Journal of Mathematics Research; Vol. 10, No. 1; February 2018
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Numerical Solution of System of Three Nonlinear Volterra Integral Equations Using Implicit Trapezoidal

Dalal Adnan Maturi[1] & Honaida Mohammed Malaikah[2]

[1] Departement of Mathematics, Faculty of Science, King Abdulaziz University,Jeddah, Saudi Arabia

[2] Departement of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

Correspondence: Dalal Adnan Maturi, Departement of Mathematics, Faculty of Science, King Abdulaziz University, P.O.Box 42664, Jeddah 21551, Saudi Arabia. E-mail: maturi_dalal2020@yahoo.com

Received: November 15, 2017 Accepted: December 1, 2017 Online Published: December 13, 2017

doi:10.5539/jmr.v10n1p44 URL: https://doi.org/10.5539/jmr.v10n1p44

Abstract

In this project, we will be find numerical solution of Volterra Integral Equation of Second kind through using Implicit trapezoidal and that by using Maple 17 program, then we found that numerical solution was highly accurate when it was compared with exact solution.

Keywords: volterra integral equation of second kind, implicit trapezoidal method, nonlinear programing

1. Introduction

We consider the Volterra integral equation of the second kind

$$x(t) = f(t) + \int_0^t k\big(t, s, x(s)\big)\, ds \tag{1}$$

Where x, f and k are vector-valued functions with m components. If f and k are continuous and $k(t, s, x(s))$ satisfies a Lipcshitz condition with respect to x, then a unique solution $x(t)$ of (1) exists (Balakumar, V. & Murugesan, K., 2013; Berenguer, M. I. & et al., 2011; Burton,T. A., 2005).

Volterra integral equations have been found to be effective to describe some application such as potential theory and Dirichlet problemsand electrostatics. Also, Volterra integral equations areapplied in the biology, chemistry, engineering, mathematical problems of radiation equilibrium,theparticle transport problems of astrophysics and reactor theory, and radiation heat transfer problems (Balakumar, V. & Murugesan, K., 2013; Maturi, D. A., 2014).

The systems of Volterra integral equations appear in two kinds. For systems of Volterra integral equations of the first kind, the unknown functions appear only under the integral sign in the form:

$$f_1(t) = \int_0^t (K_1(t,s)u(s) + \widetilde{K_1}(t,s)v(s) + \cdots)ds$$

$$f_2(t) = \int_0^t (K_2(t,s)u(s) + \widetilde{K_2}(t,s)v(s) + \cdots)ds,$$

However, systems of Volterra integral equations of the second kind, the unknown functions appear inside and outside the integral sign of the form:

$$u(t) = f_1(t) + \int_0^t (K_1(t,s)u(s) + \widetilde{K_1}(t,s)v(s) + \cdots)ds$$

$$v(t) = f_2(t) + \int_0^t (K_2(t,s)u(s) + \widetilde{K_2}(t,s)v(s) + \cdots)ds,$$

The kernels $K_i(t,s)$and$\widetilde{K_i}(t,s)$, and the functions $f_i(t), i = 1,2,\cdots,n$ are given real-valued functions.

A variety of analytical and numerical methods are used to handle systems of Volterra integral equations. The existing techniques encountered some difficulties in terms of the size of computational work, especially when the system involves several integral equations (Linz, P., 1985).

In this project, we present the computation of numerical solution of systems of Volterra integral equation of the second kind.

2. Preliminaries

In this section, we recall the main theorems (Effati, S. & NooriSkandari, M. H., 2012).

Theorem 1. Consider the equation

$$x(t) = f(t) + \int_0^t p(t,s)k(t,s)x(s)\,ds \tag{2}$$

Where

1) $f(t)$is continuous in $0 \le t \le T$.
2) $k(t,s)$ is a continuous function in $0 \le s \le t \le T$,
3) for each continuous function $h$andall
   $0 \le \tau_1 \le \tau_2 \le t$ the integrals

$$\int_{\tau_1}^{\tau_2} p(t,s)k(t,s)h(s)\,ds$$

$$\int_0^t p(t,s)k(t,s)h(s)\,ds$$

   are continuous functions of t ,
4) $p(t,s)$ is absolutely integrable with respect to s for all $0 \le t \le T$,
5) there exist points$0 = T_0 < T_1 < T_2 < \cdots < T_N = T$ such that with $t \ge T_1$

$$k \int_{T_i}^{\min(t,T_{i+1})} |p(t,s)|\,ds \le \alpha < 1,$$

Were $k = \max_{0\le s\le t\le T}|k(t,s)|$,

6) for every $t \ge 0$ such that with $t \ge T_1$

$$\lim_{\delta\to 0^+} \int_t^{t+\delta} |p(t+\delta,s)|\,ds = 0.$$

Then (2) has a unique continuous solution in $0 \le t \le T$.

Theorem 2. Consider the equation

$$x(t) = f(t) + \int_0^t p(t,s)k(t,s,x(s))\,ds \tag{3}$$

Where

1) $f(t)$is continuous in $0 \le t \le T$.
2) $k(t,s,u)$ is a continuous function in $0 \le s \le t \le T, -\infty < u < \infty$,
3) the Lipschitz condition
   $|k(t,s,y)$ - $k(t,s,z)| \le L|y-z|$

is satisfies for $0 \le s \le t \le T$ and all y and z,

4) $p(t,s)$ satisfies conditions (3)-(4) of Theroem 1 with k replaced by L and $k(t,s,h(s))$ instead of $k(t,s)h(s)$.

Then (3) has a unique continuous solution in $0 \le t \le T$.

3. The Mathematics of the Volterra Procedure

In this section, we use the technique of the Volterra equation (Balakumar, V. & Murugesan, K., 2013; Effati, S. & NooriSkandari, M. H., 2012) to find an approximates the solution $x(t)$ of (1) at the equally spaced points $t_n = t_0 + nh$ for $n = 1, \cdots, N$ where $t_0 = 0$ and $N$is the total number of steps of size $h. X_n$ denotes the approximation of $x(t)$ at $t = t_n$.

Setting $t = t_n$ in (1), we have

$$x(t_n) = f(t_n) + \int_0^{t_n} k(t_n, t, x(t))\, dt \tag{4}$$

By the composite trapezoidal rule an approximation of the integral in (4) is

$$\frac{h}{2}\left[k\big(t_n, t_0, x(t_0)\big) + 2\sum_{j=1}^{n-1} k(t_n, t_j, x\big(t_j\big)) + k(t_n, t_n, x(t_n))\right] \tag{5}$$

Replacing $x(t_n)$ in (4) and (5) by X_n, we obtain the implicit trapezoidal rule

$$X_n = f(t_n) + h\left[\frac{1}{2}k(t_n, t_0, X_0) + \sum_{j=1}^{n-1} k(t_n, t_j, X_j) + \frac{1}{2}k(t_n, t_n, X_n)\right] \tag{6}$$

Where $X_0 = f(0)$since $x(0) = f(0)$.

Defining σ_n by

$$\sigma_n = f(t_n) + h\left[\frac{1}{2}k(t_n, t_0, X_0) + \sum_{j=1}^{n-1} k(t_n, t_j, X_j)\right] \tag{7}$$

We can rewrite (6) as

$$X_n - \frac{1}{2}hk(t_n, t_n, X_n) - \sigma_n = 0, \tag{8}$$

Where 0denotes the zero vector. From (8), we see that X_n is the solution of the vector equation

$$\phi(u) = 0, \tag{9}$$

Where ϕ is the vector-valued function

$$\phi(u) = u - \frac{1}{2}hk(t_n, t_n, u) - \sigma_n \tag{10}$$

We will obtain an approximation to the solution $X_n$of (9) by way of the matrix-valued function Gdefined in (11). If $A(u)$ is an m by m matrix-valued function that is invertible in a neighborhood of X_n, then X_n is a fixed point of

$$G(u) = u - A(u)\phi(u). \tag{11}$$

Assuming the components of $G(u)$ have continuous first and second order partial derivatives and that the first order partial derivatives and that the first order partial derivatives at

$X_n$are equal to zero, it can be shown that if $A(u)$ is set equal to the Jacobian matrix of the function ϕ, the iterates $X_n^{(p)}$ defined by (13) below will usually converge quadratically to X_n provided the starting value is sufficiently close to X_n. The Jacobian matrix of ϕ is the m by m matrix $J(u)$ with the element

$$J(u)_{ij} = \frac{\partial}{\partial u_j}\phi_i(u) = \delta_{ij} - \frac{1}{2}h\frac{\partial}{\partial u_j}k_i(t_n, t_n, u) \tag{12}$$

In row i and column, where δ_{ij} is the Kronecker delta. Details of the statements made here follow from the discussion of Newton's method for nonlinear systems in (Balakumar, V. & Murugesan, K., 2013). Linz gives a brief outline of the trapezoidal rule and Newton's method for Volterra integral systems of the second kind in Section of (Effati, S. & NooriSkandari, M. H., 2012).

We obtain X_n from X_{n-1} by setting $X_n^{(0)} = X_{n-1}$ and then generating the iterates $X_n^{(p)}$ from

$$X_n^{(p)} = G\left(X_n^{(p-1)}\right) = X_n^{(p-1)} - J^{-1}\left(X_n^{(p-1)}\right)\phi\left(X_n^{(p-1)}\right) \tag{13}$$

For $p = 1,2,3,\cdots$. (This is Newton's method for nonlinear systems.) Let y denote the solution of the matrix equation

$$J\left(X_n^{(p-1)}\right)y = \phi\left(X_n^{(p-1)}\right). \tag{14}$$

Then the iteration formula (13) becomes

$$X_n^{(p)} = X_n^{(p-1)} - y. \tag{15}$$

We compute the solution $y = J^{-1}\left(X_n^{(p-1)}\right)\phi\left(X_n^{(p-1)}\right)$ using the command Linear Solve. The iterates $X_n^{(p)}$ are computed until the infinity norm of the vector y is less than a prescribed tolerance Tol. Then X_n is assigned the value of the last iterate (Balakumar, V. & Murugesan, K., 2013; Effati, S. & NooriSkandari, M. H., 2012).

4. Numerical Example

In this section, we solve some examples, and we can compare the numerical results with the exact solution.

Example1. Consider the system of Volterra integral equations

$$\mathrm{X}_1(t) = 1 - t^2 + \frac{1}{4}t^4 + \int_0^t (\mathrm{X}_1(s)+\mathrm{X}_2(s) - \mathrm{X}_3(s))ds$$

$$\mathrm{X}_2(t) = 2t + t^2 - \frac{2}{3}t^3 - \frac{1}{4}t^4 + \int_0^t (\mathrm{X}_2(s)+\mathrm{X}_3(s) - \mathrm{X}_1(s))ds$$

$$\mathrm{X}_3(t) = -t + t^2 + t^3 - \frac{1}{4}t^4 + \int_0^t (\mathrm{X}_3(s) + \mathrm{X}_1(s) - \mathrm{X}_2(s))ds$$

With the exact solution $\mathrm{X}_1(t) = 1 + t$, $\mathrm{X}_2(t) = t + t^2$ and $\mathrm{X}_3(t) = t^2 + t^3$.

Table.1 Numerical results and exact solution of systems of three Nonlinear Volterra integral equations for example 1.

$\boldsymbol{t}$	$\boldsymbol{X_1(t)}$	$\boldsymbol{X_2(t)}$	$\boldsymbol{X_3(t)}$	$\boldsymbol{Exact1 = 1 + t}$	$\boldsymbol{Exact2 = t + t^2}$	$\boldsymbol{Exact3 = t^2 + t^3}$
0.00	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.05	1.0500	0.0525	0.0026	1.0500	0.0525	0.0026
0.10	1.1000	0.1100	0.0110	1.1000	0.1100	0.0110
0.15	1.1500	0.1725	0.0259	1.1500	0.1725	0.0259
0.20	1.2000	0.2400	0.0480	1.2000	0.2400	0.0480
0.25	1.2500	0.3125	0.0781	1.2500	0.3125	0.0781
0.30	1.3000	0.3900	0.1170	1.3000	0.3900	0.1170
0.35	1.3500	0.4725	0.1654	1.3500	0.4725	0.1654
0.40	1.4000	0.5600	0.2240	1.4000	0.5600	0.2240
0.45	1.4500	0.6525	0.2936	1.4500	0.6525	0.2936
0.50	1.5000	0.7500	0.3750	1.5000	0.7500	0.3750
0.55	1.5500	0.8525	0.4689	1.5500	0.8525	0.4689

0.60	1.6000	0.9600	0.5760	1.6000	0.9600	0.5760
0.65	1.6500	1.0725	0.6971	1.6500	1.0725	0.6971
0.70	1.7000	1.1900	0.8330	1.7000	1.1900	0.8330
0.75	1.7500	1.3126	0.9844	1.7500	1.3125	0.9844
0.80	1.8000	1.4401	1.1520	1.8000	1.4400	1.1520
0.85	1.8500	1.5726	1.3366	1.8500	1.5725	1.3366
0.90	1.9000	1.7101	1.5390	1.9000	1.7100	1.5390
0.95	1.9500	1.8526	1.7599	1.9500	1.8525	1.7599
1.00	2.0000	2.0001	2.0000	2.0000	2.0000	2.0000
1.05	2.0500	2.1526	2.2601	2.0500	2.1525	2.2601
1.10	2.1000	2.3101	2.5410	2.1000	2.3100	2.5410
1.15	2.1500	2.4726	2.8434	2.1500	2.4725	2.8434
1.20	2.2000	2.6401	3.1680	2.2000	2.6400	3.1680
1.25	2.25000	2.8126	3.5156	2.2500	2.8125	3.5156
1.30	2.3000	2.9901	3.8870	2.3000	2.9900	3.8870
1.35	2.3500	3.1726	4.2828	2.3500	3.1725	4.2829
1.40	2.4001	3.3602	4.7040	2.4000	3.3600	4.7040
1.45	2.4501	3.5527	5.1511	2.4500	3.5525	5.1511
1.50	2.5001	3.7502	5.6250	2.5000	3.7500	5.6250

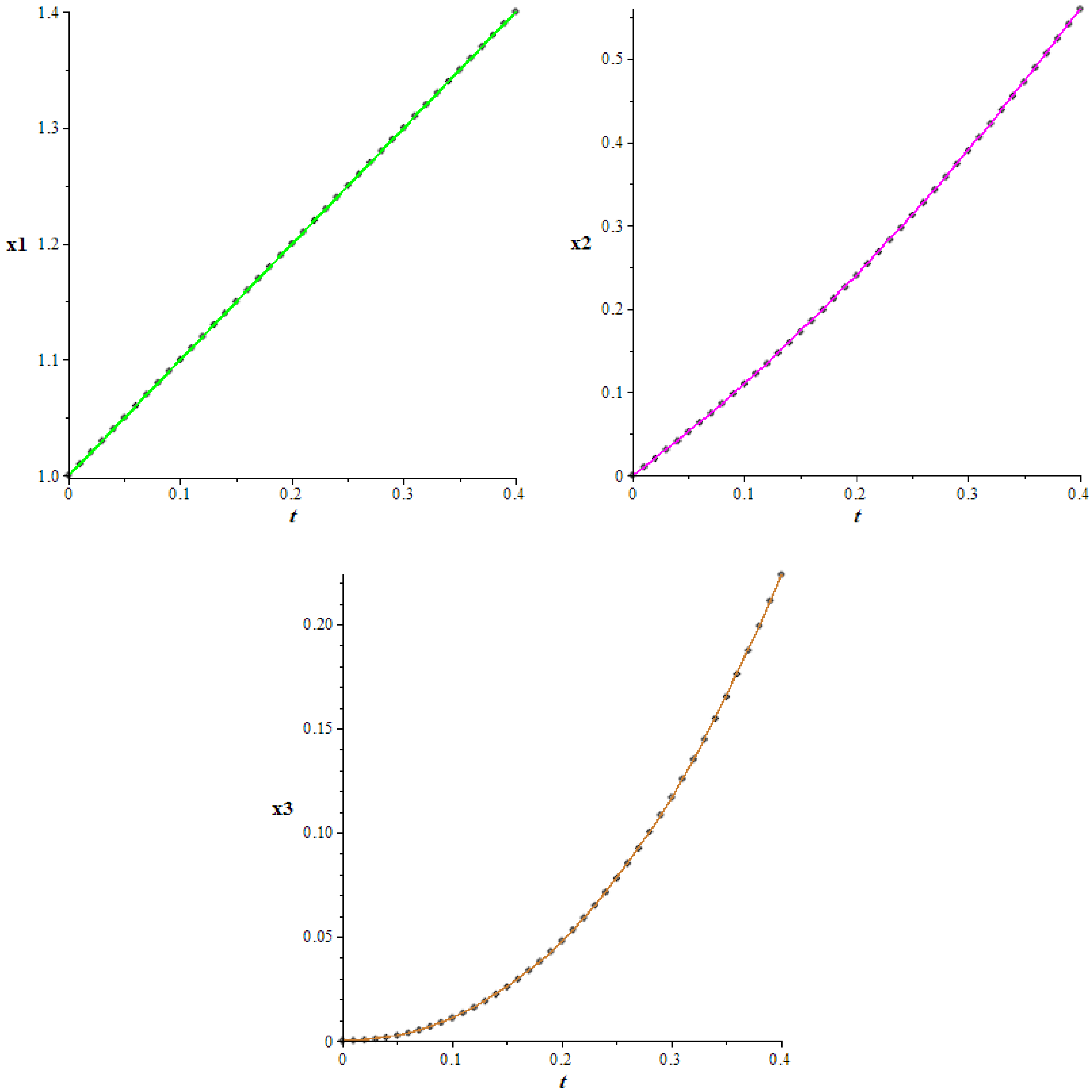

Fig. 1 The exact and approximate solutions result of systems of three Nonlinear Volterra integral equations for example 1.

Example2. Consider the system of Volterra integral equations

$$X_1(t) = \cos t - \sin t - 1 + \int_0^t (X_2(s)+X_3(s))ds$$

$$X_2(t) = 3\cos t - \sin t - 2 + \int_0^t (X_1(s)+X_3(s))ds$$

$$X_3(t) = 2\cos t - 1 + \int_0^t (X_1(s)+X_2(s))ds$$

With the exact solution $X_1(t) = \sin t$, $X_2(t) = \cos t$ and $X_3(t) = \sin t + \cos t$.

Table.2 Numerical results and exact solution of systems of three Nonlinear Volterra integral equations for example2.

t	$X_1(t)$	$X_2(t)$	$X_3(t)$	$Exact1 = \sin t$	$Exact2 = \cos t$	$Exact3 = \sin t + \cos t$
0.00	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000
0.05	0.0500	0.9987	1.0487	0.0500	0.9988	1.0487
0.10	0.0998	0.9950	1.0948	0.0998	0.9950	1.0948
0.15	0.1494	0.9888	1.1382	0.1494	0.9888	1.1382
0.20	0.1987	0.9801	1.1787	0.1987	0.9801	1.1787
0.25	0.2474	0.9689	1.2163	0.2474	0.9689	1.2163
0.30	0.2955	0.9553	1.2509	0.2955	0.9553	1.2509
0.35	0.3429	0.9394	1.2823	0.3429	0.9394	1.2823
0.40	0.3894	0.9211	1.3105	0.3894	0.9211	1.3105
0.45	0.4350	0.9004	1.3354	0.4350	0.9004	1.3354
0.50	0.4794	0.8776	1.3570	0.4794	0.8776	1.3570
0.55	0.5227	0.8525	1.3752	0.5227	0.8525	1.3752
0.60	0.5646	0.8253	1.3900	0.5646	0.8253	1.3900
0.65	0.6052	0.7961	1.4013	0.6052	0.7961	1.4013
0.70	0.6442	0.7648	1.4090	0.6442	0.7648	1.4091
0.75	0.6816	0.7317	1.4133	0.6816	0.7317	1.4133
0.80	0.7173	0.6967	1.4140	0.7174	0.6967	1.4141
0.85	0.7512	0.6600	1.4112	0.7513	0.6600	1.4113
0.90	0.7833	0.6216	1.4049	0.7833	0.6216	1.4049
0.95	0.8134	0.5816	1.3951	0.8134	0.5817	1.3951
1.00	0.8414	0.5403	1.3817	0.8415	0.5403	1.3818
1.05	0.8674	0.4975	1.3649	0.8674	0.4976	1.3650
1.10	0.8912	0.4535	1.3448	0.8912	0.4536	1.3448
1.15	0.9127	0.4084	1.3212	0.9128	0.4085	1.3213
1.20	0.9320	0.3623	1.2943	0.9320	0.3624	1.2944
1.25	0.9489	0.3152	1.2642	0.9490	0.3153	1.2643
1.30	0.9635	0.2674	1.2310	0.9636	0.2675	1.2311
1.35	0.9756	0.2189	1.1946	0.9757	0.2190	1.1947
1.40	0.9853	0.1699	1.1553	0.9854	0.1700	1.1554
1.45	0.9926	0.1204	1.1131	0.9927	0.1205	1.1132
1.50	0.9974	0.0706	1.0681	0.9975	0.0707	1.0682

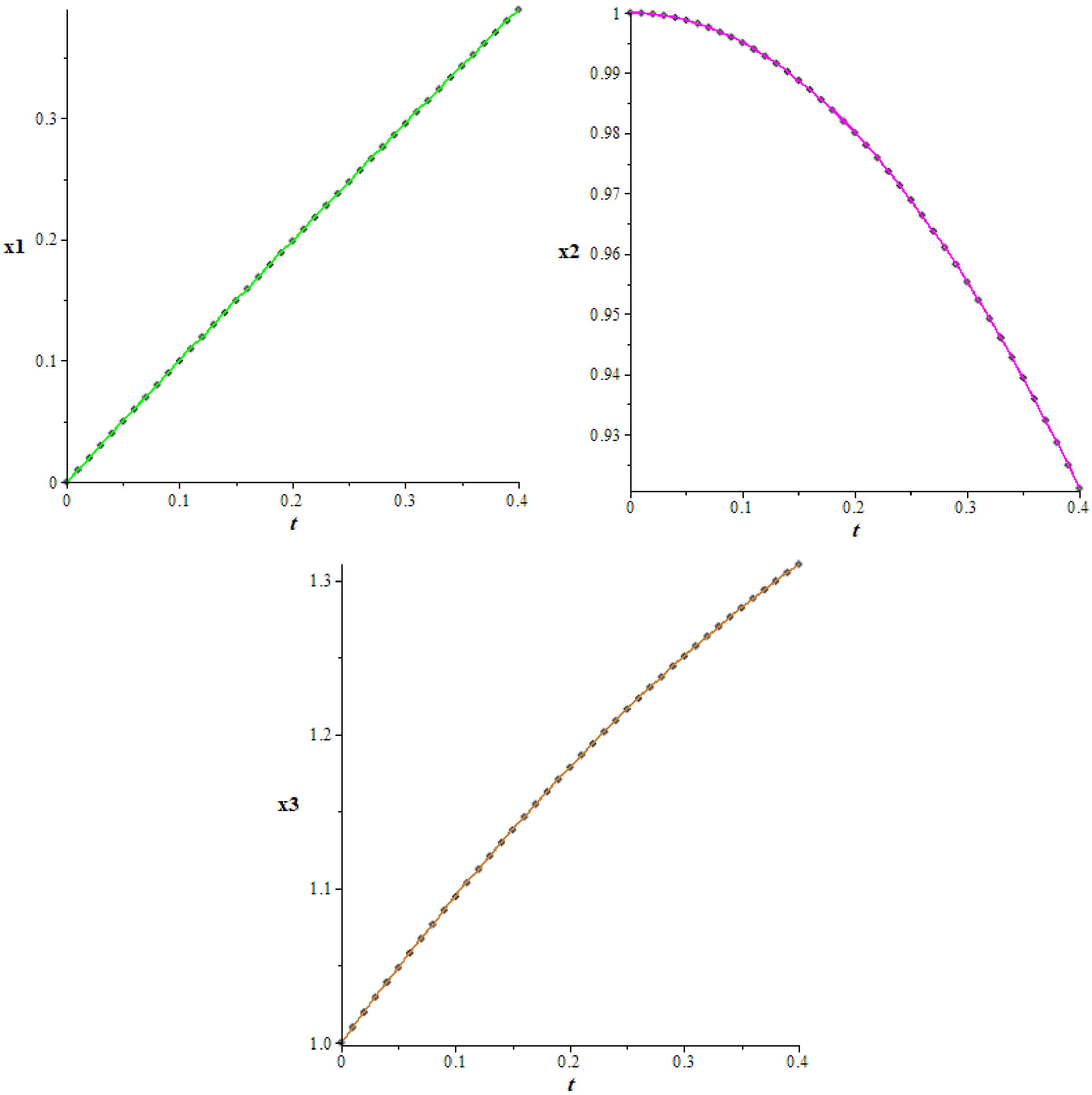

Fig. 2 The exact and approximate solutions result of systems of three Nonlinear Volterra integral equations for example 2.

Example3. Consider the system of Volterra integral equations

$$X_1(t) = 3t + \cos 2t - t^3 + \int_0^t (X_1(s) + 2X_2(s) + 3X_3(s))ds$$

$$X_2(t) = \frac{3}{2}\sin 2t - te^t + \int_0^t (tX_1(s) + (2t - 2s)X_2(s))ds$$

$$X_3(t) = t^2 - e^t + \int_0^t X_1(s)\, ds$$

With the exact solution $X_1(t) = e^t$, $X_2(t) = \sin 2t$ and $X_3(t) = t^2 - 1$.

Table.4 Numerical results and exact solution of systems of three Nonlinear Volterra integral equations for example 4.

t	$X_1(t)$	$X_2(t)$	$X_3(t)$	$Exact1 = e^t$	$Exact2 = \sin 2t$	$Exact3 = t^2 - 1$
0.00	1.0000	0.0000	-1.0000	1.0000	0.0000	-1.0000
0.05	1.0513	0.0998	-0.9975	1.0513	0.0998	-0.9975
0.10	1.1052	0.1987	-0.9900	1.1052	0.1987	-0.9900
0.15	1.1618	0.2955	-0.9775	1.1618	0.2955	-0.9775
0.20	1.2214	0.3894	-0.9600	1.2214	0.3894	-0.9600
0.25	1.2840	0.4794	-0.9375	1.2840	0.4794	-0.9375
0.30	1.3499	0.5646	-0.9100	1.3499	0.5646	-0.9100
0.35	1.4191	0.6442	-0.8775	1.4191	0.6442	-0.8775
0.40	1.4918	0.7173	-0.8400	1.4918	0.7174	-0.8400
0.45	1.5683	0.7833	-0.7975	1.5683	0.7833	-0.7975
0.50	1.6487	0.8414	-0.7500	1.6487	0.8415	-0.7500
0.55	1.7333	0.8912	-0.6975	1.7333	0.8912	-0.6975
0.60	1.8221	0.9320	-0.6400	1.8221	0.9320	-0.6400
0.65	1.9155	0.9635	-0.5775	1.9155	0.9636	-0.5775
0.70	2.0138	0.9854	-0.5100	2.0138	0.9854	-0.5100
0.75	2.1170	0.9975	-0.4375	2.1170	0.9975	-0.4375
0.80	2.2255	0.9995	-0.3600	2.2255	0.9996	-0.3600
0.85	2.3396	0.9916	-0.2775	2.3396	0.9917	-0.2775
0.90	2.4596	0.9738	-0.1900	2.4596	0.9738	-0.1900
0.95	2.5857	0.9463	-0.0975	2.5857	0.9463	-0.0975
1.00	2.7183	0.9093	0.0000	2.7183	0.9093	0.0000
1.05	2.8576	0.8632	0.1025	2.8577	0.8632	0.1025
1.10	3.0042	0.8084	0.2100	3.0042	0.8085	0.2100
1.15	3.1582	0.7457	0.3225	3.1582	0.7457	0.3225
1.20	3.3201	0.6754	0.4400	3.3201	0.6755	0.4400
1.25	3.4903	0.5984	0.5625	3.4903	0.5985	0.5625
1.30	3.6693	0.5154	0.6900	3.6693	0.5155	0.6900
1.35	3.8574	0.4273	0.8225	3.8574	0.4274	0.8225
1.40	4.0552	0.3349	0.9600	4.0552	0.3350	0.9600
1.45	4.2631	0.2392	1.1025	4.2631	0.2392	1.1025
1.50	4.4817	0.1411	1.2500	4.4817	0.1411	1.2500

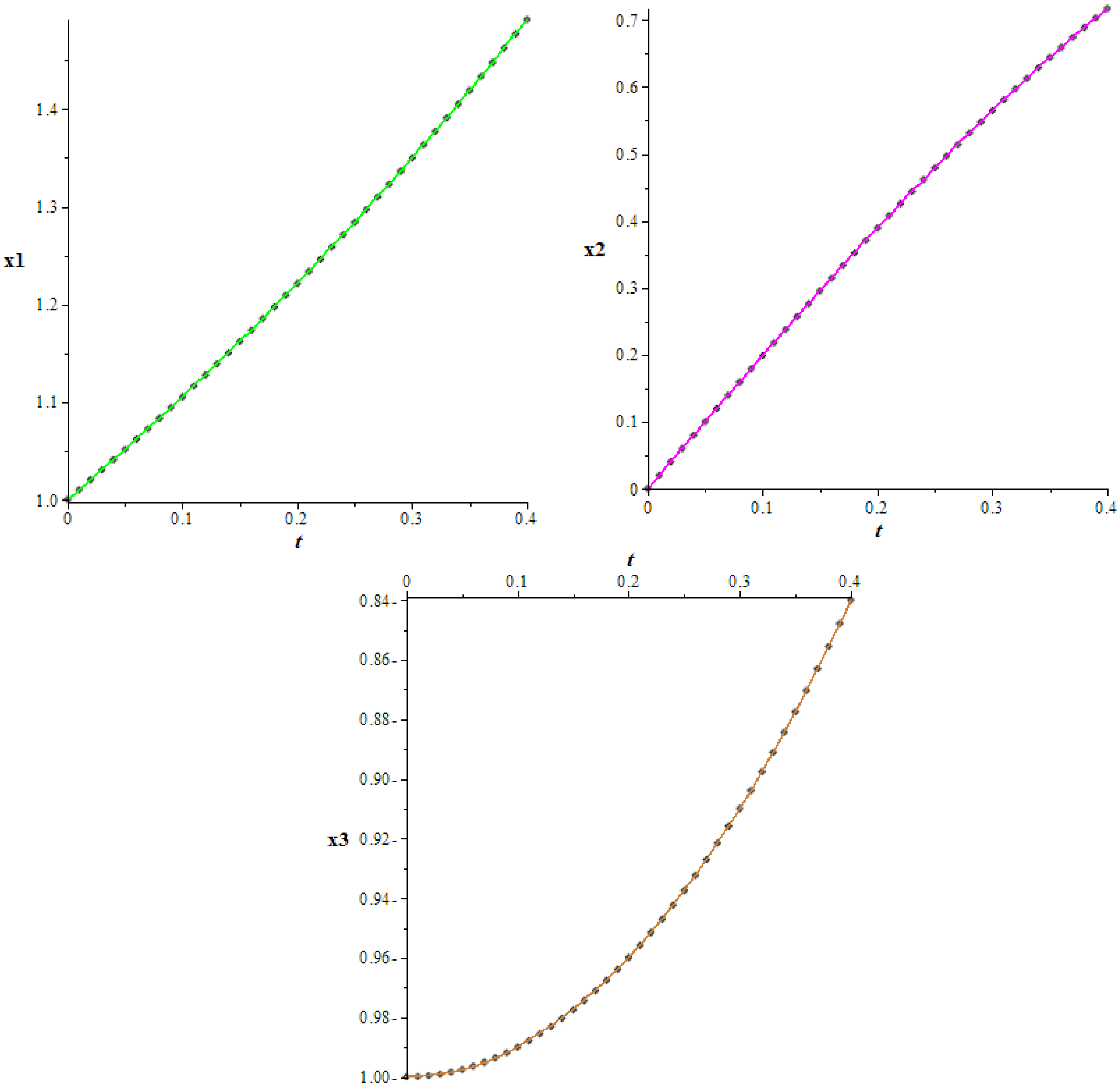

Fig. 3 The exact and approximate solutions result of systems of three Nonlinear Volterra integral equations for example 3.

Example4. Consider the system of Volterra integral equations

$$X_1(t) = t - \frac{1}{12}t^4 - \frac{1}{20}t^5 + \int_0^t ((t-s)X_2(s) + (t-s)X_3(s))ds$$

$$X_2(t) = t^2 - \frac{1}{6}t^3 - \frac{1}{20}t^5 + \int_0^t ((t-s)X_1(s) + (t-s)X_3(s))ds$$

$$X_3(t) = \frac{5}{6}t^3 - \frac{1}{12}t^4 + \int_0^t ((t-s)X_1(s) + (t-s)X_2(s))ds$$

With the exact solution $X_1(t) = t$, $X_2(t) = t^2$ and $X_3(t) = t^3$.

Table.4 Numerical results and exact solution of systems of three Nonlinear Volterra integral equations for example 4.

t	$X_1(t)$	$X_2(t)$	$X_3(t)$	$Exact1 = t$	$Exact2 = t^2$	$Exact3 = t^3$
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.05	0.0500	0.0025	0.0001	0.0500	0.0025	0.0001
0.10	0.1000	0.0100	0.0010	0.1000	0.0100	0.0010
0.15	0.1500	0.0225	0.0034	0.1500	0.0225	0.0034
0.20	0.2000	0.0400	0.0080	0.2000	0.0400	0.0080
0.25	0.2500	0.0625	0.0156	0.2500	0.0625	0.0156
0.30	0.3000	0.0900	0.0270	0.3000	0.0900	0.0270
0.35	0.3500	0.1225	0.0429	0.3500	0.1225	0.0429
0.40	0.4000	0.1600	0.0640	0.4000	0.1600	0.0640
0.45	0.4500	0.2025	0.0911	0.4500	0.2025	0.0911
0.50	0.5000	0.2500	0.1250	0.5000	0.2500	0.1250
0.55	0.5500	0.3025	0.1664	0.5500	0.3025	0.1664
0.60	0.6000	0.3600	0.2160	0.6000	0.3600	0.2160
0.65	0.6500	0.4225	0.2746	0.6500	0.4225	0.2746
0.70	0.7000	0.4900	0.3430	0.7000	0.4900	0.3430
0.75	0.7500	0.5625	0.4219	0.7500	0.5625	0.4219
0.80	0.8000	0.6400	0.5120	0.8000	0.6400	0.5120
0.85	0.8500	0.7225	0.6141	0.8500	0.7225	0.6141
0.90	0.9000	0.8100	0.7290	0.9000	0.8100	0.7290
0.95	0.9500	0.9025	0.8573	0.9500	0.9025	0.8574
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.05	1.0500	1.1025	1.1576	1.0500	1.1025	1.1576
1.10	1.1000	1.2100	1.3310	1.1000	1.2100	1.3310
1.15	1.1500	1.3225	1.5208	1.1500	1.3225	1.5209
1.20	1.2000	1.4400	1.7280	1.2000	1.4400	1.7280
1.25	1.2500	1.5625	1.9531	1.2500	1.5625	1.9531
1.30	1.3000	1.6899	2.1970	1.3000	1.6900	2.1970
1.35	1.3499	1.8224	2.4603	1.3500	1.8225	2.4604
1.40	1.3999	1.9599	2.7439	1.4000	1.9600	2.7440
1.45	1.4499	2.1024	3.0486	1.4500	2.1025	3.0486
1.50	1.4999	2.2499	3.3749	1.5000	2.2500	3.3750

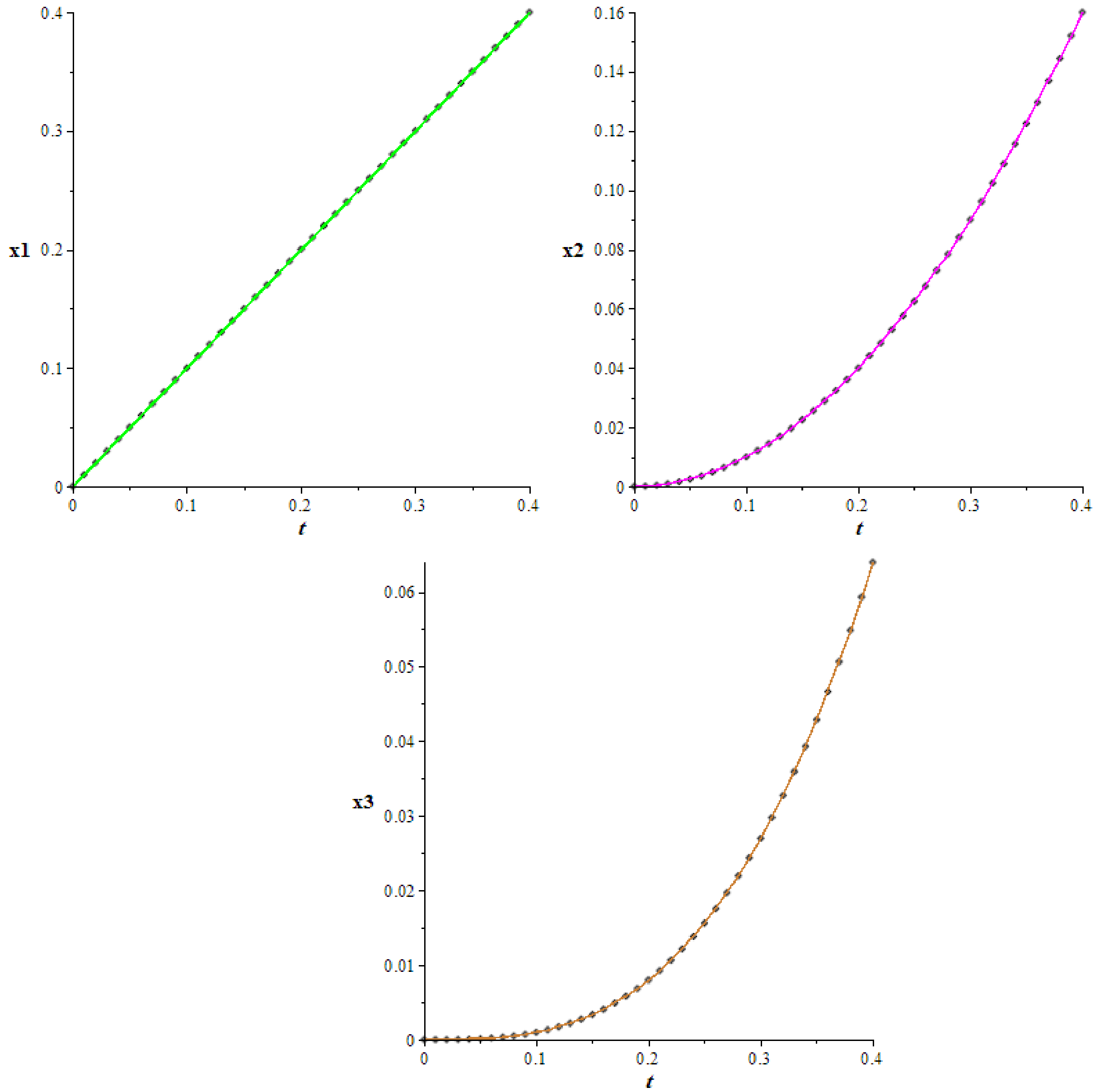

Fig. 4 The exact and approximate solutions result of systems of three Nonlinear Volterra integral equations for example 4.

Example5. Consider the system of Volterra integral equations

$$\mathrm{X}_1(t) = \cos t - 2t^2 \cos t + 4\int_0^t \mathrm{X}_1(s)\mathrm{X}_2(s)\, ds$$

$$\mathrm{X}_2(t) = t - \frac{1}{6}t^6 \cos t + \frac{1}{36}t^6 + \frac{1}{3}\int_0^t s{\mathrm{X}_3}^2(s)\mathrm{X}_1(s)\, ds$$

$$\mathrm{X}_3(t) = t^2 + \frac{1}{15}t^5 + \int_0^t s\mathrm{X}_3(s)\mathrm{X}_2(s)\, ds$$

With the exact solution $X_1(t) = \cos t$, $X_2(t) = t$ and $X_3(t) = t^2$.

Table.5 Numerical results and exact solution of systemsof three Nonlinear Volterra integral equations for example 5.

t	$X_1(t)$	$X_2(t)$	$X_3(t)$	$Exact1 = \cos t$	$Exact2 = t$	$Exact3 = t^2$
0.000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.005	1.0000	0.0050	0.0000	1.0000	0.0050	0.0000
0.010	1.0000	0.0100	0.0001	0.9998	0.0100	0.0001
0.015	0.9999	0.0150	0.0002	0.9996	0.0150	0.0002
0.020	0.9998	0.0200	0.0004	0.9992	0.0200	0.0004
0.025	0.9997	0.0250	0.0006	0.9988	0.0250	0.0006
0.030	0.9996	0.0300	0.0009	0.9982	0.0300	0.0009
0.035	0.9994	0.0350	0.0012	0.9976	0.0350	0.0012
0.040	0.9992	0.0400	0.0016	0.9968	0.0400	0.0016
0.045	0.9990	0.0450	0.0020	0.9960	0.0450	0.0020
0.050	0.9988	0.0500	0.0025	0.9950	0.0500	0.0025
0.055	0.9985	0.0550	0.0030	0.9940	0.0550	0.0030
0.060	0.9982	0.0600	0.0036	0.9928	0.0600	0.0036
0.065	0.9979	0.0650	0.0042	0.9916	0.0650	0.0042
0.070	0.9976	0.0700	0.0049	0.9902	0.0700	0.0049
0.075	0.9972	0.0750	0.0056	0.9888	0.0750	0.0056
0.080	0.9968	0.0800	0.0064	0.9872	0.0800	0.0064
0.085	0.9964	0.0850	0.0072	0.9856	0.0850	0.0072
0.090	0.9960	0.0900	0.0081	0.9838	0.0900	0.0081
0.095	0.9955	0.0950	0.0090	0.9820	0.0950	0.0090
0.100	0.9951	0.1000	0.0100	0.9801	0.1000	0.0100
0.105	0.9946	0.1050	0.0110	0.9780	0.1050	0.0110
0.110	0.9940	0.1100	0.0121	0.9759	0.1100	0.0121
0.115	0.9935	0.1150	0.0132	0.9737	0.1150	0.0132
0.120	0.9929	0.1200	0.0144	0.9713	0.1200	0.0144
0.125	0.9923	0.1250	0.0156	0.9689	0.1250	0.0156
0.130	0.9917	0.1300	0.0169	0.9664	0.1300	0.0169
0.135	0.9911	0.1350	0.0182	0.9638	0.1350	0.0182
0.140	0.9904	0.1400	0.0196	0.9611	0.1400	0.0196
0.145	0.9897	0.1450	0.0210	0.9582	0.1450	0.0210
0.150	0.9890	0.1500	0.0225	0.9553	0.1500	0.0225

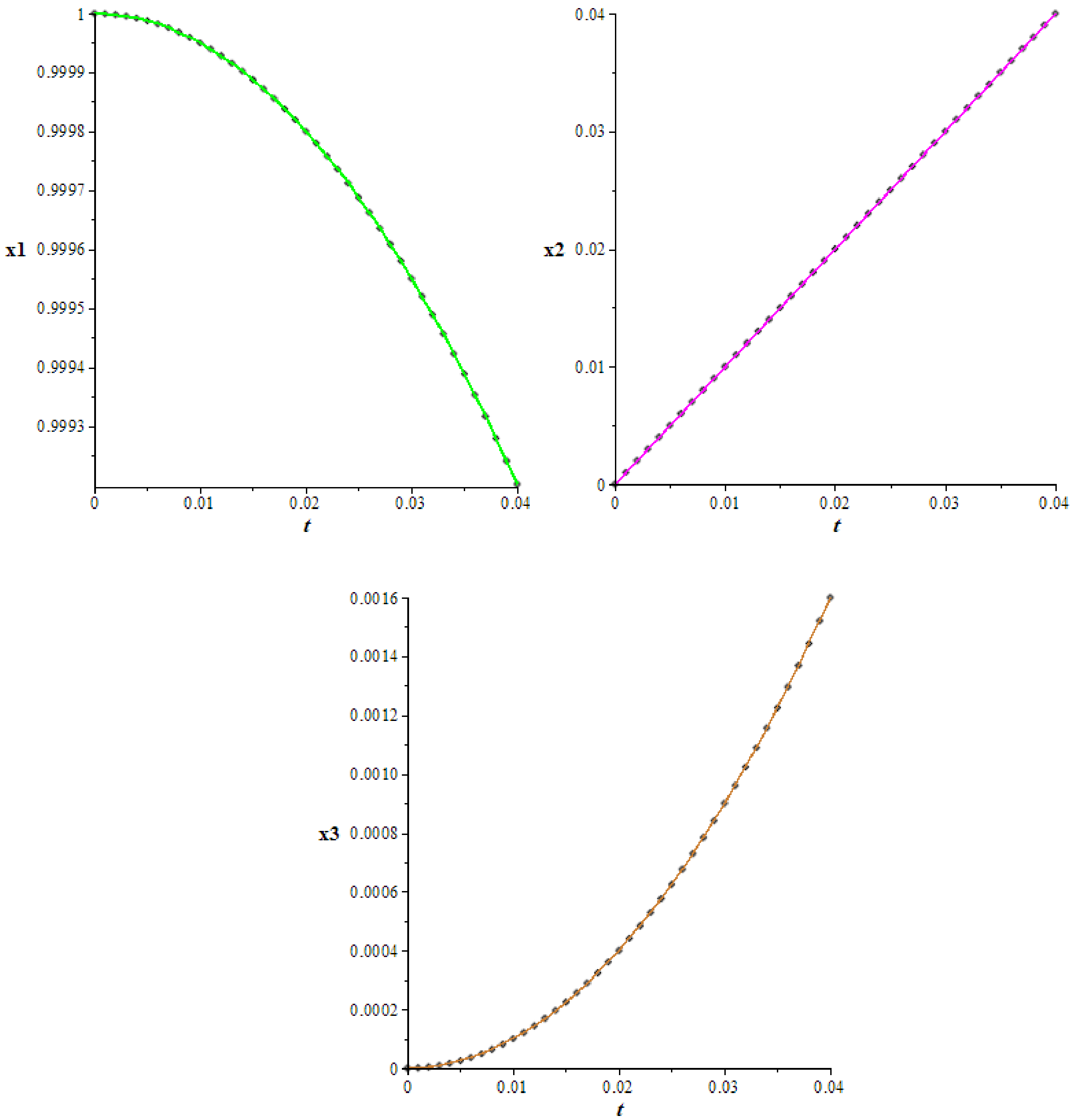

Fig. 5 The exact and approximate solutions result of systems of three Nonlinear Volterra integral equations for example 5.

4. Conclusion

In this project, we have studied system of three nonlinear Volterra integral equations with the implicit trapezoidal method. The basic goal of the present project is to mechanize the computing process of our implicit trapezoidal method by a Maple program and obtain more precise values of the solutions. The results showed that the implicit trapezoidal method is remarkably effective and performing is very easy. The computed values and graphics, illustrated by the results, agree well with the exact solution.

Acknowledgments

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, under grant no. (446-363-1436/HiCi). The authors, therefore, acknowledge with thanks DSR technical and financial support.

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