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Some New Results on Super Heronian Mean

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Abstract

Here we discuss on Some new results on Super Heronian Mean Labelling of graphs. For the present investigation on Qn ⊙ K1, D(Qn) ⊙ K1, Middle graph, Total graph, Ln ⊙ K1.We discuss briefly about the summary and the other valid information with appropriate graphs and definitions.

Keywords: Middle graph, Super Heronian mean graph, Total graph

1. Introduction

The concept of graph Labelling was introduced by Rosa in 1967. A graph Labelling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges), then the Labelling is called a vertex Labelling (or an edge Labelling). Here we consider simple, finite, undirected and connected graph G = (V, E). In this paper Q_n and $D(Q_n)$ denotes Quadrilateral snake and Double Quadrilateral snake with n vertices. For all other general expressions and symbols we follow Harary.First we will provide some definitions useful for the present work.

1.1 Definition

Let f :V(G)→{1, 2, ------, p+q} be an injective function. For a vertex Labelling "f" the induced edge Labelling f*(e=uv) is defined by,

$$f^*(e)=\left\lfloor\frac{f(u)+\ \sqrt{f(u)f(v)}\ \ +f(v)}{3}\right\rfloor \quad [OR] \quad \left\lceil\frac{f(u)+\sqrt{f(u)f(v)}\ +f(v)}{3}\right\rceil.$$

Then "f" is called a Super Heronian Mean Labelling if {f(V(G)} U {f(e): e ϵ E(G)={1, 2, p+q}}. A graph which admits Super Heronian Mean Labelling is called Super Heronian Mean Graph.

1.2 Definition

The Total graph T(G) of a graph G is the graph whose vertex set is V(G)UE(G) and two vertices are adjacent whenever they are either adjacent or incident in G.

1.3 Definition

The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)UE(G) and in which two vertices are adjacent iff either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

1. 4 Theorem

Quadrilateral snakes are Super Heronian mean graph.

1.5 Theorem

Double Quadrilateral snakes are Super Heronian mean graph.

2. Main Results

2.1 Theorem

Let G be a graph obtained by attaching pendant edges to each vertex of a Quadrilateral snake $Q_{n.}$ Then G is a Super Heronian mean graph.

Proof: Consider a graph G which is obtained by attaching pendant edges to each vertex of a Quadrilateral snake Q_n. Let Q_n be a Quadrilateral snake. Let u_i, v_i, w_i be the vertices of Quadrilateral snake. Join $u_i v_i$, $u_{i+1} w_i$ and $v_i w_i$. Let x_i, y_i, z_i be the pendant vertices. Join $u_i x_i$, $v_i y_i$ and $w_i z_i$.

Define a function $f : V(Q_n \odot K_1) \rightarrow \{1, 2, \text{-------}, p+q\}$ by

$$f(u_i)=13i-12;\ 1\leq i\leq n$$
$$f(v_i)=13i-5;\ 1\leq i\leq n-1$$
$$f(w_i)=13i-3;\ 1\leq i\leq n-1$$
$$f(x_i)=13i-10;\ 1\leq i\leq n$$
$$f(y_i)=13i-9;\ 1\leq i\leq n-1$$
$$f(z_i)=13i;\ 1\leq i\leq n$$

Edges are labeled with,

$$f(u_iu_{i+1})=13i-6;\ 1\leq i\leq n-1$$
$$f(u_iv_i)=13i-8;\ 1\leq i\leq n-1$$
$$f(v_iw_i)=13i-4;\ 1\leq i\leq n-1$$
$$f(w_iu_{i+1})=13i-2;\ 1\leq i\leq n-1$$
$$f(u_ix_i)=13i-11;\ 1\leq i\leq n$$
$$f(v_iy_i)=13i-7;\ 1\leq i\leq n-1$$
$$f(w_iz_i)=13i-1;\ 1\leq i\leq n-1$$

Obviously f is a Super Heronian mean labelling and $Q_n \odot K_1$ is a Super Heronian meangraph.

2.2. Example

A Super Heronian mean labelling of $Q_4 \odot K_1$ is displayed below.

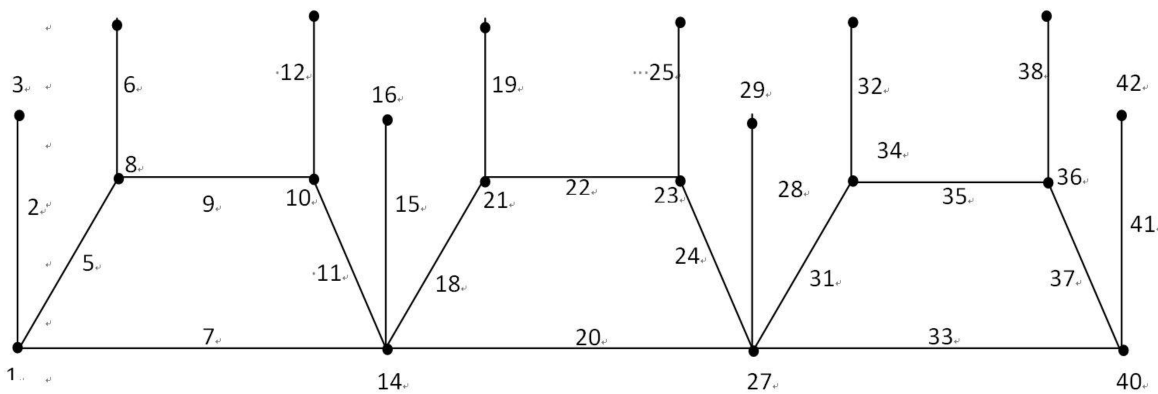

Figure 1

2.3 Theorem

$D(Q_n) \odot K_1$ is a Super Heronian mean graph.

Proof: Let D(Qn) be a Double Quadrilateral snake.Let ui, vi, wi, xi, yi be the vertices of Double Quadrilateral snake. Join ui vi, ui+1 wi, vi wi and ui xi ui+1 yi, xi yi. Let ai, bi, ci, di, si, ti be the pendant vertices.Join vi ai, wi bi, xi ci, yi di, xi ti and ui si.

Define a function $f: V(D(Q_n) \odot K_1) \rightarrow \{1, 2, \text{-------}, p+q\}$ by,

$$f(u_i)=24i-19;\ 1\leq i\leq n$$
$$f(v_i)=24i-16;\ 1\leq i\leq n-1$$
$$f(w_i)=24i-11;\ 1\leq i\leq n-1$$
$$f(x_i)=24i-12;\ 1\leq i\leq n-1$$
$$f(y_i)=24i-4;\ 1\leq i\leq n-1$$

$$f(a_i)=24i-14;\ 1\le i\le n-1$$
$$f(b_i)=24i-6;\ 1\le i\le n-1$$
$$f(c_i)=24i-3;\ 1\le i\le n-1$$
$$f(d_i)=24i;\ 1\le i\le n-1$$
$$f(s_i)=24i-22;\ 1\le i\le n-1$$
$$f(t_i)=24i-21;\ 1\le i\le n-1$$

Edges are labeled with,

$$f(u_iu_{i+1})=24i-8;\ 1\le i\le n-1;\ f(u_iv_i)=24i-18;\ 1\le i\le n-1$$
$$f(v_iw_i)=24i-14;\ 1\le i\le n-1;\ f(w_iu_{i+1})=24i-4;\ 1\le i\le n-1$$
$$f(u_ix_i)=24i-17;\ 1\le i\le n-1;\ f(x_iy_i)=24i-10;\ 1\le i\le n-1$$
$$f(u_iu_{i+1})=24i-1;\ 1\le i\le n-1;\ f(x_ic_i)=24i-7;\ 1\le i\le n-1$$
$$f(y_id_i)=24i-2;\ 1\le i\le n-1;\ f(v_ia_i)=24i-15;\ 1\le i\le n-1$$
$$f(w_ib_i)=24i-9;\ 1\le i\le n-1;\ f(s_iu_i)=24i-22;\ 1\le i\le n$$
$$f(u_it_i)=24i-20;\ 1\le i\le n$$

Clearly f is a Super Heronian mean Labelling and $D(Q_n)\odot K_1$ is a Super Heronian mean graph.

2.4 Example

A Super Heronian mean Labelling of $D(Q_4)\odot K_1$is displayed below.

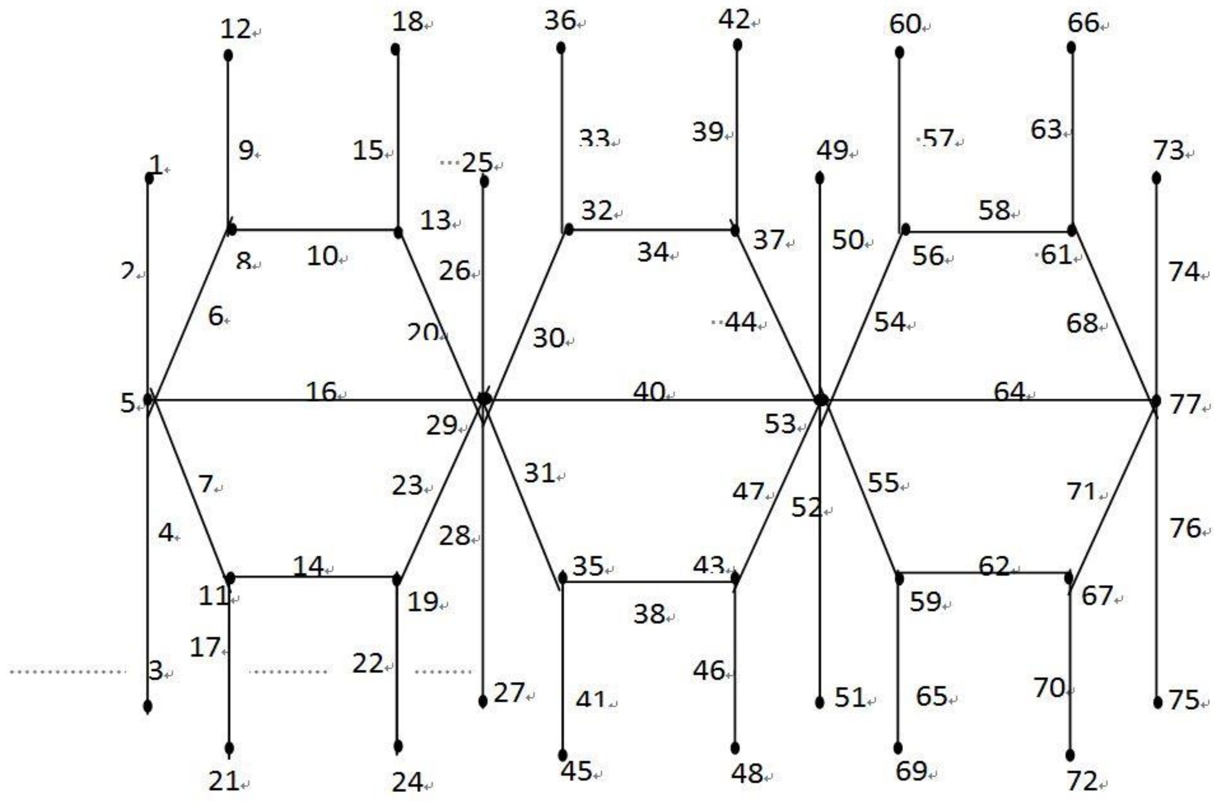

Figure 2

2.5 Theorem

Middle graph of path P_n is a Super Heronian mean graph.

Proof: Let $u_1, u_2,$ --------, u_n be the vertices and $v_1, v_2,$ ------, v_{n-1} be the edges of path P_n and $G=M(P_n)$ be the middle graph of path P_n.

Define a function $f: V(G)\rightarrow\{1, 2, ----, p+q\}$ by

$$f(u_i)=5i-4;\ 1\leq i\leq n$$
$$f(v_i)=5i-2;\ 1\leq i\leq n-1$$

Edges are labeled with,

$$f(u_iv_i)=5i-3; 1\leq i\leq n-1$$
$$f(u_iv_{i-1})=5i-1;\ 1\leq i\leq n-1$$
$$f(v_iv_{i+1})=5i; 1\leq i\leq n-2$$

Thus f provides a Super Heronian mean Labelling for $M(P_n)$.

Hence $M(P_n)$ is a Sper Heronian mean graph.

2.6 Example

Middle graph of path P_5 and its Super Heronian mean Labelling is displayed below.

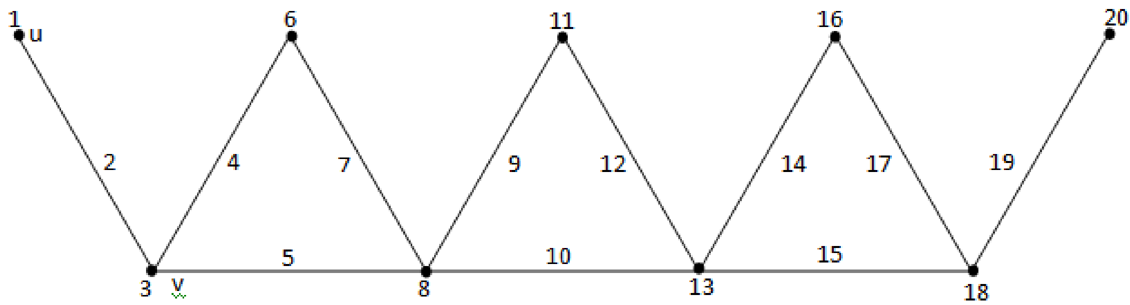

Figure 3

2.7 Theorem

Total graph of path P_n is a Super Heronian mean graph.

Proof: Let u_1, u_2, --------, u_n be the vertices and v_1, v_2, ----v_{n-1} be the edges of path P_n and $G=T(P_n)$ be the total graph of path P_n.Define a function $f: V(G)\rightarrow\{1, 2, ----, p+q\}$ by

$$f(u_1)=1;\ f(u_i)=6i-6;\ 2\leq i\leq n$$
$$f(v_i)=6i-2;\ 1\leq i\leq n-1$$

Edges are labeled with,

$$f(u_iv_i)=6i-4;\ 1\leq i\leq n-1; f(u_iv_{i-1})=6i-1;\ 1\leq i\leq n$$
$$f(v_iv_{i+1})=6i+1;\ 1\leq i\leq n-2;\ f(u_iu_{i+1})=6i-3;\ 1\leq i\leq n-1$$

The above defined function, f provides a Super Heronian mean Labelling for $T(P_n)$.

Hence $T(P_n)$ is a Super Heronian mean graph.

2.8 Example

Total graph of path P_6 and its Super Heronian mean labelling is displayed below.

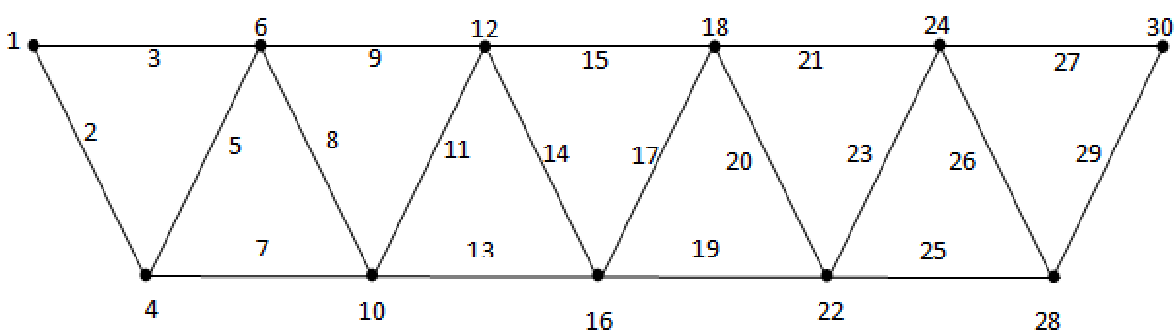

Figure 4

2.9 Theorem

$L_n \odot K_1$ is a Super Heronian mean graphs.

Proof: Let L_n be a Ladder and also w_i be the pendant vertex adjacent to v_i and x_i be the pendant vertex adjacent to u_i.

Define a function f: V($L_n \odot K_1$) →{1, 2, ..., p+q} by,

$$f(u_i)=9i-4;\ 1\leq i\leq n,$$
$$f(v_i)=9i-6;\ 1\leq i\leq n,$$
$$f(w_i)=9i-8;\ 1\leq i\leq n$$
$$f(x_1)=8,$$
$$f(x_i)=9i-2;\ 2\leq i\leq n$$

Edges are labeled with,

$$f(u_iu_{i+1})=9i;\ 1\leq i\leq n-1,\ f(u_1v_1)=7,\ f(u_iv_i)=9i-1;\ 2\leq i\leq n$$
$$f(u_ix_i)=9i-3;\ 1\leq i\leq n,\ f(v_iw_i)=9i-7;\ 1\leq i\leq n$$
$$f(v_1v_2)=7,\ f(v_iv_{i+1})=9i-1;\ 2\leq i\leq n-1$$

Hence $L_n \odot K_1$ is a Super Heronian mean graph.

2.10 Example

A Super Heronian mean labelling of $L_5 \odot K_1$ is given below

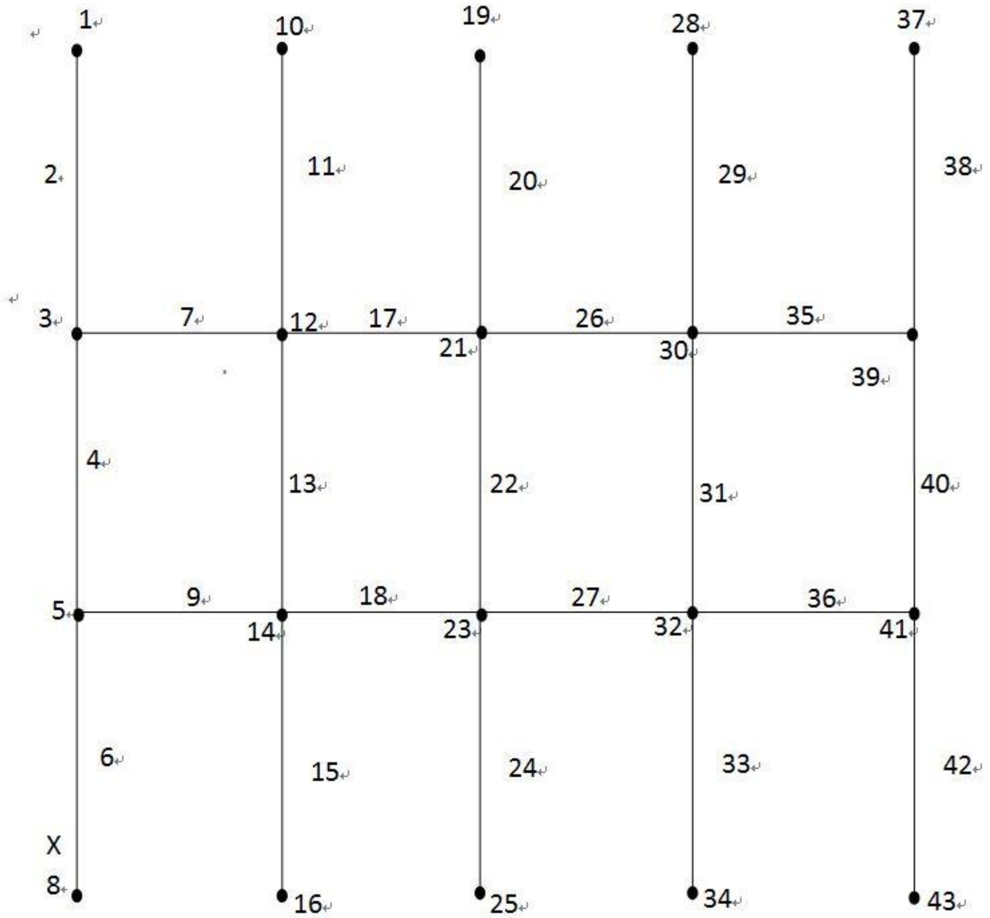

Figure 5

3. Conclusion

In this publication we discussed Some new results on Super Heronian Mean Labelling of graphs and we investigated on $Q_n \odot K_1$,$D(Q_n) \odot K_1$, Middle graph, total graph, $L_n \odot K_1$. Extending the study to other systematic formations of graph families is an open area of research.

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