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Dynamic Behaviors of the Solutions for a Class Delay Harvesting Nicholson's Blowflies Model

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Abstract

In this paper, a class of nonlinear delay harvesting Nicholson's blowflies model is considered. Some criteria to ensure the boundedness and oscillation of the solutions for this model are provided. By employing the Lyapunov function, stability of the solutions is also investigated. Moreover, a simulation is given to illustrate our main results.

Keywords: Nicholson's blowflies model, delay, boundedness, stability, oscillation

1. Introduction

In order to describe the population of the Australian sheep blowflies in agreement with the experimental data obtained by Nicholson (Nicholson, 1954), Gurney et al. (Gurney, Blythe, & Nisbet, 1980) proposed the following nonlinear time delay equation:

$$x^{'}(t) = -\delta x(t) + Px(t-\tau)e^{-ax(t-\tau)}. \tag{1}$$

where $x(t)$ is the size of the population at time t, δ and P are the per capita daily adult death rate and the maximum per capita daily egg production, respectively, and $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate. Since then the most important qualitative properties such as the existence of the solution, persistence, global attractivity, stability and oscillation have been extensively studied for model (1) and various generalized Nicholson's blowflies models (Kulenovic, Ladas, & Sficas, 1992; So, & Yu, 1994; Zhang, & Xu, 1999; Saker, & Zhang, 2002; Wei, & Li, 2005; Saker, & Agarwal, 2002; Long, 2012; Liu, & Gong, 2011; Liu, 2011; Saker, 2005; Li, & Du, 2008; Zhao, Zhu, & Zhu, 2012; Zhou, Wang, & Zhang, 2010; Rashkovsky, & Margaliot, 2007; Faria, 2011; Cherif, 2015; Wang, 2014; Ding, & Nieto, 2013; Deng, & Wu, 2015). Recently, Berezansky et al. (Berezansky, Braverman, & Idels, 2010) pointed out that a linear model of density-dependent mortality will be most accurate for populations at low densities, and marine ecologists are currently in the process of constructing new fishery models with nonlinear density-dependent mortality rates. An open problem: Describe the dynamics of the Nicholson's blowflies model with a nonlinear density-dependent mortality term given by

$$x'(t) = -\frac{ax(t)}{x(t)+b} + Px(t-\tau)e^{-x(t-\tau)} \tag{2}$$

was proposed in which a, b both are positive constants (open problem 7). Berezansky et al. also presented the following model with a harvesting function (open problem 6):

$$x'(t) = -\delta x(t) + Px(t-\tau)e^{-x(t-\tau)} - Hx(t-\sigma) \tag{3}$$

Corresponding model (2), Tang has studied a generalized Nicholson's blowflies model with a nonlinear density dependent mortality term as follows (Tang, 2015):

$$x'(t) = -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{j=1}^{m}\beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} \tag{4}$$

where $a(t), b(t), \beta_j(t), \tau_j(t)$ and $\gamma_j(t)$ are all nonnegative bounded continuous functions. By employing analysis method, some criteria to guarantee the global asymptotic stability of the zero equilibrium point for this model have been provided. Hou et al. (Hou, Duan, & Huang, 2013) have investigated the permanence and the existence of positive periodic solutions for model (4). Zhao et al. (Zhao, Zhu, & Zhu, 2012) have discussed the positive periodic solution for a class of Nicholson's blowflies model with harvesting term as follows:

$$x'(t) = -\delta(t)x(t) + \sum_{i=1}^{m}P_i(t)x(t-\tau_i)e^{-x(t-\tau_i)} - H(t, x(t)) \tag{5}$$

where $\delta(t), P_i(t)$ and $H(t,x)$ for t are positive periodic functions. By using the Krasnoselskii's fixed point theorem on a cone, the author has established some criteria to ensure that the solutions of this model converge locally exponentially to a positive periodic solution.

Motivated by the above models, in this paper, we shall consider the following model:

$$x'(t) = -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{i=1}^{m} P_i(t)x(t-\tau_i)e^{-\gamma_i(t)x(t-\tau_i)} - H(t)x(t-\sigma) \tag{6}$$

where $a(t), b(t), P_i(t), \gamma_i(t)$ and $H(t)$ are all nonnegative bounded continuous functions. In a Nicholson's blowflies model, since $x(t)$ is the size of the population at time t, not only we concern the stability of the zero equilibrium point, but more importantly the stability of the inter-equilibrium point. The boundedness, stability, and oscillation of the solutions for model (6) are obtained.

Throughout this paper, g^+ and g^- will be defined as $g^+ = \sup_{t\in R} g(t)$, $g^- = \inf_{t\in R} g(t)$ for a bounded continuous function $g(t)$. It will be assumed that $a^- > 0, b^- > 0, H^- \geq 0, \gamma_i^- > 0, P_i^- > 0$, and time delays $\tau_i \geq 0, i = 1, 2, \cdots, m$. The initial condition for (6) is $x(t) = \phi(t), t \in [-\tau, 0], \phi(t) \geq 0, \phi(0) > 0$, where $\tau = \max\{\tau_1, \tau_2, \cdots, \tau_m\}$.

2. Main Results

Theorem 1 Assume that

$$\sum_{i=1}^{m} P_i^- > \frac{a^+}{b^-} + H^+, \qquad a^- + H^-b^- > \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e}, \tag{7}$$

then all solutions of system (6) are bounded.

Proof Firstly, to prove the boundedness of the solutions of the system (6), we can consider the boundedness of the solutions for the following without delays system:

$$x'(t) = -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{i=1}^{m} P_i(t)x(t)e^{-\gamma_i(t)x(t)} - H(t)x(t) \tag{8}$$

Noting that $\sup_{u\geq 0} ue^{-u} = \frac{1}{e}$ and $x(t) \geq 0$, from (8) we get

$$\begin{aligned} x'(t) &= -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{i=1}^{m} \frac{P_i(t)}{\gamma_i(t)} \cdot \gamma_i(t)x(t)e^{-\gamma_i(t)x(t)} - H(t)x(t) \\ &\leq -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - H^- x(t) \\ &= \frac{\sum_{i=1}^{m} \frac{P_i^+ b(t)}{\gamma_i^- e} + (\sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - a(t) - H^-b(t))x(t) - H^-x^2(t)}{b(t)+x(t)} \\ &\leq \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e b^-} + \frac{1}{b^-}(\sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - a^- - H^-b^-)x(t) \end{aligned} \tag{9}$$

From (9) we have

$$\begin{aligned} x(t) &\leq \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e b^-} \cdot \frac{b^-}{a^- + H^-b^- - \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e}} \\ &= \frac{\sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e}}{a^- + H^-b^- - \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e}} \end{aligned} \tag{10}$$

On the other hand, noting that $e^{-u} \geq 1 - u$ for $u \geq 0$, from (8) we obtain

$$\begin{aligned} x'(t) &\geq -\frac{a(t)}{b(t)}x(t) + \sum_{i=1}^{m} P_i(t)x(t)(1 - \gamma_i(t)x(t)) - H(t)x(t) \\ &= \sum_{i=1}^{m}(P_i(t) - \frac{a(t)}{b(t)} - H(t))x(t) - \sum_{i=1}^{m} P_i(t)\gamma_i(t)x^2(t) \\ &\geq \sum_{i=1}^{m}(P_i^- - \frac{a^+}{b^-} - H^+)x(t) - \sum_{i=1}^{m} P_i^+\gamma_i^+ x^2(t) \end{aligned} \tag{11}$$

(11) is a Bernoulli's equation, we can easily get

$$x(t) \geq \sum_{i=1}^{m} \frac{P_i^- - \frac{a^+}{b^-} - H^+}{P_i^+ \gamma_i^+} \tag{12}$$

Now set $B_1 = \sum_{i=1}^{m} \frac{P_i^- - H^+}{P_i^+ \gamma_i^+}$, $B_2 = \frac{\sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e}}{a^- + H^- b^- - \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e}}$. Suppose that $x(t) = x(t, 0, x_0), x_0 > 0$ is a solution of (8), which passes through $(0, x_0)$. We claim that if $x_0 \in [B_1, B_2]$, then $x(t) \in [B_1, B_2]$ for all $t \geq 0$. Assume, by way of contradiction, that there exists a t^* such that $x(t^*) > B_2$ and $x'(t^*) \geq 0$. Thus, from the condition $a^- + H^- b^- > \sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e}$ we have $\sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - a^- - H^- b^- < 0$ and

$$\begin{aligned} x'(t^*) &\leq \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e b^-} + \frac{1}{b^-}\Big(\sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - a^- - H^- b^-\Big) x(t^*) \\ &< \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e b^-} + \frac{1}{b^-}\Big(\sum_{i=1}^{m} \frac{P_i^+}{\gamma_i^- e} - a^- - H^- b^-\Big) B_2 \\ &= \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e b^-} - \frac{1}{b^-} \sum_{i=1}^{m} \frac{P_i^+ b^+}{\gamma_i^- e} = 0 \end{aligned} \tag{13}$$

A contradiction with $x'(t^*) \geq 0$. By a similar argument we can show that $x(t) \geq B_1$ for all $t \geq 0$. Time delay τ is a constant, so as t is sufficiently large, $t - \tau$ is equivalent to t and the solutions of (6) and (8) will be coincided. Therefore, B_1 and B_2 also are the lower bound and the upper bound of the solutions of the time delayed equation (6).

Theorem 2 Let $\tau = \min\{\sigma, \tau_1, \tau_2, ..., \tau_m\}$. Suppose that the conditions in Theorem 1 are satisfied. In addition

$$\Big(\sum_{i=1}^{m} P_i^+ - H^-\Big) e \cdot \exp\Big(-\frac{a^-}{b^+ + B_2} \tau\Big) > 1 \tag{14}$$

Then all solutions of system (6) are oscillatory.

Proof In order to discuss the oscillatory behavior of the solutions, we first discuss the following equation:

$$x'(t) = -\frac{a(t)x(t)}{b(t) + x(t)} + \sum_{i=1}^{m} P_i(t) x(t - \tau) e^{-\gamma_i(t) x(t-\tau)} - H(t) x(t - \tau) \tag{15}$$

Based on the property of time delayed equation, we know that if the solutions of (15) are oscillatory, then for all $\sigma \geq \tau, \tau_i \geq \tau, i = 1, 2, ..., m$, the solutions of equation (6) are oscillatory. Since $x(t) \geq 0$ and $e^{-u} \leq 1 (u \geq 0)$, from the assumptions and $x(t) \in [B_1, B_2]$, we have

$$\begin{aligned} x'(t) &\leq -\frac{a(t)}{b(t) + B_2} x(t) + \sum_{i=1}^{m} P_i(t) x(t - \tau) - H(t) x(t - \tau) \\ &\leq -\frac{a^-}{b^+ + B_2} x(t) + \Big(\sum_{i=1}^{m} P_i^+ - H^-\Big) x(t - \tau) \end{aligned} \tag{16}$$

Now we consider the following equation

$$y'(t) = -\frac{a^-}{b^+ + B_2} y(t) + \Big(\sum_{i=1}^{m} P_i^+ - H^-\Big) y(t - \tau) \tag{17}$$

According to the comparison theorem of the differential equation, for every solution of (17), we have $x(t) \leq y(t)$. Thus, if the solution $y(t)$ of (17) is oscillatory, then the solution $x(t)$ of (16) is also oscillatory. Noting that the characteristic equation associated with (17) is given by

$$\lambda = -\frac{a^-}{b^+ + B_2} + \Big(\sum_{i=1}^{m} P_i^+ - H^-\Big) e^{-\lambda \tau} \tag{18}$$

We claim that the solution $y(t)$ of (17) is oscillatory under the assumption (14). Otherwise, equation (18) will have a real nonpositive root, say λ^* such that

$$\lambda^* = -\frac{a^-}{b^+ + B_2} + (\sum_{i=1}^{m} P_i^+ - H^-)e^{-\lambda^*\tau} \tag{19}$$

Since λ^* is nonpositive, then $-\lambda^* = |\lambda^*|$ and we get

$$|\lambda^*| \geq (\sum_{i=1}^{m} P_i^+ - H^-)e^{|\lambda^*|\tau} - \frac{a^-}{b^+ + B_2} \tag{20}$$

From (20), using inequality $e^u \geq eu (u \geq 0)$ we obtain

$$\begin{aligned} 1 &\geq \frac{(\sum_{i=1}^{m} P_i^+ - H^-)\exp(|\lambda^*|\tau)}{|\lambda^*| + \frac{a^-}{b^+ + B_2}} \\ &= \frac{(\sum_{i=1}^{m} P_i^+ - H^-)\exp(|\lambda^*| + \frac{a^-}{b^+ + B_2})\tau \cdot \exp(-\frac{a^-}{b^+ + B_2}\tau)}{(|\lambda^*| + \frac{a^-}{b^+ + B_2})\tau} \\ &\geq \frac{(\sum_{i=1}^{m} P_i^+ - H^-)e(|\lambda^*| + \frac{a^-}{b^+ + B_2})\tau \cdot \exp(-\frac{a^-}{b^+ + B_2}\tau)}{(|\lambda^*| + \frac{a^-}{b^+ + B_2})\tau} \\ &= (\sum_{i=1}^{m} P_i^+ - H^-)e \cdot \exp(-\frac{a^-}{b^+ + B_2}\tau) \end{aligned} \tag{21}$$

This is a contradiction with (14). Therefore, the solutions of (17) are oscillatory, implying that the solutions of (16)(or (15)) are oscillatory. Hence, for any $\sigma \geq \tau, \tau_i \geq \tau, i = 1, 2, ..., m$, the solutions of equation (6) are oscillatory.
In order to discuss the stability of the equilibrium points, the functions of $a(t), b(t), H(t), P_i(t), \gamma_i(t)$ $(i = 1, 2, ..., m)$ are restricted to positive constants respectively as the follows:

$$x'(t) = -\frac{ax(t)}{b + x(t)} + \sum_{i=1}^{m} P_i x(t-\tau_i)e^{-\gamma_i x(t-\tau_i)} - Hx(t-\sigma) \tag{22}$$

It is known that if x^* is a real equilibrium point of (22), then x^* satisfies the following algebraic equation

$$-\frac{ax^*}{b + x^*} + \sum_{i=1}^{m} P_i x^* e^{-\gamma_i x^*} - Hx^* = 0 \tag{23}$$

Obviously, zero point is an equilibrium point. Apart from zero point, there is a unique real inter-equilibrium point satisfying

$$\sum_{i=1}^{m} P_i e^{-\gamma_i x^*} = \frac{a}{b + x^*} + H \tag{24}$$

Theorem 3 Suppose that the following condition holds

$$\sum_{i=1}^{m} P_i > \frac{a}{b} + H, \qquad a + Hb > \sum_{i=1}^{m} \frac{P_i}{\gamma_i e}, \tag{25}$$

and

$$\sum_{i=1}^{m} \frac{P_i}{\gamma_i e} + \frac{ab}{b + x^*} < a + Hx^* \tag{26}$$

where x^* is the real inter-equilibrium point. Then the inter-equilibrium point of (22) is stable.
Proof Condition (25) ensures the boundedness of the solutions of equation (22). In order to prove the stability of the

real inter-equilibrium point x^* of (22) we assume that $x(t) > x^*$ for t is sufficiently large and set $z(t) = x(t) - x^*$. So for sufficiently large t we have $z(t) > 0$. Noting that $-\frac{a(z(t)+x^*)}{b+z(t)+x^*} = -a + \frac{ab}{b+z(t)+x^*}$, then

$$\begin{aligned} z'(t) &= -\frac{a(z(t)+x^*)}{b+z(t)+x^*} + \sum_{i=1}^{m} P_i(z(t)+x^*)e^{-\gamma_i(z(t)+x^*)} - H(z(t)+x^*) \\ &\leq -a + \frac{ab}{b+x^*} + \sum_{i=1}^{m} \frac{P_i}{\gamma_i e} - Hx^* \end{aligned} \tag{27}$$

To prove the stability of the real inter-equilibrium point, it suffices to show that $\lim_{t\to\infty} z(t) = 0$. We consider a Lyapunov function $V(z(t))$ as follows

$$V(t) = V(z(t)) = [e^{z(t)} - 1]^2 \tag{28}$$

The upper right derivative of $V(t)$ along the solution of (27) can be written as

$$\begin{aligned} V'(t)|_{(27)} &= 2[e^{z(t)} - 1]e^{z(t)} \cdot z'(t) \\ &= 2[e^{z(t)} - 1]e^{z(t)}[-\frac{a(z(t)+x^*)}{b+z(t)+x^*} + \sum_{i=1}^{m} P_i(z(t)+x^*)e^{-\gamma_i(z(t)+x^*)} - H(z(t)+x^*)] \\ &\leq 2[e^{z(t)} - 1]e^{z(t)}[-a + \frac{ab}{b+x^*} + \sum_{i=1}^{m} \frac{P_i}{\gamma_i e} - Hx^*] \end{aligned} \tag{29}$$

Noting that $e^{z(t)} - 1 \geq 0$ for any $z(t) \geq 0$. Condition (26) implies $V'(t)|_{(27)} \leq 0$. Therefore, we have $\lim_{t\to\infty} z(t) = 0$. The inter-equilibrium point x^* is stable. The case for $z(t) < 0$ is similar, the details are omitted.

Example 1 Consider the following Nicholson's blowflies model with a nonlinear density-dependent mortality term and delay harvesting term:

$$\begin{aligned} x'(t) &= -\frac{(0.8+e^{-t})x(t)}{5+\frac{1}{t^2+1}+x(t)} + 0.58e^{0.5\sin t}x(t-\tau_1)e^{-(2+0.2\sin t)x(t-\tau_1)} \\ &+ 0.58e^{0.5\cos t}x(t-\tau_2)e^{-(2-0.2\cos t)x(t-\tau_2)} - 0.03(2+\arctan t)x(t-\sigma) \end{aligned} \tag{30}$$

where $a(t) = 0.8 + e^{-t}, a^+ = 1.8, a^- = 0.8$; $b(t) = 5 + \frac{1}{t^2+1}, b^+ = 6, b^- = 5$; $P_1(t) = 0.58e^{0.5\sin t}, P_2(t) = 0.58e^{0.5\cos t}, P_1^+ = P_2^+ = 0.58\sqrt{e}, P_1^- = P_2^- = \frac{0.58}{\sqrt{e}}$; $\gamma_1(t) = 2 + 0.2\sin t, \gamma_2(t) = 2 - 0.2\cos t, \gamma_1^+ = \gamma_2^+ = 2.2, \gamma_1^- = \gamma_2^- = 1.8$; $H(t) = 0.03(2 + \arctan t), H^+ = 0.0394$, and $H^- = 0.03$. It is easy to check that the conditions of Theorem 1 are satisfied. The solutions of model (30) are bounded. The lower bound $B_1 = \sum_{i=1}^{2} \frac{P_i^- - H^+}{P_i^+\gamma_i^+} = 0.2970$, and the upper bound $B_2 = \frac{\sum_{i=1}^{2} \frac{P_i^+ b^+}{\gamma_i^- e}}{a^- + H^- b^- - \sum_{i=1}^{2} \frac{P_i^+}{\gamma_i^- e}} = 7.3188$. Noting that the functions e^{-t}, $\frac{1}{t^2+1}$ are not periodic, the model (30) is not a periodic system. When we select time delays as $\tau = \tau_1 = \tau_2 = \sigma = 3$, from condition (14) we have $(\sum_{i=1}^{2} P_i^+ - H^-)e \cdot \exp(-\frac{a^-}{b^+ + B_2}\tau) = 9.2642 > 1$, based on Theorem 2, the solution is oscillatory. Then for any $\sigma \geq \tau, \tau_i \geq \tau (i = 1, 2)$, the oscillatory behavior of the solution is maintained (see Fig. 1 and Fig. 2).

Fig. 1 Oscillation of the solution, delays: (3, 3, 3).

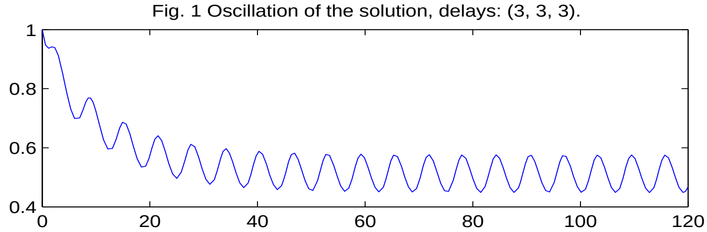


Fig. 2 Oscillation of the solution, delays: (3, 5, 7).

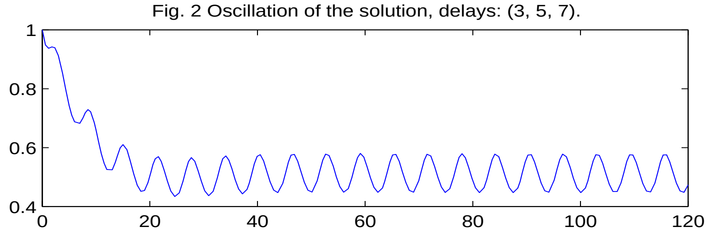


Fig. 3 Convergence of the solution, delays: (3, 5, 7, 6).

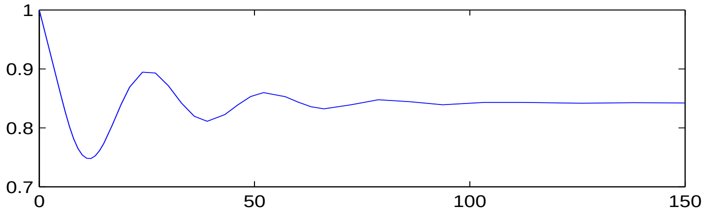
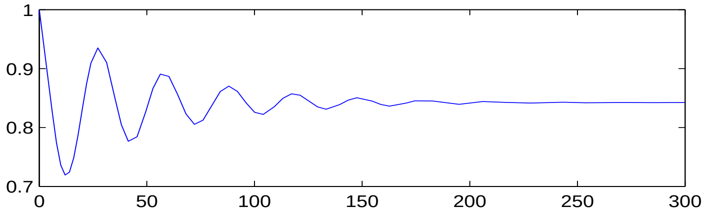


Fig. 4 Convergence of the solution, delays: (4, 6, 8, 6).



Example 2 Consider the following Nicholson's blowflies model in which the parameters are constants:

$$\begin{aligned} x'(t) &= -\frac{1.8x(t)}{4+x(t)} + 0.1x(t-\tau_1)e^{-0.3x(t-\tau_1)} + 0.2x(t-\tau_2)e^{-0.4x(t-\tau_2)} \\ &+ 0.3x(t-\tau_3)e^{-0.5x(t-\tau_3)} - 0.1x(t-\sigma) \end{aligned} \tag{31}$$

Obviously, the inter-equilibrium point $x^* = 0.8418$ is stable (see Fig. 3 and Fig. 4).

Remark 1 We have shown the boundedness of the solutions, and only given the general upper bound and the lower bound.

Remark 2 For equation

$$x'(t) = -\frac{a(t)x(t)}{b(t)+x(t)} + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))}$$

Under the following restricted conditions: $\overline{\lim}_{t\to\infty}\sum_{j=1}^{m}\frac{\beta_j(t)}{e\gamma_j(t)a(t)} < 1, \max_{1\le j\le m}\gamma_j^+ \le 1$, $\sup_{t\in R}\sum_{j=1}^{m}\frac{\beta_j(t)}{\gamma_j(t)} < \frac{a^-}{\max\{1,b^+\}}, \overline{\lim}_{t\to\infty}\sum_{j=1}^{m}\frac{\beta_j(t)}{e\gamma_j(t)} < \frac{a^-}{b^++1}$, the globally asymptotically stable of zero equilibrium point has been proved by the author of Tang (Tang, 2015, Theorem 2.1). Those conditions are different from our Theorem 3.

Remark 3 In equation (6), if $m = 1, H = 0$, then open problem 7 provided by Berezansky et al. (Berezansky, Braverman, & Idels, 2010) has been discussed. If $m = 1$, similar open problem 6 has also been investigated.

Remark 4 According to our simulation, our criteria are the only sufficient conditions.

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