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Exact Solitary Wave Solutions for the Generalized Schamel Korteweg–De Vries Equation

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Abstract

The generalized Schamel-Korteweg-de Vries (S-KdV) equation containing root of degree n nonlinearity is a very attractive model for the study of ion-acoustic waves in plasma and dusty plasma. In this work, we obtain the soliton-like solutions, the kink solutions, and the plural solutions of the generalized S-KdV equation by using the sine-cosine method. These solutions may be of important significance for the explanation of some practical physical problems. It is shown that these two methods provide a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Keywords: Schamel–Korteweg–de Vries equation; Schamel equation; Travelling wave solutions

Mathematics Subject classification: 35B10; 35Q99; 35Q53

1. Introduction

In plasma physics, the theory of one dimensional ion-acoustic waves is a typical topic in nonlinear waves. As is known, in a collisionless plasma the dynamical behavior of the ions is determined by the presence of electrons and, as a result, the ion-acoustic wave develop in the medium. The propagation of ion-acoustic wave in different types of plasma has been investigated extensively over the last two decades, starting from the work of (Washimi & Taniuti, 1966) The study of different methods for the solution of nonlinear partial differential equations has enjoyed an intense period of activity over the last 30 years from both theoretical and practical points of view. Improvements in numerical techniques, together with the rapid advances in computer technology, have meant that many of the partial differential equations arising from engineering and scientific applications, which were previously intractable, can now, be routinely solved (Jawad, et al., 2010).

Finding the exact solutions of nonlinear evolution equations (NLEEs) plays an important role in the study of many physical phenomena in various fields such as fluid mechanics, solid state physics, plasma physics, chemical physics, optical fiber, and geochemistry. Thus, it is important to investigate the exact explicit solutions of NLEEs. In recent years, various powerful methods have been presented for finding exact solutions of the NLEEs in mathematical physics, such as modified simple equation method (Bhrawy, et al., 2013), extended F-expansion method (Ma, 1993), tanh-sech method (Malfliet, 1992; Khater, et al., 2002; Wazwaz, 2006), extended tanh method (Ma & Fuchssteiner, 1996; El-Wakil & Abdou, 2007; Fan, 2000; Maliet, 2004), sine–cosine method (Wazwaz, 2004 a; Wazwaz, 2004b; Yusufoglu & Bekir, 2006) and Bäcklund transformation (Ma & Lee, 2009; Khater, et al., 2006; Khater, et al., 2004; Sayed, 2013) but solving nonlinear equations is still an important task. Some of the nonlinear models in plasma and dust plasma are described by canonical models, such as the KdV, the mKdV, and so on (Hassan, 2010).

Nonlinear wave phenomena play a major role in sciences such as fluid mechanics, plasma physics, solid state physics, optical fibers, chemical kinetics and geochemistry. The phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. The concepts like solitons, peakons, kinks, breathers, cusps and compactons (Rosenau & Hyman, 1993) are now thoroughly investigated in the scientific literature.

The ion-acoustic solitary wave is one of the fundamental nonlinear wave phenomena appearing in plasma physics. In (Hans Schamel, 1973) Hans Schamel studies a modified Korteweg-de Vries equation for ion-acoustic waves. The S-KdV equation containing a square root nonlinearity is a very attractive model for the study of ion-acoustic waves in plasmas and dusty plasmas. Hence, seeking new exact solutions of the generalized S-KdV equation is important. This paper is organized as follows. In section 2, we briefly describe the main steps of the sine–cosine ansatz. In section 3, we will study the soliton-like solutions of the generalized S-KdV equation by using the sine-cosine method. Section 4 contains the conclusion.

2. A Sine–Cosine Ansatz

1. The wave variable $\xi = x - ct$ where c is a constant, carries a nonlinear PDE in two independent variables (Dusuel, et al., 1998)

$$P(u, u_t, u_x, u_{xx}, u_{3x}, \dots\dots) = 0, \tag{1}$$

where $u(x,t)$ is the traveling wave solution, to a nonlinear ODE

$$Q(u, u^{/}, u^{//}, u^{///}, \cdots) = 0. \tag{2}$$

Notice that

$$\frac{\partial}{\partial t} = -c\,\frac{d}{d\xi},\ \ \frac{\partial^2}{\partial t^2} = c^2\,\frac{d^2}{d\xi^2},\quad \frac{\partial}{\partial x} = c\,\frac{d}{d\xi},\ \ \frac{\partial^2}{\partial x^2} = \frac{d^2}{d\xi^2}. \tag{3}$$

Eq. (2) is then integrated as long as all terms contain derivatives such that integration constants are neglected.

2. The solutions of many nonlinear equations can be expressed in the form

$$u(x,t) = \lambda\ cos^{\,m}(\mu\,\xi), \qquad |\xi| \le \frac{\pi}{2\,\mu} \tag{4}$$

Or in the form

$$u(x,t) = \lambda\ sin^{\,m}(\mu\,\xi), \qquad |\xi| \le \frac{\pi}{\mu} \tag{5}$$

where $\lambda\ and\ m$ are parameters that will be determined, $\mu\ and\ c$ are the wave number and the wave speed respectively. We then use (Ludu & Draayer, 1998)

$$u(\xi) = \lambda\ cos^{\,m}(\mu\,\xi), \tag{6}$$

$$u^{/}(\xi) = -\ \lambda\,\mu\,m\ cos^{\,m-1}(\mu\,\xi)\sin(\mu\xi), \tag{7}$$

$$u^{//}(\xi) = -\ \lambda\,\mu^2\,m^2\ cos^{\,m}(\mu\,\xi) + \lambda\,\mu^2\,m(m-1)\,cos^{\,m-2}(\mu\,\xi). \tag{8}$$

and for (5) we use

$$u(\xi) = \lambda\ sin^{\,m}(\mu\,\xi), \tag{9}$$

$$u^{/}(\xi) = \lambda\,\mu\,m\ sin^{\,m-1}(\mu\,\xi)\cos(\mu\xi), \tag{10}$$

$$u^{//}(\xi) = -\ \lambda\,\mu^2\,m^2\ sin^{\,m}(\mu\,\xi) + \lambda\,\mu^2\,m(m-1)\,sin^{\,m-2}(\mu\,\xi). \tag{11}$$

and so on for other derivatives.

3. Substituting (6) – (8) or (9) –(11) into the reduced ODE gives a trigonometric equation of $cos^{\,R}(\mu\,\xi)$ or $sin^{\,R}(\mu\,\xi)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosine or sine to determine R. We next collect all terms with same power in $cos^{\,k}(\mu\,\xi)$ or $sin^{\,k}(\mu\,\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns $\lambda,\ \mu\ and\ m$. The problem is now completely reduced to a system of algebraic equations that can be easily solved to determine $\lambda\ and\ \mu$. The solutions proposed in (4) and in (5) are then readily obtained.

3. The Generalized S-Kdv Equation

In this paper, a traveling wave solution for the generalized S-KdV equation (Yang & Tang, 2015; Figen Kangalgil 2016)

$$u_t + \left(\alpha + \beta\,u^{\frac{1}{n}}\right) u^{\frac{1}{n}}\,u_x + \gamma\ u_{3x} = 0, \qquad n \ne -2, -1, 0 \tag{12}$$

where $u(x,t)$ refers to the perturbed ion density in a plasma with nonisothermal electrons, and α, β, γ are real constants. Obviously, for $n = 2,\ \beta = 0$ Equation (12) reduces to the schamel KdV equation. The KdV equation follows for $n = 2,\ \alpha = 0$, and the mKdV equation follows for $n = \frac{1}{2},\ \beta = 0$. Presumably in Equation (12) other fractional powers of u could be obtained by looking in more detail and different orderings between kinetic fluid effects for nonisothermal behavior of trapped particles. In this paper, we will study Equation (12) by using the sine-cosine method. We first use the wave variable $\xi = x - ct$ where c is a constant, to carry a PDE in two independent variables (12) into the following ordinary differential equation

$$-c\,u' + \left(\alpha + \beta\, u^{\frac{1}{n}}\right) u^{\frac{1}{n}} u' + \gamma u''' = 0, \tag{13}$$

where $' = \frac{d}{d\xi}$, Integrating (13) once, and considering the constants of integration as zero, we can find

$$-c\,u + \left(\frac{n\,\alpha}{n+1}\, u^{\frac{1}{n}+1} + \frac{n\beta}{n+2}\, u^{\frac{2}{n}+1}\right) + \gamma u'' = 0, \tag{14}$$

Substituting (6) – (8) into (14) yields

$$-\left(c\,\lambda + \gamma\,\mu^2\, m^2\,\lambda\right)\cos^{m}(\mu\,\xi) + \frac{n\,\alpha}{n+1}\,\lambda^{\left(\frac{1}{n}+1\right)}\cos^{\left(\frac{1}{n}+1\right)m}(\mu\,\xi) +$$

$$\frac{n\,\beta}{n+2}\,\lambda^{\left(\frac{2}{n}+1\right)}\cos^{\left(\frac{2}{n}+1\right)m}(\mu\,\xi) + \gamma\,\lambda\,\mu^2\, m(m-1)\cos^{m-2}(\mu\,\xi) = 0. \tag{15}$$

Equating the exponents and the coefficients of each pair of the cosine functions, we find the following system of algebraic equations:

Case 1.

$$(m-1) \neq 0,$$

$$\left(\frac{2}{n} + 1\right) m = m - 2,$$

$$c\,\lambda + \gamma\,\mu^2\, m^2\,\lambda = 0,$$

$$\frac{n\,\alpha}{n+1}\,\lambda^{\frac{1}{n}+1} = 0,$$

$$\frac{n\,\beta}{n+2}\,\lambda^{\frac{2}{n}+1} + \gamma\,\lambda\,\mu^2\, m(m-1) = 0. \tag{16}$$

Solving the system (16) yields

$$m = -\ n\,,$$

$$\mu = \sqrt{\frac{-c}{\gamma\, n^2}}\ , \qquad \alpha = 0,$$

$$\lambda = \left(\frac{c\ (n+1)(n+2)}{\beta\, n^2}\right)^{\frac{n}{2}}. \tag{17}$$

We point out that the results (17) are valid if we also use the sine method (9)-(11). Consequently, the following solutions:

$$u_1(x,t) = \left(\frac{c\ (n+1)(n+2)}{\beta\, n^2}\right)^{\frac{n}{2}} sec^{n}\left(\sqrt{\frac{-c}{\gamma\, n^2}}\ (x-ct)\right), \quad c<0 \tag{18}$$

and

$$u_2(x,t) = \left(\frac{c\ (n+1)(n+2)}{\beta\, n^2}\right)^{\frac{n}{2}}\ csc^n\left(\sqrt{\frac{-c}{\gamma\, n^2}}\ (x-ct)\right),\quad c<0 \qquad (19)$$

are readily obtained. It is worth noting that the results (18) and (19) are valid only if $c < 0$.
However, for $c > 0$, the following solutions:

$$u_3(x,t) = \left(\frac{c\ (n+1)(n+2)}{\beta\, n^2}\right)^{\frac{n}{2}}\ sech^n\left(\sqrt{\frac{c}{\gamma\, n^2}}(x-ct)\right),\quad c>0 \qquad (20)$$

and

$$u_4(x,t) = \left(\frac{c\ (n+1)(n+2)}{\beta\, n^2}\right)^{\frac{n}{2}}\ csch^n\left(\sqrt{\frac{c}{\gamma\, n^2}}(x-ct)\right),\quad c>0 \qquad (21)$$

Case 2.

$$(m-1) \neq 0,$$

$$\left(\frac{1}{n}+1\right)m = m-2,$$

$$c\,\lambda + \gamma\,\mu^2\,m^2\,\lambda = 0,$$

$$\frac{n\,\beta}{n+2}\,\lambda^{(\frac{2}{n}+1)} = 0,$$

$$\frac{n\,\alpha}{n+1}\,\lambda^{(\frac{1}{n}+1)} + \gamma\,\lambda\,\mu^2\,m(m-1) = 0. \qquad (22)$$

Solving the system (22) yields

$$m = -\ 2\,n\,,$$

$$\mu = \sqrt{\frac{-c}{4\gamma\, n^2}}\ ,\qquad \beta = 0,$$

$$\lambda\ = \left(\frac{c\ (n+1)(2n+1)}{2\,\alpha\, n^2}\right)^n. \qquad (23)$$

We point out that the results (23) are valid if we also use the sine method (9)-(11). We can readily get the following solutions:

$$u_5(x,t) = \left(\frac{c\ (n+1)(2n+1)}{2\,\alpha\, n^2}\right)^n\ sec^{2n}\left(\sqrt{\frac{-c}{4\gamma\, n^2}}\ (x-ct)\right),\quad c<0 \qquad (24)$$

$$u_6(x,t) = \left(\frac{c\ (n+1)(2n+1)}{2\,\alpha\, n^2}\right)^n\ csc^{2n}\left(\sqrt{\frac{-c}{4\gamma\, n^2}}\ (x-ct)\right),\quad c<0 \qquad (25)$$

$$u_7(x,t) = \left(\frac{c\ (n+1)(2n+1)}{2\,\alpha\, n^2}\right)^n\ sech^{2n}\left(\sqrt{\frac{c}{4\gamma\, n^2}}(x-ct)\right),\quad c>0 \qquad (26)$$

$$u_8(x,t) = \left(\frac{c\ (n+1)(2n+1)}{2\ \alpha\ n^2}\right)^n\ csch^{2n}\left(\sqrt{\frac{c}{4\gamma\ n^2}}\,(x-ct)\right),\quad c>0 \quad (27)$$

Case 3. It is interesting to point out that for where $(m-1)=0$,
this is satisfied only when $\alpha = 0\ and\ \beta = 0$. Consequently, we obtain

$$\lambda = any\ real\ number,\quad \mu = \sqrt{\frac{-c}{\gamma}}\,, \quad (28)$$

This in turn gives the solutions

$$u_9(x,t) = a\ \cos\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right) + b\ \sin\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right),\quad c<0 \quad (29)$$

$$u_{10}(x,t) = a\ \cosh\left(\sqrt{\frac{c}{\gamma}}\ (x-ct)\right) + b\ \sinh\left(\sqrt{\frac{c}{\gamma}}\ (x-ct)\right),\quad c>0 \quad (30)$$

where $a\ and\ b$ are arbitrary constants. These solutions of (12) are solitary wave solutions. They are linear combinations of kink solitary and bell solitary wave solutions.

In particular (1) when $n=2,\ \alpha=0,$ we obtain the KdV equation

$$u_t + \beta\, u\, u_x + \gamma\ u_{3x} = 0, \quad (31)$$

we have the following formal solitary wave solutions

$$u_{11}(x,t) = \frac{3\,c}{\beta}\ sec^2\ \frac{1}{2}\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right),\quad c<0 \quad (32)$$

$$u_{12}(x,t) = \frac{3\,c}{\beta}\ \ csc^2\ \frac{1}{2}\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right),\quad c<0 \quad (33)$$

$$u_{13}(x,t) = \frac{3\,c}{\beta}\ sech^2\ \frac{1}{2}\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right),\quad c>0 \quad (34)$$

$$u_{14}(x,t) = \frac{3\,c}{\beta}\ \ csch^2\ \frac{1}{2}\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right),\quad c>0 \quad (35)$$

(2) when $n=2,\ \beta=0,$ we obtain the S-KdV equation

$$u_t + \alpha\, u^{\frac{1}{2}}\, u_x + \gamma\ u_{3x} = 0, \quad (36)$$

we have the following formal solitary wave solutions

$$u_{15}(x,t) = \frac{225\,c^2}{64\alpha^2}\ sec^4\ \frac{1}{4}\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right),\quad c<0 \quad (37)$$

$$u_{16}(x,t) = \frac{225\,c^2}{64\alpha^2}\ \ csc^4\ \frac{1}{4}\left(\sqrt{\frac{-c}{\gamma}}\ (x-ct)\right),\quad c<0 \quad (38)$$

$$u_{17}(x,t) = \frac{225\,c^2}{64\alpha^2}\, sech^4\, \frac{1}{4}\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right), \quad c>0 \tag{39}$$

$$u_{18}(x,t) = \frac{225\,c^2}{64\alpha^2}\, csch^4\, \frac{1}{4}\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right), \quad c>0 \tag{40}$$

(3) when $n = \frac{1}{2}$, $\beta = 0$, we obtain the mKdV equation

$$u_t + \alpha\, u^2\, u_x + \gamma\; u_{3x} = 0, \tag{41}$$

we have the following formal solitary wave solutions

$$u_{19}(x,t) = \sqrt{\frac{6\,c}{\alpha}}\;\; sec\left(\sqrt{\frac{-c}{\gamma}}\;(x-ct)\right), \quad c<0 \tag{42}$$

$$u_{20}(x,t) = \sqrt{\frac{6\,c}{\alpha}}\;\; csc\left(\sqrt{\frac{-c}{\gamma}}\;(x-ct)\right), \quad c<0 \tag{43}$$

$$u_{21}(x,t) = \sqrt{\frac{6\,c}{\alpha}}\;\; sech\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right), \quad c>0 \tag{44}$$

$$u_{22}(x,t) = \sqrt{\frac{6\,c}{\alpha}}\;\; csch\left(\sqrt{\frac{c}{\gamma}}(x-ct)\right), \quad c>0 \tag{45}$$

4. Conclusion

We discuss the generalized S-KdV equation which describes the nonlinear ion-acoustic waves in a collisionless plasma consisting of adiabatic warm ions, a weakly relativistic electron beam and non-isothermal electrons. In this paper, we use the sine-cosine method to study the generalized S-KdV equation. This method provides the soliton-like solutions, the kink solutions, and plural solutions (Sayed & Al-Atawi, 2017). The presented exact solutions can describe various new features of waves and then may be useful in the theoretical and numerical studies of the considered equation. The study emphasizes the fact that these two methods are reliable in handling nonlinear problems. The computer symbolic systems such as Maple and Mathematica allow us to perform complicated and tedious calculations.

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