Journal of Mathematics Research; Vol. 8, No. 6; December 2016
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

Mathematical Models of Refugee Immigration and Recommendations of Policies

Qilong Cheng[1], Tiancheng Yu[2], Jingkai Yan[2], & Ru Wang[3]

[1] Department of Mechanical Engineering, Tsinghua University, 100084, China

[2] Department of Electronic Engineering, Tsinghua University, 100084, China

[3] School of Foreign Languages, Jiangxi Normal University, 330022, China

Correspondence: Qilong Cheng, Department of Mechanical Engineering, Tsinghua University, 100084, China. E-mail: chengqlthu@hotmail.com

The team was designated as Meritorious Winner in Interdisciplinary Contest in Modeling in 2016.

Received: October 18, 2016 Accepted: November 8, 2016 Online Published: November 25, 2016

doi:10.5539/jmr.v8n6p85 URL: http://dx.doi.org/10.5539/jmr.v8n6p85

Abstract

Over the past two years, the refugee crisis resulted from the racial conflict, persecution, generalized violence and violations of human rights has forced an enormous number of refugees to flee to Europe. Aiming to address the problem caused by the flow of refugees, we analyzed the actual procedure of their movement and divide it into three major stages. We designed the gathering model, the entering model, the transferring model, even the health and safety model. Finally, we used the models described above to complete our assigned tasks. Also we put forward seven major policy recommendations to the committee. We accompanied every policy with a straightforward explanation so that people without any technical background can easily understand our insights. The main strength of our model is that it can forecast the flow of immigration and provide meaningful suggestions policies for refugees. With the help of modern computing software, we can track the current tendency and make judges efficiently.

Keywords: mathematical models, refugee immigration, policies

1. Introduction

In order to illustrate the problems caused by the influx of refugees inside or outside of their source country into Europe, the following background is worth mentioning.

1.1 Crisis Overview

With the serious and turbulent situation in the west of Asia and north of Africa in 2015, numerous people are forced to leave their homes, especially in Syria and Afghanistan, fleeing their countries through the sea and land route into Europe to find security, support and sufficient food and water to live on. This is the second biggest refugee immigration crisis that happened in Europe since the World War Two. A picture about a young baby huddling up his little body died on the beach is the epitome of lack of the safe transportation vehicles and protection. Attention also has been aroused by public that the refugee crisis has brought a series of social problems, for instance a sexual abuse in Germany and other crime cases from stealing, robbery and even murder.

1.2 International Response

The UN High Commissioner for refugees (UNHCA) has reached out hands to help these displaced people, calling up some countries to provide them with safe shelters and daily life necessities. Germany showed the most positive attitudes towards refugees and received the highest numbers of new asylum applications worldwide. On the contrast, some counties among the east of Europe displayed opposite opinions towards the flow of refugees. However, despite the harsh winter in 2016, there are still a large number of people making their dangerous journey continually across the Aegean Sea to Europe. They are provided with some blankets, pillows, and bed linens to survive the winter even below 24 ℃. And ambulances are also equipped to help take people with health problems to hospitals. But attention should also be focused that European countries may tighten asylum rules as refugee waves continue.

1.3 Our Faith

Our team, the ICM-RUN is very interested in refugee problems not only because of its large scale and fast expending trend, but also because we hold the belief that man was born to be equal, which rooted on our mind deeply and that man has his freedom to pursue his right to live and enjoy the esteem of humanity so that a better solution must be found with the aid of some scientific methods to help them.

2. The Description of the Problem

2.1 How Do Refugees Gather at Gathering Points of Travel Routes?

As a result of major political and social unrest and warfare, refugees will come at assembling places of six travel routes at a specified speed, which will be given in the model. We presume that warfare will burst in all places at the same time, so the refugees rush to the six gathering spots simultaneously. The gathering spots become models of which inputs are refugees that pour into and outputs are refugees that come into Europe through travel routes.

2.2 How Do Refugees Travel Through Routes?

Take the six travel routes as example, there are three by sea and three by land. The model is also suitable if there are more than six travel routes. What's more, routes by land is no different from routes by sea except some parameters like danger coefficient and transport capacity, so we shall restrict our discussion here to sea transport, and train transport can be analyzed just the same with some minor adjustment to values of parameters.

2.2.1 From the Perspective of Shipping Business

Obviously, all the refugees are traveling across the sea in the ships of local business illegally. The shipping business tends to load more refugees in ships and earn more money, but suffers from more danger of shipwreck and more financial loss. The shipping business, which aims at money, should have a plan on how to balance these two things.

2.2.2 From the Perspective of the Government

What is different is that the government cares more about people's lives rather than money. If the government does not supervise the shipping business at all, the shipping business will load as many refugees as they can and cause serious shipwreck. If the government do supervise the shipping business strictly, there will be many refugees waiting to be transported, who still suffer from warfare. The government should supervise to a certain extent where the number of endangered refugees is least.

2.2.3 The Compromise

Although refugees are not legal citizens of Europe so far, the government should impose a fine on the shipping business out of humanitarianism if they cause a shipwreck and make refugees die. Thus, the shipping business faces another condition of balance.

2.3 How Do Refugees Travel into Countries after Entry Points?

Considering limited speed of transportation and adaption to the local environment, we assume that refugees can only travel into adjacent countries in a time unit. We find that how they travel has something in common with Markov Random Field. We regard countries as minimum units and we define each country a parameter that stands for potential. We think that the potential relies on two sides, the national power and the number of refugees left in the nation. The potential of each country determines its energy. The energy of a country is proportionate to how attractive it is to refugees. Meanwhile, we consider each country to have a parameter called inertia probability which means the extent that refugees in one country are not willing to move. And the inertia probability will increase over time, which means refugees are more and more unwilling to move with time passing. Thus, we have a model that can simulate the flow of refugees through energy difference.

2.4 How Do Countries Grant Asylum Applications?

We know that each country will have many illegal refugees that will apply for asylum, but the country cannot grant all the applications. On the one hand, the refugees who are granted asylum will consume the country's resource. On the other hand, the refugees who are not granted asylum will be a kind of threat to the country bring with discontent, impatience, disturbance or even crimes to the country. Therefore, the country should decide how many applications will be granted in a time unit due to the maximum of its benefit.

2.5 How Do We Describe Healthcare and Security?

As for healthcare, the refugees' chronic diseases, acute diseases, infectious diseases should be considered first. Taking the poorer environment into account, refugees are more likely to suffer from the diseases mentioned above and become more susceptible to dying. Besides, both the security problems when refugees travel across the sea and the threat when they have many illegal refugees wandering about should also be considered. The most important, the extremely serious and sudden events such as the terrorist attack in Paris should be paid more attention to.

3. Models

3.1 Gathering Model

First of all, when warfare burst, refugees will choose which route to escape from their homes into Europe.

3.1.1 Assumptions

1) Six routes are considered.

2) Each refugee is independent and has his own choice of routes.

3) Refugees choose the route of the least danger he/she faces on the route.

4) The danger is proportional to the distance refugees have covered.

5) The density of refugees in the effected region is the same.

3.1.2 The Foundation of Model

1) Preparation

a. We number the routes on the map (Baronett, 2008) and set up a plane coordinate system as the following Figure 1.

Figure 1. Map of the six routes

b. We measure the x-coordinate x_i and y-coordinate y_i of the gathering point and the length of the route on the map (under a certain proportion).

We used data retrieved from charts online (Baronett, 2008) to calculate the mortality rate l on each route. Note that the data of eastern Mediterranean covers four routes of the six, therefore we take the average of the eastern Mediterranean data to be the mortality rate of all four eastern routes.

Refugees' route from their homes to the gathering point of the travel routes must be on the land. But their travel routes may not be on the land, 1, 2, 4 on the sea and 3, 5, 6 on the land. We use the parameter ε to convert the route by sea into route by land. Because shipping is often quicker than walking on foot, we propose that ε should be a figure close to 0. ($\varepsilon = 0.1$)

Route #	x_i	y_i	d	ε	l
1	25	15	20	0.1	0.39%
2	130	0	35	0.1	1.93%
3	165	65	30	1	0.0668%
4	220	55	20	0.1	0.0668%
5	175	65	40	1	0.0668%
6	210	120	40	1	0.0668%

Chart 1. Route parameters of the six routes

2) The danger

The danger contains danger on the land and danger on the sea, but we use ε to convert the sea route into land route, so the danger contains the total distance and the danger parameter l. So we have:

$$f_i = l_i(\sqrt{(x-x_i)^2+(y-y_i)^2}+\varepsilon_i d_i) \quad (1)$$

As we can see from the Fig 1, Route 1 and 2 are associated, Route 3 and 5 are associated and Route 4 and 6 are associated. Take Route 1 and 2 for an example.

$$f_1 = f_2 \quad (2)$$

$$l_1\left[\sqrt{(x-x_1)^2+(y-y_1)^2}+\varepsilon_1 d_1\right] = l_2\left[\sqrt{(x-x_2)^2+(y-y_2)^2}+\varepsilon_2 d_2\right] \quad (3)$$

Thus, we have an equation which stands for the border where refugees can either choose Route 1 or 2. It is the same with Route 4 and 6. Note that Route 4 and 6 are naturally separated because of Black Sea. Note that Route 3 and 5 are different because they are inside of Europe and have convenient transportation, so we do not consider them here.

3.1.3 Solution and Result

The solution of the equations is showed as curves in the Figure 2, as follows

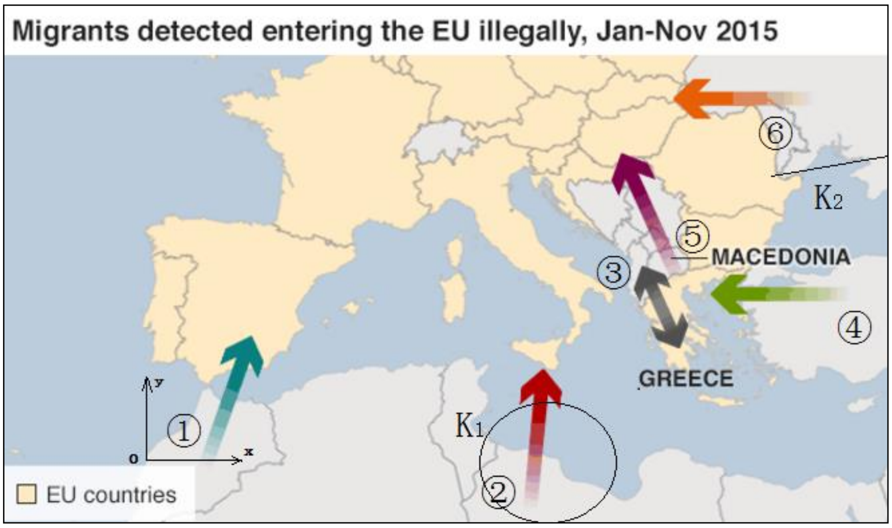

Figure 1. Map partition of the source of refugees

Figure 2 give us a partition of map, which indicates that refugees inside of K_1 should choose Route 2 and the others of North Africa should choose Route 1.

The border K_2 shows that the refugees north of K_2 should choose Route 6 and the refugees south of K_2 should choose Route 4.

The result is understandable because Route 2 is so dangerous that people should not choose it. But the fact is that there are quantities of refugees risking their lives across Central Mediterranean. We do not advocate the choice of routes. Besides, the border K_2 is basically consistent with the natural trend of terrain.

Our result is not sensitive to the parameter ε, so the model is stable itself.

3.2 Entering Model

After arriving at the gathering points, refugee wait for boats or trains to make the vital step of entering the Europe continent.

3.2.1 Assumptions

1) All refugees enter Europe via one of the six routes, setting off from a certain gathering point of each route.

2) The number of refugees to arrive at a assembling place is limited.

3) Each route has a limited capacity. The traffic amount of refugees cannot exceed the capacity.

4) Refugees either leave in batches by boat or train or leave on foot, such that the number of refugees leaving per batch is relatively small versus the frequency of their departure. Therefore the flow of refugees leaving for Europe can

be regarded as a constant continuous flow.

5) When a train ride is required, refugees manage to sneak onto trains without the normal procedure of buying tickets. Therefore the trains may be overloaded and thus brings a chance for chaos on trains.

6) When crossing a body of water (e.g. the Mediterranean) is required, refugees have to take boats owned by individuals who make a profit out of this. Therefore the boat owners may risk overloading the boat in order to make more profits.

7) Refugees are not well-informed about how overpopulated their destinations are, and are willing to go even if the destinations are overpopulated.

8) Each assembling place has limited resources for refugees to live on.

9) Refugees are endangered by local unrest, malnutrition and deadly diseases in the assembling place.

10) Refugees can reproduce while held up in the assembling place.

3.2.2 The Foundation of Model

1) Influence of resources

As assumed, the gathering point possesses resources enough to support S refugees. At some point, if the number of refugees exceeds S, then adverse conditions and lack of resources would cause the population to decrease. On the other hand, if the number of refugees is less than S, this would mark a surplus of resources and reproduction would become the dominant factor and cause the population to expand. This makes a factor $1-\frac{S}{X}$.

Also in either case, this term of the influence of resources is proportional to the current population X (because the rates of reproduction and mortality are both proportionate to it), and also proportional to the rate of resource insufficiency or surplus. Set the coefficient as α, and the term becomes

$$-\alpha X\left(1-\frac{S}{X}\right)$$

If X is greater than S, this marks an excessive population, and the term becomes negative which marks a decrease term for X.

2) Influence of arrival

When a local unrest breaks out, instantly people rush to seek refuge, thus the rate of refugee influx would soar. As time passes, the impact of the unrest recedes and fewer people make new arrivals at the gathering points. Considering that the total number of refugees produced by an unrest is limited, we model the arrival of refugees and an exponential term including time t. Let A be the total number of refugees to come and β the initial rate of incoming refuges, we have the expression of the rate of arrival

$$\beta e^{-\beta t/A}$$

3) Influence of departure

As assumed, we model the departing flow of refugees as a constant continuous flow L. In terms of designed load capacity and overload rate, this decrease term becomes

$$-L=-L_0(1+\theta)$$

4) The differential equation

Therefore, we have the differential equation for X, the number of refugee at the gathering point as follows:

$$\frac{\mathrm{d}X}{\mathrm{d}t}=-\alpha X\left(1-\frac{S}{X}\right)+\beta e^{-\beta t/A}-L \tag{4}$$

The equation has a natural initial condition:

$$X|_{t=0}=0 \tag{4'}$$

5) Accidents caused by overloading

Due to overloading of boats and trains, there exists a positive correlation between the overload rate θ and the accident rate R.

Since a very minimal overload hardly makes any difference, and the accident rate is has a natural upper bound 1, therefore the curve of P versus θ has to be an S-curve. Here we apply and modify the most widely adopted S-curve, the logistic function. Considering that the function should be a mapping from $[0,\infty)$ to $[0,1]$, we therefore modify the logistic function by translation and stretching to be

$$R=\frac{1}{2}\left(1+\frac{1-e^{-k(\theta-m)}}{1+e^{-k(\theta-m)}}\right) \tag{5}$$

Here k is a parameter indicating the slope of the curve, and m is a parameter indicating the point at which the curve attains the steepest increase.

For our purpose of describing the shipwreck probability, we set $m=3$ and $k=5$. This corresponds well to statistics from the marine databases. (Vickers, 2001; Kosko, 1990) This set of parameters gives the following graph by MATLAB:

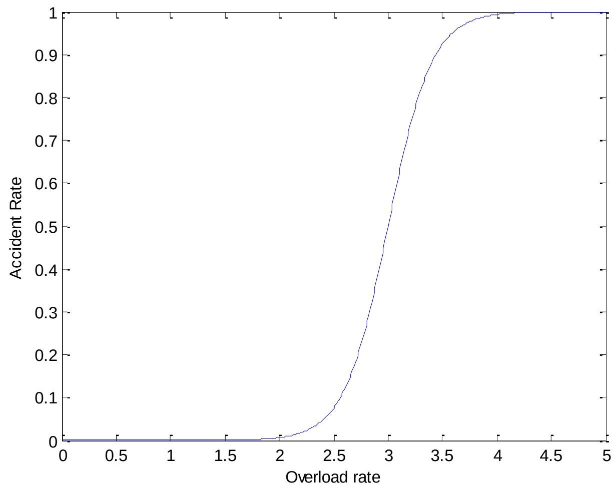

Figure 3. S-curve of the accident rate

6) Analyzing the motive of boat owners

Considering that the mortality rate is much higher on the routes overseas according to statistics (Baronett, 2008), we focus particularly on the overloading on sea, where the boat owners are driven by a desire for profit, and where the surroundings is much more hostile.

The boat owners commonly charges a fixed sum a for each passenger. When unfortunately a boat sinks, the loss for the boat owner comes from two sources – the value of the boat itself b, and less future income due to reduced boat number. Ironically he does not have to compensate for the drown refugees, since there'll be nobody to ask for the compensation.

Hence, there exists a balance between higher boat trip revenues and the risk of losing the boat. The profit function for the boat owners is

$$P(\theta)=La-Rb$$

that is

$$P(\theta)=L_0(1+\theta)a-\frac{1}{2}\left(1+\frac{1-e^{-k(\theta-m)}}{1+e^{-k(\theta-m)}}\right)b \tag{6}$$

The boat owner would seek an optimum overload rate θ to maximize his profit.

3.2.3 Solution and Result

1) Solution to the differential equation

Solve equation (4) with (4'), and we get:

$$X(t)=\left(S-\frac{L}{\alpha}\right)(1-e^{-\alpha t})+\frac{\beta}{A(\alpha-\beta)}\left(e^{-\beta t}-e^{-\alpha t}\right) \tag{7}$$

This is a composite of exponential decay functions. After the time period of 5τ, where $\tau = \max\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ is the time constant , we can regard X as stable henceforth.

2) Optimal choice for boat owners

To find the maximum of $P(\theta)$, we set values for each parameter and use MATLAB for our calculation.

Set $a = 1$ as the fare as the unit, and $b = 1000$ as the loss of a sunken boat.

Set $L_0 = 20$ as the rated load capacity of a boat.

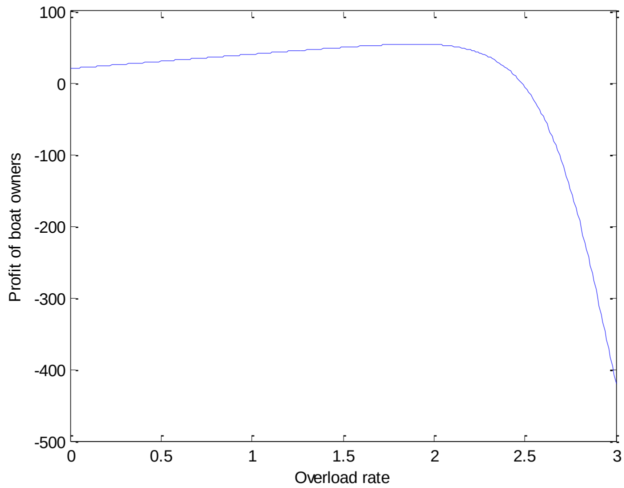

Figure 4. Optimum overload rate at 1.90

Numerical calculations give $\theta = 1.90$ as the maximum point.

3.2.4 Analysis of the Result

1) The pattern of population at gathering points

From the solution(7), we can see a pattern of exponential decay in the population, with a limit of $S - \frac{L}{\alpha}$ when $t \to \infty$, namely

$$\lim_{t \to \infty} X = S - \frac{L}{\alpha}$$

Since all parameters above are positive, this naturally yields $\lim_{t\to\infty} X < S$. This result corresponds to reality, because under the assumption that gathering points have the capacity for a certain number of refugees to live on, the stable condition must be a value less than S. Also in this particular case, combining the departure rate L and reproduction rate index α gives the stable size of the population.

2) Optimal choice for boat owners

The optimal overload rate solved above $\theta = 1.90$ indicates very crowded boats for the refugees. Boats on average would carry twice its capacity, which poses considerable risks for refugees on the vast seas.

3) Policy for less fatality over sea

However, we cannot let the lives of human beings be grasped in the hands of boat owners. International organizations ought to set up laws and regulations to reverse this tragic trend.

A simple way is to follow the logic of boat owners and give them penalties for every sinking accident that happens on their hand. This would add a weight on the scales against overloading.

Formally, an occurrence of sinking accident would bring a loss of b for boat owners. Adding a penalty is equivalent to raising the loss value. Assume penalty makes the loss ten times its original value, at $b = 10000$. Repeat the previous steps.

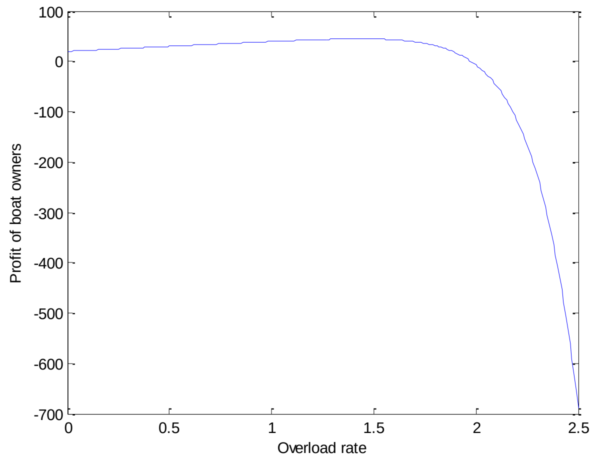

Figure 5. Optimum overload rate at 1.44 with penalty

Comparing Figure 4 (with penalty) with Figure 3 (without penalty), we see that the pattern of the figure is similar but the optimal overload rate becomes remarkably smaller. Besides, the slope of the initial increasing part becomes smaller in the model with penalty, which also means that boat owners may be less willing to risk overloading since there is no notable increase in profits. Therefore, applying penalty to boat owners can indeed lower the average overload rate of refugee boats and help save their souls.

3.3 Transferring Model

As we have already modeled the process of gathering and entering Europe, we now focus on the immigration flow inside Europe. We only focus on the main accepting countries in the below diagram and the transferring among them.

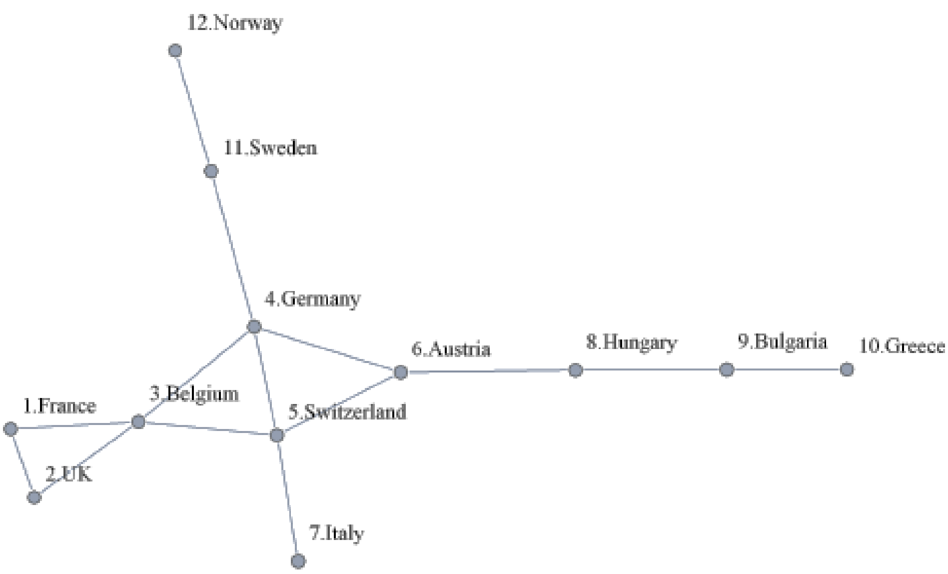

Figure 6. Transferring graph in Europe

3.3.1 Assumptions

1) Refugees can either come into Europe through customs or illegally.

2) In a single period, the refugees can only move among nearby countries. This is because the velocity of immigration is relatively slow.

3) The refugees are attracted by nearby countries considering both whether it is livable and how many refugees are already there. The attraction will increase as refugees accumulate first but finally decrease because of limited environmental capacity. The flow is also constrained by the transportation capacity.

4) The flow of immigration is radical at first but will finally become gentle. Refugees not accepted by any country will just stray in the country.

5) The EU will force each country to accept a certain number of refugees every period and EU will provide an amount of subsidy. The accumulation of unaccepted refugees will lead to crime and even riot so every country will accept an optimal number of refugees to keep a balance.

6) Based on the above assumptions, we model the flow by a Markov Random Field.

3.3.2 The Foundation of Model

1) The output of entry points

Refugees can either come into Europe through customs or illegally, so

$$O = O_1 + O_2$$

Illegal refugees into Europe has a proportion μ in the refugees that have not been accepted at entry points, so

$$O_2 = \mu(y - O_2)$$

Thus, we have a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = L - O \tag{8}$$

$$y|_{t=0} = 0 \tag{8'}$$

Solving this equation gives the following

$$O = (1 - e^{-\mu t})L + (1 - \mu)e^{-\mu t}O_1 \tag{9}$$

We can apparently see that as time flies or as L changes, we can change O_1 to guarantee O at a relatively stable value, so European countries will not suffer from intense variation of refugees that pour in.

2) The transfer equation

Based on the assumptions above, we can construct an equivalent potential for each country to describe the related attraction. A proper option is the logistic:

$$\phi_i = -(\lambda_i + x_i - b_i {x_i}^2)$$

According to the statistical mechanics principle in physics, the partition of is proportional to the exponential of potential. Considering the time factor and inertia of immigration, we also include an inert parameter

$$\alpha = 1 - e^{-\mu\tau}$$

Thus we can compute the flow from the i_{th} country to the j_{th} country

$$f_{ij} = x_i(1 - \alpha)\frac{e^{-\phi_j}}{\sum_k e^{-\phi_k}} \tag{10}$$

$$f_{ii} = \mathrm{x}_i\left[\alpha + (1 - \alpha)\frac{e^{-\phi_i}}{\sum_k e^{-\phi_k}}\right] \tag{11}$$

Finally, considering the limited transportation capacity, we must compute the minimum of the transportation capacity and the above flow

$$f_{ij} = \min\left\{x_i(1 - \alpha)\frac{e^{-\phi_j}}{\sum_k e^{-\phi_k}}, l_{ij}\right\} \tag{12}$$

$$f_{ii} = x_i - \sum_{j \neq i} f_{ij} \tag{13}$$

Finally we can compute the recursive relation of the current period and the next period

$$x'_i = \sum_j f_{ji} \tag{14}$$

3) The acceptance strategy

As refugees accumulates, they bring risk of crime and riot. The EU force each country to accept a certain numbers of refugees. At the same time, every country will find an optimal strategy to accept refugees and it is our focus in this section.

We conquer the problem by minimize a cost function, namely

$$C_i = (q-r)n_i + s(x_i - n_i)^2 + tn_i(x_i - n_i) \tag{15}$$

Here we give a brief explanation of the meaning of the loss function. The linear form is the resettlement cost and the EU subsidy. The quadratic form is a metric of the risk of crime because both the probability and the destructive power is proportional to the accepted refugees. The last cross term is a metric of the riot because the dissatisfaction of the accepted refugee is proportional to the number of refugee accepted so the sum is a proper metric.

Not surprisingly, the cost of the every country is independent so they will do the best decision for themselves. Finding the minimum of the cost function gives

$$n_i = \frac{t-2s}{2t-2s}x_i - \frac{q-r}{2t-2s} \tag{16}$$

Finally, considering the quota

$$n_i = \max\left\{\frac{t-2s}{2t-2s}x_i - \frac{q-r}{2t-2s}, quota_i\right\} \tag{17}$$

The accepted number is a linear function of the total refugee. Taking this in account, we can give a better recursive equation of the refugees:

$$f_{ij} = \min\left\{(x_i - n_i)(1-\alpha)\frac{e^{-\phi_j}}{\sum_k e^{-\phi_k}}, l_{ij}\right\} \tag{12'}$$

$$f_{ii} = x_i - \sum_{j \neq i} f_{ij}$$

$$x'_i = \sum_j f_{ji}$$

3.3.3 Solution and Result

1) The asymptotic solution

Unfortunately, because of the appearance the minimum function, it is impossible for us to give an explicit solution of the recursive equation. We will utilize numerical simulation software later we will first use a physical principle to give an asymptotic solution and thus get some insight into the essence of the problem.

From the recursive equation, we know that the total potential of the system will always decrease therefore after a long time it will definitely reach a minimum point of potential(at least a local minimum, but as we prove later, the extreme point is unique so it is also the global minimum).

The minimum problem can be transformed into a constrained optimization problem

$$\text{minimize} \sum_i \phi_i$$

$$\text{s.t.} \sum_i x_i = X$$

This problem can be tackled by the Lagrange multiplier technique and the corresponding unique solution is

$$x_i = \frac{X}{\left(\sum_k \frac{1}{b_k}\right) b_i}$$

2) The simulation result

Using the current statistics, a quintessential diagram of the refugee in each country and the accumulating number of refugees accepted is as below.

The first row describe the refugee in the twelve major countries and each number represent a country as in Figure 6 above. The second row describe the accumulating number of refugee accepted in the twelve countries. The unit of the horizontal coordinate is a week and the vertical coordinate is a thousand persons.

We can easily find that the countries in north Europe only accept very few refugees and it is just that case. In the contrary, countries in the middle like Germany accepted a lot of refugees.

Another insight we can give is that the tendency of the flow. In the boundary countries, the number of refugee accumulates very fast at first but later decreases quickly. For most other countries, there is at least one extreme point.

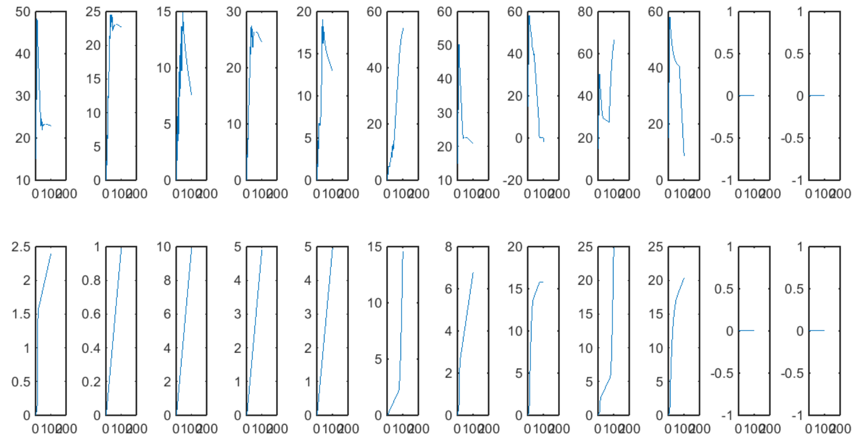

Figure 7. A quintessential diagram of the refugee in each country

3.4 Health/Security Model

In this section, we model the two crucial elements, namely the disease and crime.

3.4.1 Assumptions

1) Disease are divided into infectious diseases and non-infectious diseases. For both kind, we consider the different curing rate and lethal rate between permanent residents and refugees. For the first kind, we also consider the infectious rate

2) The riot is a small probability event. The probability is uniform in time and is proportional to the number of unaccepted refugees.

3.4.2 *The Foundation of Model*

1) The infectious diseases model

In this section, as there are so many qualities to compute, we use a short-hand form. We corporate the same quality with only different i into a vector, for instance,

$$u = [u_1, u_2, \dots \dots, u_n]$$

When we use this notation, the multiply operation should be regarded as operated between the corresponding terms between the two vectors.

Another notation we will utilize is when we add to any variable, that it means the value in the next period. Regarding this notation, we can write the evolutionary equation in a very explicit manner. We first list the equations below and then explain the meaning of each term.

$$\begin{aligned} u' &= u - h_1 v_1 \\ {v_1}' &= (1 - h_1 - f_1) v_1 + g(u - v_1)(v_1 + v_2) \\ {w_1}' &= w_1 + h_1 v_1 \\ x' &= x - h_2 v_2 \\ {v_2}' &= (1 - h_2 - f_2) v_2 + g(x - v_2)(v_1 + v_2) \\ {w_2}' &= w_2 + h_2 v_1 \end{aligned} \tag{18}$$

For x and u, we need only minus the dead people in the current period. For v_1 *and* v_2, we minus the dead and cured people and add the infectious people in the current period. Finally, for w_1 *and* w_2, we accumulate the dead people.

2) The non-infectious disease model

Different from infectious disease, non-infectious disease will not propagate, so we only need to calculate the dead rate. Similar notation gives

$$u' = u - h_1 v_1$$

$$\begin{aligned} {v_1}' &= (1 - h_1 - f_1)v_1 \\ {w_1}' &= w_1 + h_1 v_1 \\ x' &= x - h_2 v_2 \\ {v_2}' &= (1 - h_2 - f_2)v_2 \\ {w_2}' &= w_2 + h_2 v_1 \end{aligned} \tag{19}$$

A major difference is that the flow of refugee's influence here is not as crucial as in the previous situation.

3) The riot model

Different from disease, riots are small probability events and must be modeled stochastically. Based on the motivations stated in the assumption, we model the riots by a Compound Poisson Process.

In a period of time t, the probability that happened k riots is

$$P(N_i = k) = \frac{(\xi_i x_i t)^k}{k!} e^{-\xi_i x_i t} \tag{20}$$

Where we have already used the fact that the coefficient is proportional to the scale of refugees.

When a riot happen, we model the destructive power the scale of refugee multiplying a standard Rayleigh random variable R

$$R_i = x_i R$$

According to the Wald equation, the expectation

$$E_i = \xi_i {x_i}^2$$

Therefore, again, our problem is transformed into an optimization problem:

$$\text{minimize} \sum_i \xi_i {x_i}^2$$

$$\text{s.t.} \sum_i x_i = X$$

3.4.3 Solution and Result

1) The simulation of the infectious and non-infectious diseases model

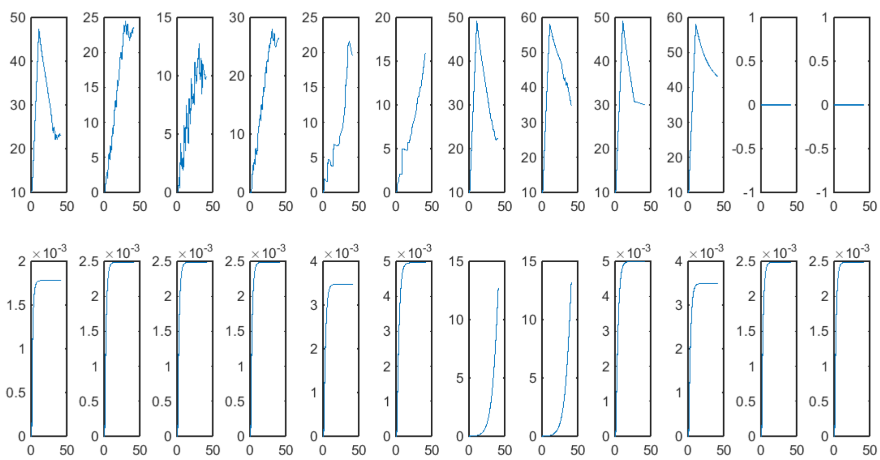

Figure 8. Simulation results with infectious disease

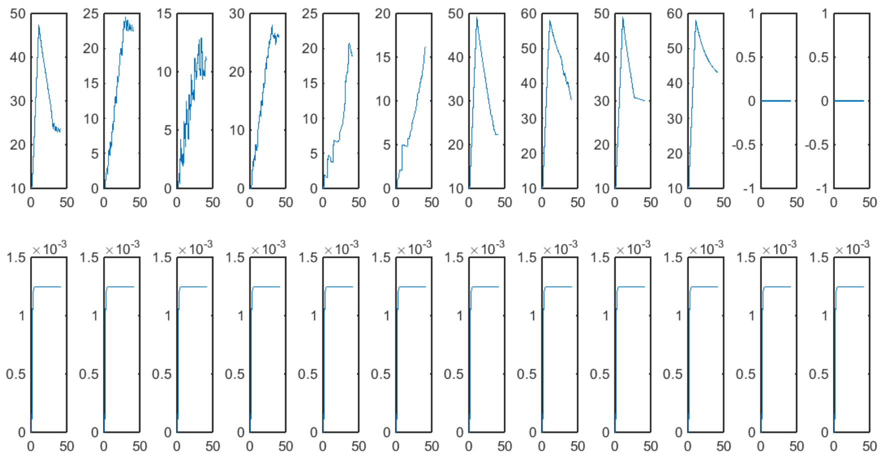

Figure 9. Simulation results with noninfectious disease

The two diagrams above describe the simulation results when there is an infectious disease or a noninfectious disease. The detailed information can be found in the appendix.

Comparing with infectious disease, the influence of non-infectious disease is much weaker. In fact, the major difference is the infectious rate. Therefore, it is a very crucial parameter for the system and we will argue later efficient policy is needed to limit the infectious disease in order not to let the disease outbreak.

2) The minimization of average destructive power

Fortunately, this time we can compute the minimum explicitly. By Cauchy's inequality

$$\sum_i \xi_i {x_i}^2 \sum_i \frac{1}{\xi_i} \geq \left(\sum_i x_i\right)^2 = X^2 \tag{21}$$

$$\sum_i \xi_i {x_i}^2 \geq \frac{X^2}{\sum_i \frac{1}{\xi_i}} \tag{22}$$

When the equality holds

$$x_i = \frac{X}{\xi_i \sum_i \frac{1}{\xi_i}} \tag{23}$$

4. Solutions to Tasks

4.1 Metrics of Refugee Crises

We classify the factors relevant to a refugee crisis into four categories: demographic data, route information, traffic condition, and resource capacity.

The specific measures and parameters are as follows:

- Total numbers of incoming refugee – *A (the total number of refugees that suffered from an outbreak of crisis)*

This measures the size of impact of a refugee crisis, which is a basic factor of crisis intensity.

- Rate of the refugee influx – β *(the initial speed of incoming refugees when a crisis breaks out)*

This measures the intensity of a refugee crisis. An abrupt crisis when people flood out to seek refugee is more troublesome than a chronic one provided the total number of refugees is the same.

- Constitution of the refugee population, including gender distribution, age distribution, etc. – *(a series of variables indicating proportions)*

This measures the refugees' ability to withstand exogenous adversities, such as diseases and lack of food and resources. Besides, there could be some factors that specially arise from the population's constitution.

- Load capacity of each route – L *(rated transport capacity of a necessary route for refugees to escape to safety),* l_{ij} *(transport capacity for refugees to move around between countries)*

This measures rate at which refugees are able to reach safe countries. This is a vital index about the outward flow of refugees.

- Danger on each route – ε *(index for the specific danger on a route), k (indicating the danger of overload of vehicles)*

This measures the safety of refugees on their way of migration. This factor is directly related to the well-being of the refugees. This can also significantly affect the condition and choices of the refugees.

- Resource capacity – *S (the amount of resource at a gathering point for refugees to cross over to safe regions), b (index for the capacity of a country to accommodate refugees)*

This measures the amount of environmental and social resource available to the refugees. Similar to the danger index, the resource capacity index strongly influences the refugees' survival and well-being.

4.2 Flow of Refugees

We divided the flow of refugees into 2 parts. One is the route from the refugees' effected homes to the gathering point. The other is the route from the gathering point to Europe.

- From Homes to the Gathering Point

We have made best options of route for refugees according to the model in 3.1. Taking the danger that refugees will meet on the route to Europe into account, our model assumes that refugees will choose the route with the least danger. After calculation, we suggest that refugees from North Africa choose Route 1 or 2 according to the border K1, and that refugees from West Asia choose Route 4 or 6 according to the border K2.

- From the Gathering Point to Europe

We have built a model of how refugees arrive in Europe in 3.2. We estimate the optimal option of boat owners considering their profits. Also, we estimate the minimum fatality of refugees.

4.3 Dynamics of the Crisis

A crucial character which have been considered thoroughly in our model is that the environmental change over time. As refugees move across Europe, the attraction of each country will definitely change, thus influence the flow. More detailed information can be found in the modeling section above.

- Forecasting

An essential advantage of our model is that it is rather easy to forecast the move the refugees. According to the results of simulation, we can preposition all kind of resources. According to the simulation results, the crucial resource include living necessities (clean water, food and clothes), medicine and medical care.

- Capacity and availability

The parameter b in our model is proportional to the reciprocal of the capacity. As the refugees accumulate in certain place, the situation will deteriorate exponentially which incorporate the limit of capacity. Although we do not include an absolute capacity which can never be exceed, but our assumptions are more realistic and easy to compute. Also, the availability of other typical resource is tackled similarly, like the transportation capacity and the availability of living necessities (already incorporated into b).

- The role of the government and the NGOs

We have not shown explicitly how government and NGOs work in the model before, so we will give an in-depth discussion here. As the government, it should give necessary resources to refugee unaccepted and incorporate the accepted refugee into the society. This is exactly what we do when finding the optimistic strategy for governments by minimizing the cost function.

For NGOs, their work can be considered global optimization. For NGOs who need to allocate their resource in advance, they can utilize the result of minimize the total potential and the results says that the asymptotic distribution is proportional to the capacity in each country so we recommend the NGOs to allocate their resource according to the current capacity in each country.

- Extensibility

Our assumptions above are fairly general that it can be easily utilized by non-European countries like Canada and China. We only need the remove the influence of quota because there is no such powerful local union.

4.4 Policy to support refugee model

We list our recommendations to both government, UN and NGOs below and then explain our reasons. As instructed, we prioritize the security and body health of both refugees and permanent residents in Europe.

- Set up regulations to control the phenomenon of overloading refugee boats, which poses a major threat to refugees' lives.

While it is totally understandable that refugees are extremely eager to reach Europe, even putting aside the danger of drowning themselves, efforts still need to be done to control the excessive overload. We call for some international organizations to set up a regulation that sets the maximum overload rate. We note that slight overloading actually meets the demand of refugees, so total elimination of overloading is unpractical, but certain restrictions on the upper limit would be beneficial nonetheless. An alternative choice would be to cooperate with some local boat owners, offering money to them to let them keep the overload rate at a relatively rational level.

- Supervise the output of refugees that come past customs at the entry points into Europe to supervise the total refugees into Europe.

From our model of entry points in 3.3.3, the number of refugees that enter Europe via entry points is influenced by the intensity of the inflow, the rate of illegally sneaking, and time. In order to control the rate of the entrance of refugees at a relatively stable level, customs need to frequently adjust the number of applications approved. For example, when the number of the incoming refugees sees a sharp rise, customs should not rise the number of approved applications along with it, otherwise the rate of refugees entering Europe would rise notably. Also, governments of those countries should devise more effective ways to settle the refugees held up at entry points, since it is unreasonable to admit too many refugees into the country at a time.

- The government and NGOs should pay enough attention to decrease the infectious rate, especially between refugee and permanent residents.

Based on our infectious disease model, we conducts over 100 simulation in different conditions and find the result is really sensitive to the infectious rate g. In fact, when g increases 10 times, the infectious and dead people in a year increase 10000 times. Comparing with the infectious rate, the model is rather robust to the change of curing and lethal rate of the disease. So we strongly recommend the government and NGOs to provide enough facilities to limit the infectious rate.

- The EU and NGOs should guide the refugees to come to countries less crowded like those in North Europe for better stability and less riot destruction.

Both the analysis of expectation of riot destruction and global stability shows that the best distribution of the refugee is uniform in some sense. Our first result is the most stable (least potential) distribution is proportional to the capacity and the second result shows the least potential riot destruction happens when the distribution is proportional to the square of population in each country. Our simulation also found that the refugee in North Europe is a lot less than in the middle and west part of Europe. Guiding refugee to these districts will moderate the stress in other parts.

- The EU should adjust the refugee subsidy, weaken the quota system and convince the European countries there will be harmful if not accepting any refugee.

According to our model and the corresponding simulation results, the best subsidy which make the global situation most stable and decrease more radical immigration is a little higher than the resettlement cost of each refugee. The current subsidy is proper in this regard. Also, we find the performance of the quota policy is rather weak, especially in the long run. Rather, they should convince the European countries of the harm and guide them to use the optimal strategies.

4.5 Exogenous Events

In general, our following procedure can deal with all kinds of exogenous events but to be concrete, we assume like in the task that Belgium was placed in a lockdown after the Paris raids in attempts to capture possible terrorists. We assume a protest campaign happens in Belgium and Belgium forbids all the refugee to enter the country. The following are our simulation, analysis and policy recommendation.

- Parameters shifts

The livable degree and environmental capacity in Belgium is completely changed. It can be considered as if λ decreases and b increases both dramatically. A rational simplification is to consider $\lambda \to 0$ while $b \to \infty$.

- Cascading effects

Again, we simulate the resulting refugee flow in the following diagram.

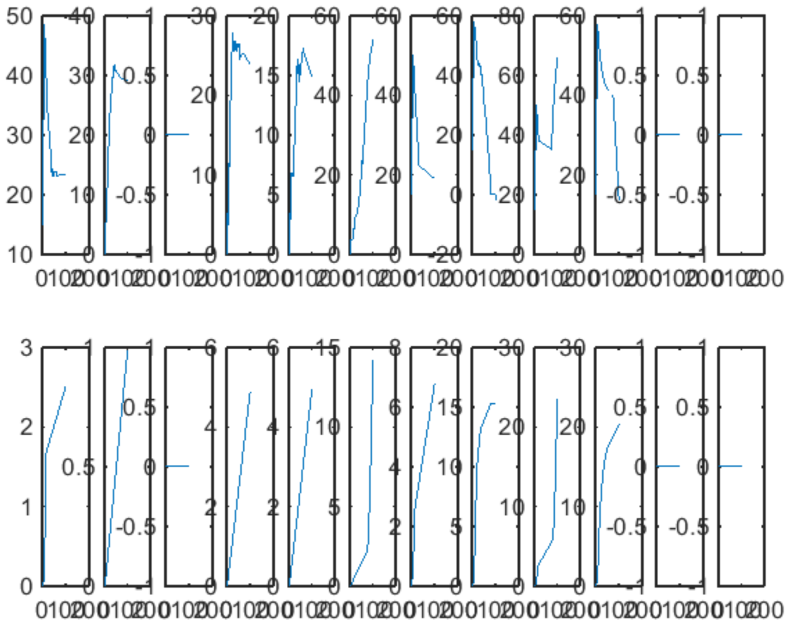

Figure 10. Simulation results with exogenous event - shutdown of city

Comparing with the diagram when Belgium functions well, we can see that the stress of the neighboring countries, namely, the accumulated refugee and refugee accepted, increased a lot and the extreme points come earlier.

- Policy recommendation

As we can see, if an important country is paralyzed, the result is serious. As EU or NGOs, we suggest they allocate the resources more uniformly, which we have already suggested before. This can make the policy more resilient. Also, we suggest the EU to moderately increase the subsidy for the neighboring countries to moderate the stress.

4.6 Scalability

When the scale have increased by ten times, the framework of our model do not need any further major adjustments. Only the number of refugees will increase. We simulate the new scenario and the result is as below

Comparing the diagram with the diagram before, we can see that the peak have been dramatically increased and the number of refugee in the boundary countries have increased more than 10 times. This is the result of relatively limited transportation capacity. The time for the unaccepted refugee scale to be decreased to the same scale has been obviously prolonged and this bring us new challenges in health and security, just like we discussed before. Finally, we suggest the EU give some extra support to help facilitate the flow. If the transportation capacity also increases by 20 times, we can get the new simulation results below.

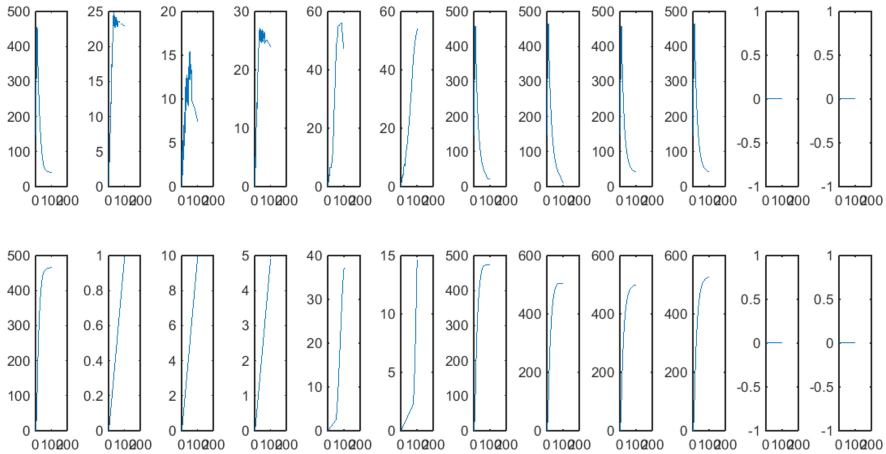

Figure 11. Simulation result of refugee population expanded by 10 times

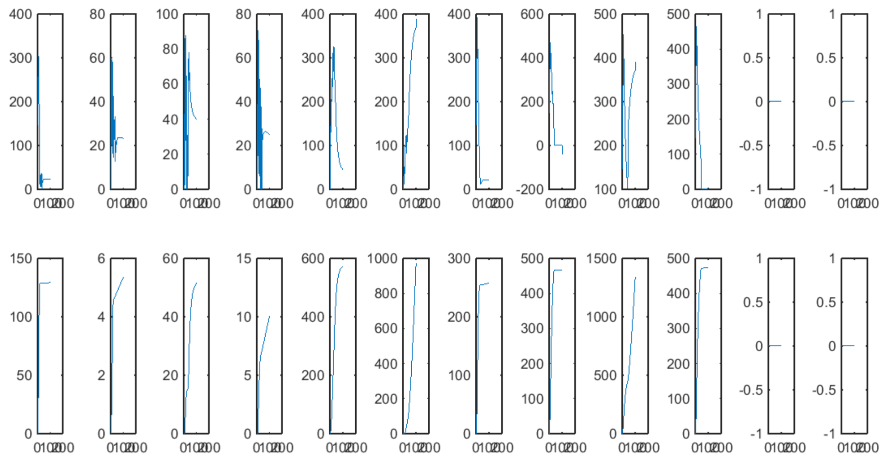

Figure 12. Simulation result with traffic capacity expanded along with population

We can see that this time, the refugee distribution is comparatively more uniform and more controllable.

5. Evaluations of Models

5.1 Sensitivity

5.1.1 Assembling Model

The partition of North Africa and West Asia relies on the parameter ε . And our result is not sensitive to the parameter ε , so the model is stable itself.

5.1.2 Entering Model

This model of population size includes many contributing factors and none of them is sensitive. The calculation of optimum overload rate is quite sensitive to the parameter k which indicates the shape of the S-curve and is not sensitive to load capacity L_0.

5.1.3 Transferring Model

The flow of immigration relies on the parameter b asymptotically but not too sensitive to the parameter. The flow of immigration is relatively more sensitive to the transportation capacity and the other parameter's influence is rather weak.

5.1.4 Health/Security Model

The most sensitive parameter is the infectious rate g and the other parameters' influence is a lot weaker than g.

5.2 Strength/Weakness

5.2.1 Assembling Model

The model is prioritize refugees' lives. The model only considers about six routes and we can use the same way to solve it if there are more routes. But we do not consider about population distribution of different regions.

5.2.2 Entering Model

The model is inclusive of many aspects, including resource capacity and internal attributes of refugees. This model well explains the overloading phenomenon. A point to be improved is that we consider the sinking accidents as evenly distributed instead of discrete incidents.

5.2.3 Transferring Model

The model's principal advantage is that we can easily forecast the flow of the immigration and get many insights into the nature of the problem. The weakness should be attributed to the abstract and simplified nature of the model and thus ignore some potential influential effects like the inherent structure of the refugee.

5.2.4 Health/Security Model

The model's major advantage is that we can find the propagation tendency of certain disease and thus we can preposition our resource to moderate the stress. The weakness is that we do not include the correlation between different diseases and riots, which is crucial when considering the global situation.

6. Conclusion

Aiming to address the problem caused by the flow of refugees, we analyzed the actual procedure of their movement and divide it into three major stages. In addition, health and security and threat of refugees and local people should also be considered.

In the assembling model, under the basic assumption to minimize risk, our model proposes a partition of Northern Africa and Western Asia, which proposes the choice of the six main routes for the refugees.

In the entering model, we modeled the gathering points with input of refugees from nearby regions, output of refugees to Europe and the influence of danger and resource limitations. In particular, we studied the pervasive phenomenon of boat overload, and found that moderate overloading could both yield higher profits for boat owners and meet the refugees' demand. We determined the optimal value of overload rate from the perspective of boat owners. Also our model gives advice that international organizations should cooperate with boat owners and lower the overload rate to a safe level via contractual means.

In the transferring model, upon arriving at entry places, refugees seek to go through customs, whether with granted asylum applications or with stealth. We modeled this process and gave an insight into the flow control policy. Then we considered the transfer of refugees between European countries. This is modeled as a Markov Random Field to forecast the immigration flow together with a constrained optimization technique to choose the optimal strategy for countries to accept refugees.

In the health and security model, we modeled two major concerns about refugees as well as permanent residents in Europe, namely, diseases and riots. We evaluated the lethal rate, curing rate and infectious rate of the diseases and gave a policy recommendation. For the riots, we modeled them stochastically and used an optimization technique to give the most stable distribution of unaccepted refugees.

Finally, we used the models described above to complete our assigned tasks. Also we put forward seven major policy recommendations to the committee. We accompanied every policy with a straight forward explanation so that people without any technical background can easily understand our insights.

The main strength of our model is that it can forecast the flow of immigration and provide meaningful suggestions policies for refugees. With the help of modern computing software, we can track the current tendency and make judges efficiently. Our major weakness, due to the abstract and simplified nature of our model, is that it does not give enough attention to some less essential factors like inherent structure of the refugees.

References

Baronett, S. (2008). Logic. Upper Saddle River, NJ: Pearson Prentice Hall.[M] p. 321–325.

Copi, I. M., Cohen, C., & Flage, D. E. (2007). Essentials of Logic (Second ed.). Upper Saddle River, NJ: Pearson Education.

Gray, P. (2001). *Psychology* (Sixth ed.).[M] New York: Worth.

Herms, D. (2010). Logical Basis of Hypothesis Testing in Scientific Research. Stanford Encyclopedia of Philosophy: Kant's account of reason.

Vickers, J. (2011). *The Problem of Induction*.[M] The Stanford Encyclopedia of Philosophy.

Kosko, B. (1990). Fuzziness vs. Probability. *International Journal of General Systems*. p: 211–240.

Popper, K. R., & Miller, D. W. (1983). A proof of the impossibility of inductive probability. *Nature*. 687–688.

Rathmanner, S., & Hutter, M. (2011). A Philosophical Treatise of Universal Induction. *Entropy*.

Tymoczko, T. (1980).The Four-Color Problem and its Mathematical Significance.

Trans. R.G. Bury. (1933). Harvard University Press, Cambridge, Massachusetts, p. 283.

Vickers, J. (2010). The Problem of Induction (Section 2). Stanford Encyclopedia of Philosophy.

Appendix

A chart of symbols and definitions

1. Gathering model

Symbol	Definition
d	Distance of the i-th route into Europe
ε	Parameter that stands for the route by sea or by land
l	Parameter that stands for the probability of meeting danger
f	Parameter that stands for the danger of different routes

2. Entering model

Symbol	Definition / Explanation
X	The number of refugees at a certain gathering point
t	Time elapsed since the beginning of the refugee crisis
S	The amount of resources at the gathering point, measured by the number of refugees it can stably support
A	The total number of refugees to come to a gathering point because of a crisis outbreak
α	A parameter indicating the impact of resources on the number of refugees
β	The initial rate of incoming refugees when a crisis breaks out
L_0	The designed load capacity of vehicles
L	The actual load of vehicles
θ	The overload rate of vehicles
R	The accident rate of vehicles, as caused by overloading
k	A parameter indicating the correlation between fatality rate and overload rate
a	The price the boat owners charges per person
b	The loss of a boat owner when a boat sinks
P	The profit of boat owners in a time unit

3. Transferring model

Symbol	Definition
y	The number of refugees at entry points

O	The output of refugees who come into Europe from entry points
O_1	The output of refugees who come into Europe through customs
O_2	The output of refugees who come into Europe illegally
μ	The proportion of illegal refugees into Europe in the refugees that have not been accepted at entry points
λ_i	The livable degree of the *i*-th country
x_i	Refugee in the *i*-th country in the current period
x'_i	Refugee in the *i*-th country in the next period
b_i	The decay coefficient of attraction in the *i*-th country
ϕ_i	The equivalent potential of the *i*-th country
α	The inert parameter of the refugee flow
l_{ij}	The transportation capacity between the *i*-th and the *j*-th country
f_{ij}	The refugee flow between the *i*-th and the j-th country
n_i	The number of refugee the *i*-th country accept in the current period
C_i	The loss function of the *i*-th country
q	The subsidy EU provide for each refugee accepted
r	The resettlement cost for each refugee accepted
s	The crime parameter for unaccepted refugees
t	The riot parameter for unaccepted refugees
$quota_i$	The quota for the *i*-th country

4. Health/Security Model

Symbol	Definition
$h_{1,2}$	The lethal rate for permanent resident and refugee
g	The infectious rate
$f_{1,2}$	The curing rate for permanent resident and refugee
u_i	The population of the *i*-th country
$v_{1,2i}$	The number of infected permanent resident/ refugee in the *i*-th country
$w_{1,2i}$	The number of permanent resident/ refugee dead in the *i*-th country
N_i	Riots happened in a certain period of time in the *i*-th country
ξ_i	The risk coefficient in the *i*-th country
R	Standard Rayleigh random variable
R_i	The destructive power for each riot in the *i*-th country