) \in E$ for $m = 1, 2, \ldots, k$. A path is simple if all its vertices are different. In an undirected graph, a path $\langle r_0, r_1, \ldots, r_k \rangle$ forms a *cycle* if $r_0 = r_k$ and $r_1, r_2, \ldots, r_k$ are different (Kocuk et al, 2016).

A graph $G' = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. Given a set $V' \subseteq V$, the subgraph of G *induced* by V' is the graph $G' = (V', E')$ where $E' = \{(i, j) \in E : i, j \in V'\}$.

2.3 Cycle basis

Let $G = (V, E)$ be an unmanaged graph with m links and n vertices. A *cycle* of G is a subgraph of G. The vector space generated by the incident vectors of cycles is called *space of cycles* of G, which has as dimension:

$$m - n + \alpha(G),$$

where m is the number of links or corridors of G, n is the number of nodes or vertices and $\alpha(G)$ is the number of related components of G. The set *maximal* of linearly independent cycles are called *cycle basis*.

It is important to note that the G links have an assigned weight or value. Therefore, a base of cycles where the sum of the cycle weights is minimal is called *minimum cycle basis* of G. In our case, the G graph is not connected and, through an iterative process, nodes not connected to the system can be added if this allows lower cost solutions. Given that the final system $G' = (V', E')$ obtained is connected but of smaller dimension than G, we will denote the dimension of the final space of cycles as:

$$N = m - n + 1,$$

where m is the number of links in the final solution of G', n is the number of nodes or vertices in the final solution G' and $\alpha(G') = 1$ because the The resulting subgraph in the final solution is always connected in the problem of planning the expansion of transmission systems.

2.4 Solution Methodology

The methodology used to solve the planning problem is presented below, replacing Kirchhoff's second law with the concept of critical cycles.

A cycle is called a critical cycle if it meets one of the following two conditions:

- It presents circuits with power flows at its maximum capacity, forms cycles with existing and/or added circuits, and the sum of the link weights is minimal.
- It presents additions of circuits in new corridors, forms cycles with existing and/or added circuits, and the sum of the weights of the links is minimal.

Since the critical cycles replace the effect of Kirchhoff's second law, in the planning model, the weights of the links are associated with the bus-angle difference $(\theta_i - \theta_j)$ of the power system corridors. If C is a critical system cycle, then:

$$\sum_{(i,j)\in C} (\theta_i - \theta_j) = \sum_{(i,j)\in C} w_{ij} f_{ij} x_{ij} = 0, \tag{10}$$

where C is any oriented cycle and x_{ij} is the reactance in the path (i, j). Therefore, the mathematical model for the planning problem of the long-term expansion of transmission systems involving Kirchhoff's second law can be modified using the model given by the equations (1) – (9) plus the *minimum cycles* given by (10).

Associated with each link $e \in E$, of a critical cycle C, there is a value w_{ij} defined as:

$$w_{ij} = \begin{cases} 1, & \text{if the link } ij \in C \text{ and has the orientation of } C \\ -1, & \text{if the link } ij \in C \text{ and has an opposite orientation to the } C \\ 0, & \text{if the link } ij \notin C \end{cases}$$

2.5 Procedure for Generating Critical Cycles

In an expansion planning problem, given the initial network of the system and the future generation data, future demand and electrical characteristics of the investment options in lines and transformers, the problem is solved using the disjunctive transport model (which does not includes Kirchhoff's second law), and it is verified in the response if circuits appear in their maximum capacity or circuits added in new corridors. The cycle containing these circuits is then determined and is a cycle with minimum sum of weights. These cycles, called critical cycles, are added to the transport model and the process is repeated until no new circuits appear at their upper limit or no new circuits are added in expansion corridors.

When the critical cycles are formed exclusively by existing corridors, their general form is:

$$\sum_{(i,j)\in C} w_{ij}\left(\frac{f_{ij}^0}{n_{ij}^0}\right)x_{ij} = 0. \tag{11}$$

As an example of application, in the *South-Brazilian system* one of the critical cycles assumes the following form:

$$\left(\frac{f_{12}^0}{n_{12}^0}\right)x_{12} + \left(\frac{f_{25}^0}{n_{25}^0}\right)x_{25} + \left(\frac{f_{58}^0}{n_{58}^0}\right)x_{58} - \left(\frac{f_{78}^0}{n_{78}^0}\right)x_{78} - \left(\frac{f_{17}^0}{n_{17}^0}\right)x_{17} = 0,$$

that corresponds to the succession of vertices: $\langle 1, 2, 5, 8, 7, 1\rangle$. The positive terms correspond to links that have the same orientation of the cycle and the negative ones to the links that have opposite orientation to the cycle.

When critical cycles include circuits in new corridors, the general form of Eq.(10) involves a disjunctive constraint:

$$\sum_{(i,j)\in C_1} w_{ij}\left(\frac{f_{ij}^0}{n_{ij}^0}\right)x_{ij} + \sum_{(i,j)\in C_2} w_{ij}f_{ij,1}x_{ij} \le M\left(z - \sum_{(i,j)\in C_2} y_{ij,1}\right),\ \forall (i,j)\in C, \tag{12}$$

where C_1 represents the subset of links in the critical cycle C, associated with circuits in existing corridors, C_2 represents the subset of links in the critical cycle C, associated with circuits in new brokers, z is the number of links contained in C_2 to which the binary decision variables $y_{ij,1}$ are associated.

For the South-Brazilian system, one of the critical cycles involving new corridors corresponds to the succession of vertices:

$$\langle 5, 6, 46, 19, 18, 13, 8, 5\rangle.$$

This cycle assumes the following form:

$$\left\{(f_{5-6,1})\cdot x_{5-6} + (f_{6-46,1})\cdot x_{6-46} - \left(\frac{f_{19-46}^0}{n_{19-46}^0}\right)x_{19-46} - \left(\frac{f_{18-19}^0}{n_{18-19}^0}\right)x_{18-19} - \right.$$

$$\left.\left(\frac{f_{13-18}^0}{n_{13-18}^0}\right)x_{13-18} - \left(\frac{f_{8-13}^0}{n_{8-13}^0}\right)x_{8-13} - \left(\frac{f_{5-8}^0}{n_{5-8}^0}\right)x_{5-8}\right\} \le M\left(2 - y_{5-6,1} - y_{6-46,1}\right)$$

3. Results

The results obtained are presented below, applying the methodology to the *South-Brazilian system* and the *Colombian system* of 83 nodes and 155 corridors.

3.1 South-Brazilian system

By applying the proposed methodology to the South-Brazilian system, the critical cycles that are presented below are obtained:

Table 1. Critical cycles South-Brazilian system

Cycle 1	$\left(\frac{f^0_{12}}{n^0_{12}}\right)x_{12} + \left(\frac{f^0_{25}}{n^0_{25}}\right)x_{25} + \left(\frac{f^0_{58}}{n^0_{58}}\right)x_{58} - \left(\frac{f^0_{78}}{n^0_{78}}\right)x_{78} - \left(\frac{f^0_{17}}{n^0_{17}}\right)x_{17} = 0$
Cycle 2	$\left(\frac{f^0_{45}}{n^0_{45}}\right)x_{45} + \left(\frac{f^0_{59}}{n^0_{59}}\right)x_{59} - \left(\frac{f^0_{49}}{n^0_{49}}\right)x_{49} = 0$
Cycle 3	$\left(\frac{f^0_{9\,14}}{n^0_{9\,14}}\right)x_{9\,14} + \left(\frac{f^0_{14\,18}}{n^0_{14\,18}}\right)x_{14\,18} - \left(\frac{f^0_{13\,18}}{n^0_{13\,18}}\right)x_{13\,18} + \left(\frac{f^0_{8\,13}}{n^0_{8\,13}}\right)x_{8\,13} - \left(\frac{f^0_{58}}{n^0_{58}}\right)x_{58} + \left(\frac{f^0_{59}}{n^0_{59}}\right)x_{59} = 0$
Cycle 4a	$(f_{56,1})\cdot x_{56} + (f_{6\,46,1})\cdot x_{6\,46} - \left(\frac{f^0_{19\,46}}{n^0_{19\,46}}\right)x_{19\,46} - \left(\frac{f^0_{18\,19}}{n^0_{18\,19}}\right)x_{18\,19} - \left(\frac{f^0_{13\,18}}{n^0_{13\,18}}\right)x_{13\,18} -$ $\left(\frac{f^0_{8\,13}}{n^0_{8\,13}}\right)x_{8\,13} - \left(\frac{f^0_{58}}{n^0_{58}}\right)x_{58} \leq 1000\,(2 - y_{56,1} - y_{6\,46,1})$
Cycle 4b	$(f_{56,1})\cdot x_{56} + (f_{6\,46,1})\cdot x_{6\,46} - \left(\frac{f^0_{19\,46}}{n^0_{19\,46}}\right)x_{19\,46} - \left(\frac{f^0_{18\,19}}{n^0_{18\,19}}\right)x_{18\,19} - \left(\frac{f^0_{13\,18}}{n^0_{13\,18}}\right)x_{13\,18} -$ $\left(\frac{f^0_{8\,13}}{n^0_{8\,13}}\right)x_{8\,13} - \left(\frac{f^0_{58}}{n^0_{58}}\right)x_{58} \geq -1000\,(2 - y_{56,1} - y_{6\,46,1})$
Cycle 5	$\left(\frac{f^0_{14\,22}}{n^0_{14\,22}}\right)x_{14\,22} + \left(\frac{f^0_{22\,26}}{n^0_{22\,26}}\right)x_{22\,26} - \left(\frac{f^0_{14\,26}}{n^0_{14\,26}}\right)x_{14\,26} = 0$
Cycle 6	$\left(\frac{f^0_{16\,17}}{n^0_{16\,17}}\right)x_{16\,17} + \left(\frac{f^0_{17\,19}}{n^0_{17\,19}}\right)x_{17\,19} + \left(\frac{f^0_{19\,46}}{n^0_{19\,46}}\right)x_{19\,46} - \left(\frac{f^0_{16\,46}}{n^0_{16\,46}}\right)x_{16\,46} = 0$
Cycle 7	$\left(\frac{f^0_{13\,18}}{n^0_{13\,18}}\right)x_{13\,18} + \left(\frac{f^0_{18\,20}}{n^0_{18\,20}}\right)x_{18\,20} - \left(\frac{f^0_{13\,20}}{n^0_{13\,20}}\right)x_{13\,20} = 0$
Cycle 8	$\left(\frac{f^0_{18\,19}}{n^0_{18\,19}}\right)x_{18\,19} + \left(\frac{f^0_{19\,21}}{n^0_{19\,21}}\right)x_{19\,21} - \left(\frac{f^0_{20\,21}}{n^0_{20\,21}}\right)x_{20\,21} - \left(\frac{f^0_{18\,20}}{n^0_{18\,20}}\right)x_{18\,20} = 0$
Cycle 9	$\left(\frac{f^0_{14\,26}}{n^0_{14\,26}}\right)x_{14\,26} + \left(\frac{f^0_{26\,27}}{n^0_{26\,27}}\right)x_{26\,27} + \left(\frac{f^0_{27\,38}}{n^0_{27\,38}}\right)x_{27\,38} + \left(\frac{f^0_{38\,42}}{n^0_{38\,42}}\right)x_{38\,42} + \left(\frac{f^0_{42\,43}}{n^0_{42\,43}}\right)x_{42\,43} -$ $\left(\frac{f^0_{32\,43}}{n^0_{32\,43}}\right)x_{32\,43} - \left(\frac{f^0_{19\,32}}{n^0_{19\,32}}\right)x_{19\,32} - \left(\frac{f^0_{18\,19}}{n^0_{18\,19}}\right)x_{18\,19} - \left(\frac{f^0_{14\,18}}{n^0_{14\,18}}\right)x_{14\,18} = 0$
Cycle 10	$\left(\frac{f^0_{24\,34}}{n^0_{24\,34}}\right)x_{24\,34} - \left(\frac{f^0_{33\,34}}{n^0_{33\,34}}\right)x_{33\,34} - \left(\frac{f^0_{24\,33}}{n^0_{24\,33}}\right)x_{24\,33} = 0$
Cycle 11	$\left(\frac{f^0_{20\,23}}{n^0_{20\,23}}\right)x_{20\,23} + \left(\frac{f^0_{23\,24}}{n^0_{23\,24}}\right)x_{23\,24} + \left(\frac{f^0_{24\,34}}{n^0_{24\,34}}\right)x_{24\,34} + \left(\frac{f^0_{34\,35}}{n^0_{34\,35}}\right)x_{34\,35} + \left(\frac{f^0_{35\,38}}{n^0_{35\,38}}\right)x_{35\,38} -$ $\left(\frac{f^0_{27\,38}}{n^0_{27\,38}}\right)x_{27\,38} - \left(\frac{f^0_{26\,27}}{n^0_{26\,27}}\right)x_{26\,27} - \left(\frac{f^0_{14\,26}}{n^0_{14\,26}}\right)x_{14\,26} + \left(\frac{f^0_{14\,18}}{n^0_{14\,18}}\right)x_{14\,18} + \left(\frac{f^0_{18\,20}}{n^0_{18\,20}}\right)x_{18\,20} = 0$

Cycle 4 presented in the previous table involves two new corridors, and corresponds to a disjunctive constraint that occurs in two parts cycle 4a and cycle 4b. When the methodology is applied, 11 cycles are obtained and adding them to the disjunctive transport model allows obtaining the best known solution for this system of $US\$72,870$ million dollars, with an execution time of 0.48 s and 591.56 ticks. It is important to note that the computation times with this methodology is much more efficient than using the traditional disjunctive model (including the second Kirchhoff law), in effect, with this model we obtain an execution time of 0.86 s and 1188.97 ticks. The *ticks* are a computer-independent measure of how much algorithmic work is required to obtain a provable optimum, independently of the computer on which it is run on.

3.2 Colombian System

By applying the proposed methodology to the Colombian system, the critical cycles that are presented below are obtained:

Table 2. Critical cycles Colombian system

Cycle 1*	$\langle 52, 88, 43, 39, 68, 86, 19, 82, 55, 57, 56, 54, 47, 52 \rangle$
Cycle 2*	$\langle 57, 81, 56, 57 \rangle$
Cycle 3	$\langle 25, 28, 29, 25 \rangle$
Cycle 4	$\langle 14, 31, 60, 14 \rangle$
Cycle 5	$\langle 14, 18, 20, 13, 14 \rangle$
Cycle 6	$\langle 2, 83, 9, 69, 70, 34, 4, 2 \rangle$
Cycle 7	$\langle 15, 18, 20, 15 \rangle$
Cycle 8	$\langle 19, 61, 68, 86, 19 \rangle$
Cycle 9	$\langle 61, 68, 37, 61 \rangle$
Cycle 10	$\langle 24, 75, 12, 76, 17, 23, 24 \rangle$
Cycle 11	$\langle 27, 35, 44, 27 \rangle$
Cycle 12	$\langle 45, 50, 54, 45 \rangle$
Cycle 13	$\langle 59, 67, 68, 61, 19, 66, 69, 9, 8, 59 \rangle$
Cycle 14	$\langle 8, 59, 1, 8 \rangle$
Cycle 15	$\langle 1, 3, 71, 1 \rangle$
Cycle 16	$\langle 55, 62, 82, 55 \rangle$
Cycle 17	$\langle 66, 69, 60, 14, 18, 66 \rangle$
Cycle 18	$\langle 9, 69, 70, 34, 4, 2, 9 \rangle$
Cycle 19	$\langle 31, 32, 34, 31 \rangle$
Cycle 20	$\langle 16, 18, 21, 16 \rangle$
Cycle 21	$\langle 31, 34, 33, 72, 31 \rangle$
Cycle 22	$\langle 31, 33, 72, 31 \rangle$
Cycle 23	$\langle 31, 60, 69, 70, 34, 32, 31 \rangle$
Cycle 24	$\langle 18, 20, 13, 23, 16, 18 \rangle$
Cycle 25	$\langle 18, 66, 19, 58, 18 \rangle$
Cycle 26	$\langle 19, 22, 21, 18, 58, 19 \rangle$
Cycle 27	$\langle 17, 23, 24, 15, 17 \rangle$
Cycle 29	$\langle 1, 8, 71, 3, 1 \rangle$
Ciclo 30	$\langle 19, 58, 18, 22, 19 \rangle$
Cycle 31	$\langle 27, 64, 29, 27 \rangle$
Cycle 32	$\langle 27, 44, 80, 27 \rangle$
Cycle 33	$\langle 26, 27, 28, 26 \rangle$
Cycle 34	$\langle 27, 29, 25, 28, 27 \rangle$
Cycle 35	$\langle 64, 65, 30, 72, 73, 74, 64 \rangle$
Cycle 36	$\langle 4, 34, 33, 72, 30, 64, 27, 35, 36, 4 \rangle$
Cycle 37	$\langle 8, 9, 77, 79, 87, 8 \rangle$
Cycle 38	$\langle 23, 24, 15, 18, 14, 13, 23 \rangle$
Cycle 39	$\langle 6, 10, 78, 7, 90, 3, 6 \rangle$
Cycle 40	$\langle 45, 81, 56, 54, 45 \rangle$
Cycle 41	$\langle 64, 74, 73, 72, 30, 64 \rangle$
Cycle 42	$\langle 72, 73, 62, 60, 31, 72 \rangle$
Cycle 43	$\langle 19, 82, 62, 60, 69, 66, 19 \rangle$
Cycle 44	$\langle 82, 85, 83, 9, 69, 60, 62, 82 \rangle$
Cycle 45	$\langle 1, 93, 92, 11, 1 \rangle$
Cycle 46	$\langle 9, 83, 85, 91, 90, 3, 71, 8, 9 \rangle$
Cycle 47	$\langle 29, 31, 72, 30, 64, 29 \rangle$
Cycle 48	$\langle 85, 91, 92, 93, 1, 8, 9, 83, 85 \rangle$
Cycle 49*	$\langle 27, 89, 74, 64, 27 \rangle$
Cycle 50	$\langle 4, 5, 6, 3, 71, 8, 9, 2, 4 \rangle$

Cycles 1, 2 and 49 presented in the previous table involve new corridors, and correspond to a disjunctive constraint. With the proposed methodology applied to the Colombian electricity system (using the disjunctive transport model), 50 critical cycles were obtained, which allow finding the best known solution for this system of US 562,417 million dollars. The execution time was 62.79 s with 96484.45 s. As in the previous system, the methodology proposed in this work allows to obtain better results in computation times, that is, with the traditional disjunctive model the same optimal solution is obtained but with execution time of 99.25 s with 147153.02 ticks, which shows the effectiveness of the proposal with cycles.

4. Conclusion

The DC model considered the ideal model to solve the planning problem of the transmission expansion can be replaced by the disjunctive transport model plus the set of critical system cycles. In medium and high complexity systems, this sample is a promising technique to reduce computation times.

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