Journal of Materials Science Research; Vol. 11, No. 1; 2022
ISSN 1927-0585 E-ISSN 1927-0593
Published by Canadian Center of Science and Education

New Interferometric Method and Device for Measuring the X-Ray Train Length

H. R. Drmeyan[1,2] & S. A. Mkhitaryan[1]

[1] Institute of Applied Problems of Physics at the NAS of the RA. Hr. Nersessian Str. 25, Yerevan, RA, 0014 Armenia

[2] Shirak State University after M. Nalbandyan. Paruyr Sevak str. 4, Gyumri, Armenia

Correspondence: S. A. Mkhitaryan, Institute of Applied Problems of Physics at the NAS of the RA. Hr. Nersessian Str. 25, Yerevan, RA, 0014 Armenia. E-mail: sammkhitar@yahoo.com

Received: March 21, 2022 Accepted: May 13, 2022 Online Published: May 23, 2022

doi:10.5539/jmsr.v11n1p85 URL: https://doi.org/10.5539/jmsr.v11n1p85

Abstract

A new interferometric method for measuring the length of the X-ray train is proposed, and a special interferometer, a device for measurement of the X-ray train length has been developed, created, and tested. The length of the X-ray train and the duration of coherent radiation are estimated and it has been proven that the interference pattern disappears when the difference of paths between the superimposing waves becomes greater than the length of the coherent X-ray train. The bound of a disappearance of the X-ray interference pattern is determined depending on the magnitude of the path differences. The length of the X-ray train is determined, which is close to the theoretically determined value.

Keywords: X-ray, interference, intensity, wave train, train length, coherence duration, dynamic scattering

1. Introduction

When solving various interference tasks using X-rays, it is assumed that the entire irradiated volume of the crystal, regardless of its size, scatters the X-rays coherently at the same time. If we take into account that the waves are coherent if they belong to the same act of emission of an atom, then the interference pattern cannot be observed even if the waves emitted by the same atom, but at different times, are forced to meet, moreover, with a delay and long duration of coherent radiation (the duration of one emission event). Therefore, for an accurate determination of the width and intensity of diffraction maximums, it is necessary to take into account the duration of the coherent radiation, i.e., the train length.

A train of waves is called an oscillation of such type, which is described by a simple sinusoidal curve with a constant or slightly varying amplitude in a certain section when the amplitude is everywhere equal to zero outside of that section.

Precisely by the train length are determined the dimensions of the coherently scattering regions of the irradiated volume. In this regard, determining the length of the X-ray train is important not only from the point of view of a theory of interference but also from the point of view of X-ray structural analysis in general. This is especially important for X-ray interferometry and holography, as well as for testing theoretical ideas about a train (Bezirganyan & Gasparyan, 1970; Fezzaa & Lee, 2001).

In our opinion, the interferometric method is quite suitable to use for measuring the length of a train of X-ray coherent radiations. To do this, it is necessary to divide the radiation of the X-ray source into two streams, then make them meet after they have passed different paths, and thus obtain an interference pattern. Then it's necessary gradually increase the path difference between these two streams until this interference pattern disappears. Obviously, the interference pattern disappears from the moment when the path difference between these two rays exceeds the length of the coherent X-ray train. Thus, by measuring the above path difference at the moment of disappearance of the interference pattern, we can measure the length of the train, i.e., the duration of the coherent radiation that creates this interference pattern. This experiment is quite complex and requires the use of X-ray interferometry methods.

It is known (Dichburn, 1965; Franson & Slawskij, 1967; Born & Wolf, 1973; Landsberg, 1965) that the interference patterns in optics disappear at sufficiently large differences of the paths between the superimposed waves. However, before the designing and creation of X-ray interferometers, this effect was not observed with the interference of X-rays. X-ray interferometers appeared after the discovery of anomalous transmission (the Bormann effect) and the subsequent rapid development of the dynamic theory of X-ray interference (Knowles, 1956; Kato, 1960; Takagi, 1962; Homma, Ando, & Kato, 1966; Afanasyev & Kohn, 1977; Authier, 2004; Lider, 2014). Based on the experience in designing and creating X-ray interferometers of various designs and applications (Bonse & Hart, 1965; 1966; Besirganyan, Eiramdshyan, & Truni, 1973; Aboyan, 1996; Drmeyan, 2003; Drmeyan, 2004; Drmeyan, Aboyan, & Knyazyan, 2016), we have designed and created a special interferometer, as well as proposed a new X-ray interferometric method for measuring the train length (coherent radiation duration) of X-ray radiation, which was the goal of this work.

2. Estimation of Wave Train Length and of the Duration of Coherent Radiation

As can be seen from the above, since only the waves that are part of one train are coherent, the duration of coherent radiation is the duration of one radiation event (the duration of the radiation of a train of waves). The finiteness of the wave train and, hence, the finiteness of the duration of coherent radiation, as well as of the natural width of the spectral line are stipulated in the classical theory by the gradual decrease in oscillator amplitude (damping due to the emission of radiation), or in the quantum theory by decreasing the probability of staying in the initial state (Heitler, 1956; Sobel′man, 1967).

According to the classical concepts, each transition of an atom from one level to the other may be associated with one classical oscillator (Sobel′man, 1967), which, from the point of view of classical electrodynamics, produces damped oscillations during the emission event. The damping of the oscillator due to radiation occurs according to the law $W_t = W_0 e^{-\gamma t}$, where W_0 and W_t are the energies of the oscillator (oscillating electron) at the moments $t = 0$ and t, respectively, and the damping factor (γ) is determined by the below-presented relation (Frish, 1963):

$$\gamma = \frac{8\pi^2 e^2}{3mc}\frac{1}{\lambda^2}, \tag{1}$$

Hence, we have the following for the effective lifetime or the decay time of the oscillator:

$$\tau = \frac{1}{\gamma} = \frac{3mc}{8\pi^2 e^2}\lambda^2. \tag{2}$$

From the viewpoint of the quantum theory (Heitler, 1956) we will have the following relation in energy-time uncertainty:

$$\Delta W \cdot \Delta t = \hbar \tag{3}$$

In this case under consideration, the ΔW is the width of the level and Δt is the lifetime of the level or the duration of the emission event (duration of coherent radiation). For the duration of coherent radiations, we obtain from (3)

$$\tau = \frac{\hbar}{\Delta W}. \tag{4}$$

It is not difficult to determine that the durations of coherent radiation in both classical and quantum mechanics have the same level, which we can confirm also based on our specific example. Thus, substituting the numerical values of e, m and c in (2), we will obtain

$$\tau = 4.53\lambda^2 \text{ c.} \tag{5}$$

The width of the gold's $K_{\alpha\ (K-LIII)}$ line is 58 eV (the width for the K-th level is 54 eV and for the L_{III} level is – 4 eV), the wavelength $\lambda_{K_{\alpha 1}} = 0.17982 Å$ (Heitler, 1956). For the duration of coherent radiation, we obtain respectively from (5) and (4) with due regard for the above values $\tau_{cl} = 0.146 \cdot 10^{-16}$ c and $\tau_{quant} = 0.114 \cdot 10^{-16}$ c, which shows that the classical and quantum values of coherent radiation duration agree sufficiently well.

3. Task Statement and Method Substantiation

At first sight, it could appear that the difference of phases (paths) of interfering waves can be increased simply by adding in the path of one of the waves a medium with a refractive index different from unity. However, this may lead to other difficulties. As the refractive index of X-rays for media (other than vacuum or air) is somewhat different from unity, in order to obtain optical path length differences more than the train length, it is necessary to introduce a medium with a length of about 10 cm. Because of this, firstly, the dimensions of the interferometer would be increased extremely and, secondly, due to absorption in the medium, the wave would be so damped that the interference pattern would disappear due to the above reasoning. A detailed discussion of this problem showed that in order to measure the length of the X-ray train, it is necessary to create a phase difference between the interfering waves, ensuring the difference in their geometric paths.

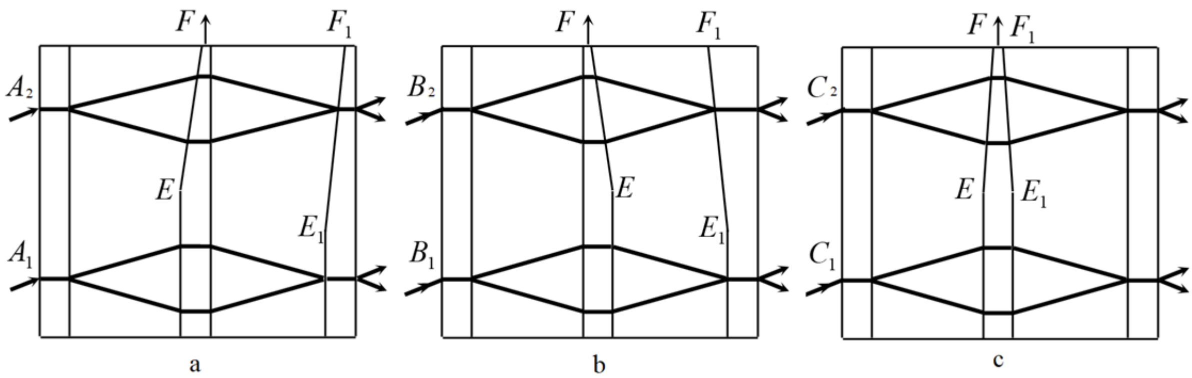

Figure 1. Some variants of interferometers for ensuring the phase changes of superimposed waves

In order to solve this problem, we considered some variants of three-block interferometers with wedge blocks (Figure 1) that provide phase changes of superimposed waves and are designed so, that the waves superimposed in these interferometers overlapped exactly on the input surface of the third block, regardless of their exit points from the first block.

In the interferometers *a, b,* and *c* (Figure 1), when the primary wave is incident on the points A_1, B_1, and C_1 of the first blocks, the phase difference at the inputs of the third blocks between the interfering waves is zero. In the same interferometers, when the primary wave falls at the points A_2, B_2, and C_2 of the first blocks, the phase differences at the inputs of the third blocks between the interfering waves first retain zero values, and when the interferometer is scanned, they gradually increase and, finally, take on a maximum and constant value.

Thus, to measure the length of the X-ray train, it is advisable to use three-block interferometers with wedge-shaped blocks, i.e., the *a, b,* and *c* type interferometers (Figure 1). The advantage of the *a, b,* and *c* type interferometers is that in the interferometers the path difference between the interfering waves changes continuously so that it will allow detecting the moment when the path difference exceeds the train length. At first glance, the disadvantage of these interferometers is that the Bragg angles can be changed here due to the inclination of the incident surfaces. But the slope of the surfaces is very small (about 1^{o}–2^{o}), and for this reason, the values of the angles of reflection are stored with high accuracy, and the blocks retain their reflective positions.

4. Theoretical Analysis

Let us assume that the lower part of the second block of the interferometer has a form of a parallelepiped and the upper part - a wedge shape form (Figures 1 and 2). For calculation of the path difference that arises between the interfering waves in the interferometer, we use the scheme shown in Figure 2.

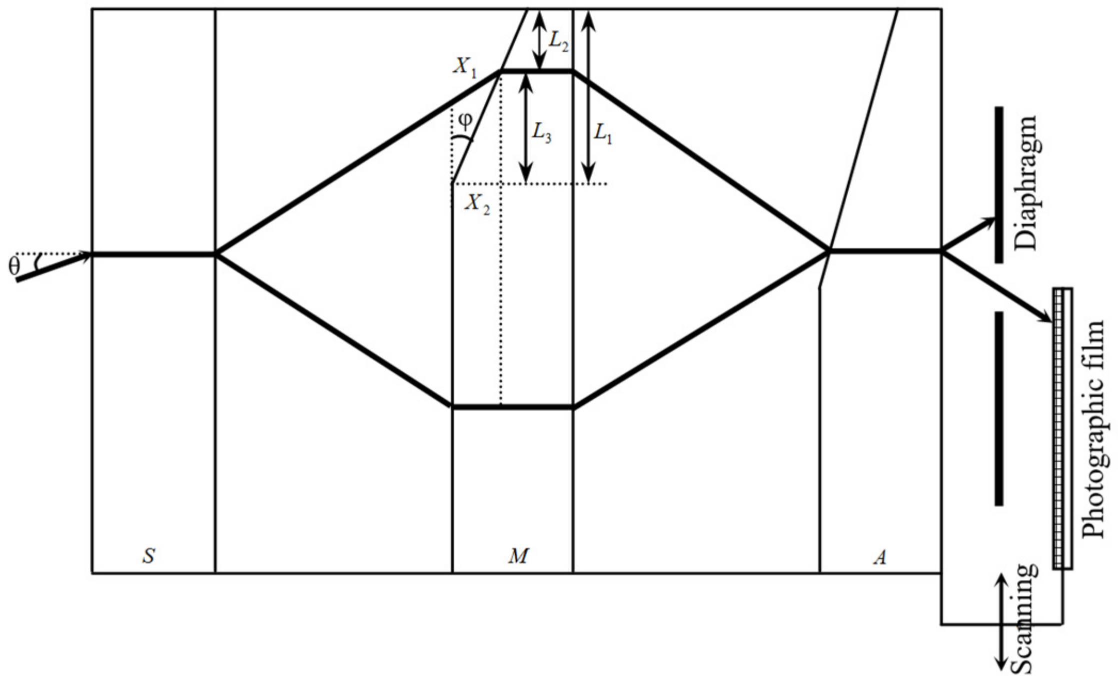

Figure 2. A scanning scheme for calculating the path difference between interfering waves in the train length meter

The following simple relation is obtained from Figure 2 for the path differences of interfering waves Δ:

$$\Delta = L_3 tg\varphi \cdot (\sec\theta - 1), \tag{6}$$

where L_3 is the distance of incidence point on the surface of the second block from the wedge`s base.

As we can see from (6), in the case of $\varphi=0$, the path difference is zero and increases with increasing of the distance L_3. When deriving the relation (6), the path differences are calculated as $\Delta = X_1 - X_2(1-\delta) = X_1 - X_2 + X_2\delta$, where δ is the unit decrement of refractive index ($\delta \sim 10^{-6}$). Keeping in mind the smallness of the $X_2\delta$, this difference was replaced by expression $\Delta = X_1 - X_2$.

5. Experimental Results and Their Discussion

To measure the length of the X-ray train, a special interferometer was designed and created from a dislocation-free silicon single crystal (Figure 2). In it, the $(1\bar{1}0)$ planes were perpendicular to the large surfaces and the base of the interferometer. The geometry of the interferometer was as follows: the distance between the blocks of the interferometer was 12.5 mm, the blocks had a thickness of 0.6 mm, width - 17.5 mm, height - 11.7 mm, L_1 = 4.84 mm, angles $\varphi = 2^o20'20''$, $\propto = 1°10'50''$at $(2\bar{2}0)$ reflection of the CuK_{α_1} radiation.

In the experiment, the increase in the difference between the paths of interfering waves is achieved by the interferometer scanning, the scheme of which and the corresponding interference topograms are shown in Figures 2 and 3.

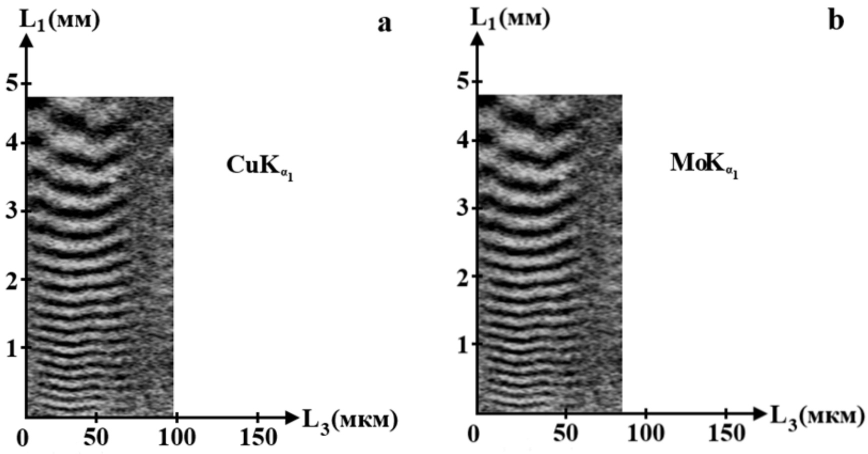

Figure 3. Interference topograms obtained by scanning: a) with CuK_{α_1} radiation, b) with MoK_{α_1} radiation

The value of the parameter L_3 =0.098mm, corresponding to the disappearance of the interference pattern (Figure 3a), was determined by formula (5), and for the value of the train length during the CuK_{α_1} radiation, $l = 3.76 \cdot 10^{-5}$cm was obtained.

The experiments were also carried out for MoK_{α_1} radiation (Figure 3b) and for the train length (dimensions of the coherent scattering volume) $l_{MoK_{\alpha_1}} = 6.87 \cdot 10^{-6}$cm was obtained.

Now let to compare the experimental results with theoretical results. As it is known (Born & Wolf, 1973), the length of train l is determined by the below relation:

$$l = c\tau, \tag{7}$$

where c is the propagation speed of electromagnetic waves.

The dimensions of coherently scattering domains of the irradiated volume are determined precisely by the length of train. If the dimensions of irradiated volume exceed the length of train, then the irradiated volume does not scatter the rays coherently as a whole and for this cause, methods for calculation of the intensity and width of reflexes are incorrect. As is seen from (7), the length of X-ray wave train is of the order of $10^{-5} \ldots 10^{-6}$ cm. Indeed, the values of τ for e.g., CuK_{α_1} and MoK_{α_1}, are respectively $1.077 \cdot 10^{-15}$ c and $2.29 \cdot 10^{-16}$ c, whence we obtain for sizes of coherently scattering volumes $l_{CuK_{\alpha 1}} = 3.23 \cdot 10^{-5}\,cm$ and $l_{MoK_{\alpha 1}} = 6.87 \cdot 10^{-5}\,cm$.

As can be seen, the experimental and theoretical results for the determination of the X-ray train length agree quite well.

The experiments were also carried out for MoK_{α_1} radiation (Figure 3b) and for the train length (dimensions of the coherent scattering volume) $l_{MoK_{\alpha_1}} = 6.87 \cdot 10^{-6}$cm was obtained.

6. Conclusion

Thus, as a result of the conducted experimental and theoretical studies, we can state the following:

1) The designed and developed interferometer of a special type is indeed an interferometer - X-ray train length meter.
2) The proposed interferometric method can be used to measure the X-ray train length with high accuracy.
3) Gradually increasing the path difference between the interfering waves using the scanning method, it’s possible to obtain an interference pattern, and, depending on the size of the path differences, determine the point of disappearance of this pattern, i.e., measure the X-ray train length.

Comparison of the experimental results obtained for determination of the X-ray train length with the theoretical value allows us to confirm that the proposed method provides high accuracy and reliable results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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