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Reverse Definite Integral of Algebraic Functions

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Abstract

In this work, an algebraic function is considered and integrated the function with some particular boundaries to obtain the area. Through the help of the area obtained and given boundaries, we determined different functions of different degree. Also, found a relationship between them.

Keywords: Degree, Area, Integration, Upper bound, Lower bound, Function, Reverse etc.

1. Introduction

We consider a function and boundaries with x-axis and calculate the area occupied by that function. What would be the case if we go reverse of it? We mean, we are given area and boundaries, can we predict the function? In my opinion yes we can predict the function. But, the degree of the function will be in our own decision. Assuming fixed area and boundaries, we are finding different functions. What can be the relationship between those functions? What can be the good interpretation of this? What may be the application of this approach?

2. Determination of Functions

In general any algebraic functions can be in the form of:

$y = mx^z + c$

where,
m = Slope or coefficient
z = Degree of the function
c = Constant

In the above function $y = mx^z + c$, when $z = 1$

$y = mx^1 + c$

$\Rightarrow Area = \int_0^7 y.dx$

$\Rightarrow Area = m\left[\frac{x^2}{2}\right]_0^7 + 7c$

$\therefore Area = 24.5 \times m + 7c$

Now,
Consider $Area = 24.5 \times m + 7c$
Boundaries with x-axis: $(0, 0) - (7, 0)$
Function=??

Let one degree function be $y = mx + c$ then,

$\int_0^7 y.dx = m\left[\frac{x^2}{2}\right]_0^7 + 7c$

$\therefore Area = m\left[\frac{x^2}{2}\right]_0^7 + 7c$

$\Rightarrow 24.5 \times m + 7c = m\left[\frac{x^2}{2}\right]_0^7 + 7c$, which is true.

Again, $\frac{d\frac{x^2}{2}}{dx} = \frac{1}{2}.2x = x$

$\therefore y = mx + c$ i.e the required one degree function is $y = mx + c$

Similarly, let two degree function be $y = mx^2 + c$ then

$\int_0^7 y.dx = m\left[\frac{x^3}{3}\right]_0^7 + 7c$

$\therefore Area = m\left[\frac{x^3}{3}\right]_0^7 + 7c$

$\Rightarrow 24.5 \times m + 7c = m\left[\frac{x^3}{3}\right]_0^7 + 7c$

$RHS = m\left[\frac{x^3}{3}\right]_0^7 + 7c$

$= \frac{7^3}{3}m + 7c$

$= 114.33 \times m + 7c$ (which is not equal to $24.5 \times m + 7c$)

Now, here m and $7c$ are common in both the result. Therefore, we will match the value by trying to reduce 114.33 to 24.5 and this can be done by either multiplying or dividing. We cannot match the value by addition or subtraction. It is because on derivative that will be zero.

Here, to get the actual result ($24.5 \times m + 7c$) we will divide $(\frac{x^3}{3})$ by $\frac{114.3}{24.5} = 4.66$

i.e $= \frac{\frac{x^3}{3}}{4.66}$

$= \frac{x^3}{13.99} \approx \frac{x^3}{14}$

$\therefore y = m\frac{d\frac{x^3}{14}}{dx} + c$

$= \frac{m}{14}.3x^2 + c$

$\therefore y = \frac{3m}{14}x^2 + c$ i.e the required second degree function is $y = \frac{3m}{14}x^2 + c$

Likewise, the third degree function is:

$y = \frac{4m}{98}x^3 + c$

Similarly, when $z = 2$ i.e., for function $y = mx^2 + c$

On applying above method for the function $y = mx^2 + c$, within the boundary $(0 - 7)$ the first three functions giving same result on integration are:

1. $y = mx^2 + c$
2. $y = \frac{4m}{21}x^3 + c$
3. $y = \frac{5m}{147}x^4 + c$

2.1 Checking if Determined Functions are True or False

(1) $y = mx + c$

Solution:

$Area = \int_0^7 y.dx$

$\Rightarrow Area = \int_0^7 (mx + c).dx$

$\Rightarrow Area = m\left[\frac{x^2}{2}\right]_0^7 + 7c$

$\therefore Area = 24.5 \times m + 7c$ (which is true)

(2) $y = \frac{3m}{14}x^2 + c$

Solution:

$Area = \int_0^7 y.dx$

$\Rightarrow Area = \int_0^7 (\frac{3m}{14}x^2 + c).dx$

$\Rightarrow Area = \frac{3m}{14}\left[\frac{x^3}{3}\right]_0^7 + 7c$

$\therefore Area = 24.5 \times m + 7c$ (which is true)

(3) $y = \frac{4m}{98}x^3 + c$

Solution:

$Area = \int_0^7 y.dx$

$\Rightarrow Area = \int_0^7 (\frac{4m}{98}x^3 + 2.5).dx$

$\Rightarrow Area = \frac{4m}{98}\left[\frac{x^4}{4}\right]_0^7 + 7c$

$\therefore Area = 24.5 \times m + 7c$ (which is true)

3. Determination of Relation Between Functions

First, three functions giving same result on integration of function $y = mx^1 + c$ are:

1. $y = mx + c = \frac{2m}{2}x + c = \frac{2}{\frac{2}{m}}x + c$
2. $y = \frac{3m}{14}x^2 + c = \frac{3}{\frac{14}{m}}x^2 + c$
3. $y = \frac{4m}{98}x^3 + c = \frac{4}{\frac{98}{m}}x^3 + c$

Now, taking only denominator part of above functions, we get: $\frac{2}{m}, \frac{14}{m}, \frac{98}{m}, ...$, in G.P

$\therefore T_n = a.r^{n-1}$

$\Rightarrow T_n = \frac{2}{m}.7^{n-1}$

again, $y = \frac{(n+1).m}{\frac{2}{7}.7^n}x^n + c$

$\Rightarrow y = \frac{(n+1).m.7^{1-n}}{2}x^n + c$

Since, difference of boundaries is 7 i.e $7 - 0 = 7$, let us denote 7 in above relation by $\triangle N$ for general. then above relation can be written as,

$y = \frac{(n+1).m.(\triangle N)^{1-n}}{2}x^n + c.$

Thus, relation is found as:

$$y = \frac{(n+1).m.(\triangle N)^{1-n}}{2}x^n + c \tag{1}$$

Similarly, first three functions giving same result on integration of function $y = mx^2 + c$ are:

1. $y = mx^2 + c = \frac{3m}{3}x^2 + c = \frac{3}{\frac{3}{m}}x^2 + c$
2. $y = \frac{4m}{21}x^3 + c = \frac{4}{\frac{21}{m}}x^3 + c$
3. $y = \frac{5m}{147}x^4 + c = \frac{5}{\frac{147}{m}}x^4 + c$

Now, taking only denominator part of above functions, we get: $\frac{3}{m}, \frac{21}{m}, \frac{147}{m}, ...$, in G.P

$\therefore T_n = a.r^{n-2}$

Here, instead of using $(n - 1)$ we have used $(n - 2)$. It is because in the above mentioned list of functions the power of variable x starts to increase from 2 and also in this G.P: $n = 2, 3, 4, ...$

$\Rightarrow T_n = \frac{3}{m}.7^{n-2}$

again, $y = \frac{(n+1).m}{\frac{3}{7^2}.7^n}x^n + c$

$\Rightarrow y = \frac{(n+1).m.7^{2-n}}{3}x^n + c$

$\Rightarrow y = \frac{(n+1).m.(\triangle N)^{2-n}}{3}x^n + c.$

Thus, relation is found as:

$$y = \frac{(n+1).m.(\triangle N)^{2-n}}{3}x^n + c \tag{2}$$

From above relations (1) and (2), we can derive more general relation as:

$y = \frac{(\triangle N)^{(\xi-n)}.(1+n).m.x^n}{(\xi+1)} + c$

$\Rightarrow y = (\triangle N)^{(\xi - n)}.(1 + n).m.(\xi + 1)^{-1}.x^n + c$
where, ξ = degree of variable in original function.

Thus, the required relation between functions is found to be: $y = (\triangle N)^{(\xi - n)}.(1 + n).m.(\xi + 1)^{-1}.x^n + c$

3.1 Checking if Determined Relation is True or False

Let us suppose the original function,

$$y = \frac{5}{x^4} + 2.5 = 5x^{-4} + 2.5 \tag{3}$$

now using above determined relation, let's determine the function of power 8 within the boundary (0 − 9.78):

here, given:

$\triangle N = 9.78 - 0 = 9.78$

$\xi = -4$

$n = 8$

$m = 5$

$c = 2.5$

now, we have

$y = (\triangle N)^{(\xi - n)}.(1 + n).m.(\xi + 1)^{-1}.x^n + c$

$\Rightarrow y = (9.78)^{(-4-8)}.(1 + 8).5.(-4 + 1)^{-1}.x^8 + 2.5$

$\Rightarrow y = \frac{-45}{2.30 \times 10^{12}} x^8 + 2.5$

therefore, the required function is:

$$y = \frac{-45}{2.30 \times 10^{12}} x^8 + 2.5 \tag{4}$$

Similarly, let's determine the function of power −2 within the boundary (0 − 9.78):

here, given:

$\triangle N = 9.78 - 0 = 9.78$

$\xi = -4$

$n = -2$

$m = 5$

$c = 2.5$

now, we have

$y = (\triangle N)^{(\xi - n)}.(1 + n).m.(\xi + 1)^{-1}.x^n + c$

$\Rightarrow y = (9.78)^{(-4+2)}.(1 - 2).5.(-4 + 1)^{-1}.x^{-2} + 2.5$

$\Rightarrow y = \frac{5}{286.95} x^{-2} + 2.5$

therefore, the required function is:

$$y = \frac{5}{286.95} x^{-2} + 2.5 \tag{5}$$

Again, let's determine the function of power $\frac{9}{5}$ within the boundary (0 − 9.78):

here, given:

$\triangle N = 9.78 - 0 = 9.78$

$\xi = -4$

$n = \frac{9}{5}$

$m = 5$

$c = 2.5$

Now, we have

$y = (\triangle N)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.x^n + c$

$\Rightarrow y = (9.78)^{(-4-\frac{9}{5})}.(1+\frac{9}{5}).5.(-4+1)^{-1}.x^{\frac{9}{5}} + 2.5$

$\Rightarrow y = \frac{-14}{9.78^{\frac{29}{5}} \times 3} x^{\frac{9}{5}} + 2.5$

therefore, the required function is:

$$y = \frac{-14}{1663744.42} x^{\frac{9}{5}} + 2.5 \tag{6}$$

Integrating functions (3), (4), (5) and (6) and seeing the result if they give same or not!

(1) $y = 5x^{-4} + 2.5$

Solution:

$Area = \int_0^{9.78} y.dx$

$\Rightarrow Area = \int_0^{9.78} (5x^{-4} + 2.5).dx$

$\Rightarrow Area = 5\left[\frac{x^{-3}}{-3}\right]_0^{9.78} + 2.5 \times 9.78$

$\therefore Area = 24.45$

(2) $y = \frac{-45}{2.30\times10^{12}} x^8 + 2.5$

Solution:

$Area = \int_0^{9.78} y.dx$

$\Rightarrow Area = \int_0^{9.78} (\frac{-45}{2.30\times10^{12}} x^8 + 2.5).dx$

$\Rightarrow Area = \frac{-45}{2.30\times10^{12}} \left[\frac{x^9}{9}\right]_0^{9.78} + 2.5 \times 9.78$

$\therefore Area = 24.45$

(3) $y = \frac{5}{286.95} x^{-2} + 2.5$

Solution:

$Area = \int_0^{9.78} y.dx$

$\Rightarrow Area = \int_0^{9.78} (\frac{5}{286.95} x^{-2} + 2.5).dx$

$\Rightarrow Area = \frac{5}{286.95} \left[\frac{x^{-1}}{-1}\right]_0^{9.78} + 2.5 \times 9.78$

$\therefore Area = 24.45$

(4) $y = \frac{-14}{1663744.42} x^{\frac{9}{5}} + 2.5$

Solution:

$Area = \int_0^{9.78} y.dx$

$\Rightarrow Area = \int_0^{9.78} (\frac{-14}{1663744.42} x^{\frac{9}{5}} + 2.5).dx$

$\Rightarrow Area = \frac{-14}{1663744.42} \cdot \frac{5}{14} \left[x^{\frac{14}{5}}\right]_0^{9.78} + 2.5 \times 9.78$

$\therefore Area = 24.45$

Since, the integration of functions (3), (4), (5) and (6) within the same boundary gave same result.

Therefore, we can conclude that the relation we derived is true.

4. Possible Work

The main drawback of this work is that for: $\triangle N$ = upper bound - lower bound, the lower bound must always be equal to zero. I am still in the process to make this work more elegant and vast. But, if lower bound is not equal to zero, we can solve by alternative method using following relation:

$\int_L^U y.dx = [(U)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.\int_0^U x^n.dx + U \times c] - [(L)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.\int_0^L x^n.dx + L \times c]$

NOTE:-In above relation:
U = Upper Bound
L = Lower Bound

Example::
Let us suppose a function,

$$y = 2x^9 + 12 \tag{7}$$

Now, using above relation let's determine the function of power 5 within the boundary (4 − 15):

Solution, here:

$U = 15$

$\xi = 9$

$n = 5$

$m = 2$

$L = 4$

$c = 12$

now, we have

$y = [(U)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.x^n + c] - [(L)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.x^n + c]$

$\Rightarrow y = [(15)^{(9-5)}.(1+5).2.(9+1)^{-1}.x^5 + 12] - [(4)^{(9-5)}.(1+5).2.(9+1)^{-1}.x^5 + 12]$

$\Rightarrow y = [60750.x^5 + 12] - [\frac{1536}{5}.x^5 + 12]$

thus, the required function is:

$$y = [60750.x^5 + 12] - [\frac{1536}{5}.x^5 + 12] \tag{8}$$

Now, integrating functions (7) and (8) and seeing the result if they give same or not!

(1) $y = 2x^9 + 12$

Solution:

$Area = \int_4^{15} y.dx$

$\Rightarrow Area = \int_4^{15} (2x^9 + 12).dx$

$\Rightarrow Area = 2\left[\frac{x^{10}}{10}\right]_4^{15} + 12 \times (15-4)$

$\therefore Area = 1.15 \times 10^{11}$

(2) $y = [60750.x^5 + 12] - [\frac{1536}{5}.x^5 + 12]$

Solution:

$\int_4^{15} y.dx = \int_0^{15} (60750.x^5 + 12).dx - \int_0^4 (\frac{1536}{5}.x^5 + 12).dx$

$\Rightarrow Area = 60750\left[\frac{x^6}{6}\right]_0^{15} + 12 \times 15 - \frac{1536}{5}\left[\frac{x^6}{6}\right]_0^4 - 12 \times 4$

$\Rightarrow Area = 1.15 \times 10^{11}$

$\therefore Area = 1.15 \times 10^{11}$

Since, the integration of functions (7) and (8) within the same boundary gave same result.

Therefore, we can conclude that the alternative relation is true.

5. Conclusion

Thus, we successfully determined different functions of different degree when area and boundaries are provided. we also determined the relation between number of functions and it is found to be:

$y = (\triangle N)^{(\xi-n)}.(1+n).m.(\xi+1)^{-1}.x^n + c$

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