ISSN: 1916-9795
E-ISSN: 1916-9809

Journal of Mathematics Research
Vol. 2, No. 2, May 2010

Blow up Property of the Solution for Quasi Linear Parabolic Equations and Its Application

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This work was partially supported by NSF of China, KJF(1011xx) of CQEC

Abstract

This dissertation is to discuss the initial-boundary value problem under the third nonlinear boundary condition for a kind of quasi-linear parabolic equations. To apply the maximum value theory and convex method, it is proved that the blowing up of solution in the definite time. And for application, this paper research into a mathematics model of fluids in porous medium. And have got the blowing up behavior for the problem in limited time.

Keywords: Quasi linear parabolic equation, Initial-boundary balue problem, Blowing up of solution, Flow dynamics

AMS Mathematics Subject Classification (2000): 35K57

1. Introduction

It is significant practically for the research that the solution of mathematics physical partial differential coefficient equation (Developmental Equation) which is related to the time variable t and appear in engineering dynamics such as the limited swing wave spreading, mixed gas burning, fluent mechanics and reaction diffusing, blows up in limited time. Because, whether the research of unitary characters (e g. stability) of fixed solution problem for nonlinear developmental equation or finding the methods of numerical value computing, both base on unitary existence of the solution. If the obtained solution blows up in limited time (the solution is to infinite in limited time), however, such blowing up behavior is not permit by the relevant engineer dynamics model, this shows the engineer dynamics model is questionable and it should be modified. If such blowing up behavior permit the studied engineer dynamics problem, we must compute in a more extensive function since the relevant engineering processes will keep on developing by no means of ending at a certain moment.

This dissertation discuss the original boundary value problem for the quasi-lineal parabolic equation with the third type nonlinear boundary condition:

$$\begin{cases} \frac{\partial u}{\partial t} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}(a_{ij}(x)\frac{\partial u}{\partial x_j}) + \sum_{i,j=1}^{n}(a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j} + b(x)u + f(x,t,u,\nabla_x u), & D\times(0,T) \\ \beta(x)\frac{\partial u}{\partial \nu} + u = g(x,t,u,\nabla_x u) & \partial D\times(0,T) \\ u(x,0) = u_0(x) & \bar{D} \end{cases} \quad (1)$$

D is the limited domain within R^n, $x = (x_1, x_2, \cdots, x_n)$, $\beta(x) > 0$, $a_{ij}(x) = a_{ji}(x)$ is the continual differentiable on $\bar{D}$, to $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_n) \in R^n$, it exists constant $\alpha > 0$, cause

$$\sum_{i,j=1}^{n} a_{ij}\zeta_i\zeta_j \geq \alpha \sum_{i=1}^{n} \zeta_i^2 \quad (2)$$

$\frac{\partial u}{\partial \nu} = \sum_{i,j=1}^{n} a_{ij}\frac{\partial u}{\partial x_j}n_i$ is the normal differential coefficient related to matrix $(a_{ij}(x))$ on suitable slick boundary∂D. $n = (n_1, n_2, \cdots, n_n)$ is the unit outer normal of ∂D,$\nabla_x u = (\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \cdots, \frac{\partial u}{\partial x_n})$. It can also simplify to $f'_{\nabla_x u} = (\frac{\partial f}{\partial q_1}, \frac{\partial f}{\partial q_2}, \cdots, \frac{\partial f}{\partial q_n})$. $q_1 = \frac{\partial u}{\partial x_1}, q_2 = \frac{\partial u}{\partial x_2}, \cdots, q_n = \frac{\partial u}{\partial x_n}, \frac{\partial(\nabla_x u)}{\partial t} = (\frac{\partial q_1}{\partial t}, \frac{\partial q_2}{\partial t}, \cdots, \frac{\partial q_n}{\partial t})$.

Dissertation (Pao C.V., 1980) is to discuss the blowing up character for the solution of problem (1) while $f = \lambda(e^{au} - b)$ (where λ, a, b are non-negative constant) and $g = 0$. Suppose $b(x) \equiv 0$ and $f(u)$ is just the function of u, literatures (Deng Jucheng, 1987; Chipot M., 1989) studies when $g = 0$ and $g = g(u)$, solution of problem (1) blow up in limited time; on the bases of $b(x) \neq 0$ and f is a function of x, t, and u , literature (Zheng Zongmu, 1987) discusses blow up problem of boundary condition $g \equiv 0$. The problem discussed in literatures (Gomez J.L., 1991; Migoguchi N., 1997; Zhang Hailiang, 2002; Li Junfeng, 2002) is special situation of (1). This dissertation extends the problem studied in literature (Zha Zhongwei, 1992), accordingly original boundary value problem has more comprehensive physical backgrounds.

Suppose:

ISSN: 1916-9795
E-ISSN: 1916-9809

$A_1)$ $u_0(x) \in C^2(\bar{D}), u_0(x) \geq 0.$

$A_2)$ $a_{ij}(x)$, $b(x) > 0$ is a suitable slick function, function $f(x,t,p,q) \in C^1[R^n \times [0,T) \times R \times R], f, f'_t > 0$, quantitative product $f'_q \cdot q'_t = \frac{\partial f}{\partial q_1} \cdot \frac{\partial q_1}{\partial t} + \frac{\partial f}{\partial q_2} \cdot \frac{\partial q_2}{\partial t} + \cdots + \frac{\partial f}{\partial q_n} \cdot \frac{\partial q_n}{\partial t}$ is non-negative, and exists constant $h > 1$, which lead to $f'_p - (h-1)f \geq 0$ is available to any$p \geq 0$.

$A_3)$ $g(x,t,p,q) \in C^1[R^n \times [0,T) \times R \times R], g'_t \geq 0$, quantitative product $g'_q \cdot q'_t$ is non-negative; when $p \geq 0, g \leq 0$; when $p < 0, g > 0$. to the same h, when $p \geq 0$, it has the inequality $g'_p < 1$ and $g'_p + (h-1)p - 1 \geq 0$.

$A_4)$ when $x \in \bar{D}, \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial e^{u_0}}{\partial x_j} + e^{u_0}[bu_0 + f(x,0,u_0,\nabla_x u_0)] \geq 0.$

2. The non-negative character of solution

In order to prove the solution of problem (1) blowing up in limited time, we should apply some lemmas as follows.

Lemma 1 If A1)—A3) satisfied on $\bar{D} \times [0,T)$, (1)'s solution $u(x,t)$ is non-negative.

Proof Take constant $K > max\{b(x), x \in \bar{D}\}$, suppose $u(x,t) = V(x,t)e^{Kt}$, then (1) become (3)

$$\begin{cases} \frac{\partial V}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial V}{\partial x_j}) + \sum_{i,j=1}^n a_{ij}\frac{\partial V}{\partial x_i}\frac{\partial V}{\partial x_j}e^{Kt} - (K-b)V + f(x,t,Ve^{Kt},\nabla_x Ve^{Kt})e^{-Kt}, & D \times (0,T) \\ \beta(x)\frac{\partial V}{\partial \nu} + V = g(x,t,Ve^{Kt},\nabla_x Ve^{Kt})e^{-Kt}, & \partial D \times (0,T) \\ V(x,0) = u_0(x). & \bar{D} \end{cases} \quad (3)$$

Obviously, we just need to prove solution of problem (3) $V(x,t)$ is non-negative on $\bar{D} \times [0,T)$. To apply counter evidence, providing $V(x,t)$ can take negative, then it must have a point $M_0(x_0,t_0)$ result in $V(M_0)$ the negative minimum value , according to (3) $V(x,0) = u_0(x)$ to know $t_0 \neq 0$.

If $M_0 \in D \times (0,T)$, then $\frac{\partial V}{\partial t}|_{M_0} \leq 0$. But since $\frac{\partial V}{\partial x_i}|_{M_0} \leq 0, \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial V}{\partial x_j})|_{M_0} \geq 0$ and condition A2), we can know the right of (3)' first equation should be positive, which is illogical.

If $M_0 \in \partial D \times (0,T)$, according to the maximum value principle we know $\frac{\partial V}{\partial \gamma}|_{M_0} \leq 0$, the left of second equation in (3) is negative here, which is illogical with condition A3). So, it must have $u(x,t) \geq 0$ on $\bar{D} \times [0,T)$.

In order to discuss easily, we make function transform with $W = e^{u(x,t)}$, then problem (1) became (3):

$$\begin{cases} \frac{\partial W}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial W}{\partial x_j}) + bWlnW + Wf(x,t,lnW,\nabla_x lnW), & D \times (0,T) \\ \beta(x)\frac{\partial W}{\partial \nu} + WlnW = Wg(x,t,lnW,\nabla_x lnW), & \partial D \times (0,T) \\ W(x,0) = e^{u_0(x)}. & \bar{D} \end{cases} \quad (4)$$

Through Lemma 1 to know $W(x,t) \geq 1$.

Lemma 2 If condition A1)—A4) is satisfied, $W(x,t)$ is the solution of problem(2.2), and $Sup\{W(x,t),(x,t) \in \bar{D} \times [0,T)\} = N < +\infty$ }, then it must has $\frac{\partial W}{\partial t} \geq 0$ on $\bar{D} \times [0,T)$.

Proof Let $Z(x,t) = \frac{\partial W}{\partial t}$, then $Z(x,t)$ satisfies original boundary value problem as follows:

$$\begin{cases} \frac{\partial Z}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial Z}{\partial x_j}) + [b(lnW+1) + f + f'_{lnW}]Z + Wf'_t + Wf'_{\nabla_x lnW} \cdot \frac{\partial(\nabla_x lnW)}{\partial t}, & D \times (0,T) \\ \beta(x)\frac{\partial Z}{\partial \nu} + (lnW + 1 - g - g'_{lnW})Z = Wg'_t + Wg'_{\nabla_x lnW} \cdot \frac{\partial(\nabla_x lnW)}{\partial t}, & \partial D \times (0,T) \\ Z(x,0) = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial e^{u_0}}{\partial x_j}) + [bu_0 + f(x,0,u_0,\nabla_x u_0)]e^{u_0}. & \bar{D} \end{cases} \quad (5)$$

Suppose $Sup\{bp + f + f'_p,\ 0 \leq p \leq N, (x,t) \in \bar{D} \times [0,T)\} = M < +\infty$, take $K > 2max\{max\{b(x), x \in \bar{D}, M\}$, then transform into $Z = V(x,t)e^{Kt}$, then we get the original boundary value problem satisfied by $V(x,t)$:

$$\begin{cases} \frac{\partial V}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial V}{\partial x_j}) - [K - b - (blnW + f + f'_{lnW})]V + W[f'_t + f'_{\nabla_x lnW} \cdot \frac{\partial(\nabla_x lnW)}{\partial t}]e^{-Kt}, & D \times (0,T) \\ \beta(x)\frac{\partial V}{\partial \nu} + (lnW + 1 - g - g'_{lnW})V = W[g'_t + g'_{\nabla_x lnW} \cdot \frac{\partial(\nabla_x lnW)}{\partial t}]e^{-Kt}, & \partial D \times (0,T) \\ V(x,0) = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial e^{u_0}}{\partial x_j}) + [bu_0 + f(x,0,u_0,\nabla_x u_0)]e^{u_0}. & \bar{D} \end{cases} \quad (6)$$

Similar with Lemma1's proving, it is easy to know the solution of problem (6), $V(x,t) \geq 0$ $((x,t) \in \bar{D} \times [0,T))$, then $\frac{\partial W}{\partial t} = V(x,t)e^{kt} \geq 0$.

Moreover, one of conclusions in literature (Guan Zhicheng, 1984) will be applied, that is

Lemma 3 Suppose $J(t) \in C^2[t_0,+\infty), J(t_0) > 0, J'(t_0) < 0, J''(t) \leq 0(t_0 \leq t+\infty)$, then exists $T_0(t_0 < T_0 < t_0 - \frac{J(t_0)}{J'(t_0)})$,result in $J(T_0) = 0$.

3. Blowing up Property of solution

With the available condition of A1)—A4), if unitary slick solution of original boundary value problem (1), $u(x,t) \in C^{3,2}[D\times(0,T)]\cap C^{2,1}[\bar{D}\times[0,T)]$, then T can not excess a certain value, that is

Theorem Suppose condition A1)-A4) is satisfied, $u(x,t) \in C^{3,2}[D\times(0,T)]\cap C^{2,1}[\bar{D}\times[0,T)]$ is a solution of (1) , then exists $T_0(0 < T_0 \le T)$, result in

$$\lim_{t\to T_0^-} Sup\{u(x,t), x\in\bar{D}\} = +\infty.$$

Proof We just need to prove existing limited moment T_0 , result in the solution of question (4) $W(x,t)$ blow up at T_0 . That is

$$\Phi(t) = \int_D \frac{1}{h+1} W^{h+1} dx \tag{7}$$

There into, $h > 1$ is the constant in condition A2) . Finding differential coefficient with both sides of equation (7), we obtain

$$\Phi'(t) = \int_D W^h \frac{\partial W}{\partial t} dx \tag{8}$$

Through lemma 1, lemma 2 to know

$$\Phi(0) = \int_D \frac{1}{h+1} W^{h+1}|_{t=0} dx > 0 \tag{9}$$

$$\Phi'(0) = \int_D W^h \frac{\partial W}{\partial t}|_{t=0} dx > 0 \tag{10}$$

To take first equation of (4) into (8), then

$$\Phi'(t) = \int_D W^h \sum_{i,j=1}^n \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial W}{\partial x_j})dx + \int_D W^{h+1}[blnW + f(x,t,lnW,\nabla_x lnW)]dx$$

Applying subsection integral to the first integral of the right side of above equation, we obtain:

$$\Phi'(t) = \int_{\partial D} W^h \sum_{i,j=1}^n (a_{ij}\frac{\partial W}{\partial x_j})n_i ds$$

$$-h\int_D W^{h-1}\sum_{i,j=1}^n a_{ij}\frac{\partial W}{\partial x_j}\frac{\partial W}{\partial x_i}dx + \int_D W^{h+1}[blnW + f(x,t,lnW,\nabla_x lnW)]dx \tag{11}$$

there into ds is the area element of ∂D . To find differential coefficient with equation (11) concerning t, we obtain

$$\Phi''(t) = h\int_{\partial D} W^{h-1}\frac{\partial W}{\partial t}\sum_{i,j=1}^n a_{ij}\frac{\partial W}{\partial x_j}n_i ds + \int_{\partial D} W^h \sum_{i,j=1}^n (a_{ij}\frac{\partial^2 W}{\partial x_j \partial t})n_i ds$$

$$-h(h-1)\int_D W^{h-2}\frac{\partial W}{\partial t}\sum_{i,j=1}^n a_{ij}\frac{\partial W}{\partial x_j}\frac{\partial W}{\partial x_i}dx - 2h\int_D W^{h-1}\sum_{i,j=1}^n \frac{\partial^2 W}{\partial x_j\partial t}\frac{\partial W}{\partial x_i}dx$$

$$+(h+1)\int_D bW^h\frac{\partial W}{\partial t}lnW dx + (h+1)\int_D W^h\frac{\partial W}{\partial t} f dx + \int_D bW^h \frac{\partial W}{\partial t}dx$$

$$+\int_D W^{h+1} f'_t dx + \int_D W^h \frac{\partial W}{\partial t} f'_{lnW} dx + \int_D W^{h+1} f'_{\nabla_x lnW}\cdot\frac{\partial(\nabla_x lnW)}{\partial t}dx \tag{12}$$

In addition, if to find differential coefficient with both sides of (8) directly, we obtain

$$\Phi''(t) = h\int_D W^{h-1}(\frac{\partial W}{\partial t})^2 dx + \int_D \frac{\partial^2 W}{\partial t^2}dx \tag{13}$$

To take equation of (6) into (13) and apply subsection integral, we obtain

$$\Phi''(t) = h\int_D W^{h-1}(\frac{\partial W}{\partial t})^2 dx + \int_{\partial D} W^h \sum_{i,j=1}^n (a_{ij}\frac{\partial^2 W}{\partial x_j\partial t})n_i ds$$

ISSN: 1916-9795
E-ISSN: 1916-9809

$$-h\int_D W^{h-1}\sum_{i,j=1}^{n} a_{ij}\frac{\partial^2 W}{\partial x_j\partial t}\frac{\partial W}{\partial x_i}dx+\int_D bW\frac{\partial W}{\partial t}lnWdx$$
$$+\int_D W^h\frac{\partial W}{\partial t}fdx+\int_D bW^h\frac{\partial W}{\partial t}dx+\int_D W^{h+1}f_t'dx$$
$$+\int_D W^h\frac{\partial W}{\partial t}f_{lnW}'dx+\int_D W^{h+1}f_{\nabla_x lnW}'\cdot\frac{\partial(\nabla_x lnW)}{\partial t}dx \tag{14}$$

Using 2 to multiply (14) and then subtract (12), with attention to the boundary condition in the (6) then obtain

$$\Phi''(t)=2h\int_D W^{h-1}(\frac{\partial W}{\partial t})^2dx+\int_{\partial D}\frac{W^h}{\beta}[g_{lnW}'+(h-1)lnW-1]\frac{\partial W}{\partial t}ds$$
$$+(1-h)\int_{\partial D}\frac{W^h}{\beta}g\frac{\partial W}{\partial t}ds+\int_{\partial D}\frac{W^{h+1}}{\beta}g_t'ds+\int_{\partial D}\frac{W^{h+1}}{\beta}g_{\nabla_x lnW}'\cdot\frac{\partial(\nabla_x lnW)}{\partial t}ds+\int_D bW^h\frac{\partial W}{\partial t}dx$$
$$+h(h-1)\int_D W^{h-2}\frac{\partial W}{\partial t}\sum_{i,j=1}^{n}a_{ij}\frac{\partial W}{\partial x_j}\frac{\partial W}{\partial x_i}dx+\int_D W^{h+1}f_t'dx$$
$$+\int_D W^{h+1}f_{\nabla_l nW}'\cdot\frac{\partial(\nabla_x lnW)}{\partial t}dx+\int_D W^h\frac{\partial W}{\partial t}[f_{lnW}'-(h-1)f]dx+(h-1)\int_D W^h\frac{\partial W}{\partial t}blnWdx$$

According to supposed condition A1)—A4) and lemma1, lemma2 concerning the conclusion of non-negative W, $\frac{\partial W}{\partial t}$, we know

$$\Phi''(t)\geq 2h\int_D W^{h-1}(\frac{\partial W}{\partial t})^2dx$$

that is

$$\Phi''(t)\cdot\Phi(t\geq\frac{2h}{h+1}\int_D W^{h-1}(\frac{\partial W}{\partial t})^2dx\cdot\int_D W^{h+1}dx\geq\frac{2h}{h+1}(\int_D W^h\frac{\partial W}{\partial t}dx)^2=\frac{2h}{h+1}[\Phi'(t)]^2 \tag{15}$$

cause

$$J(t)=[\Phi(t)]^{-\frac{h-1}{h+1}} \tag{16}$$

then through (9), (10) to know

$$J(0)=[\Phi(0)]^{-\frac{h-1}{h+1}}>0, J'(0)=-\frac{h-1}{h+1}[\Phi(0)]^{-\frac{2h}{h+1}}\Phi'(0)<0.$$

On the other side, through in equation (15) to know $J''(t)\leq 0$, According to Lemma 3, exists $T_0(0<T_0<-\frac{J(0)}{J'(0)})$, cause $J(T_0)=0$, we obtain

$$\lim_{t\to T_0^-}J(t)=\lim_{t\to T_0^-}[\Phi(t)]^{-\frac{h-1}{h+1}}=0,\quad or\quad \lim_{t\to T_0^-}\Phi(t)=+\infty$$

then we obtain

$$\lim_{t\to T_0^-}sup\{W(x,t),x\in\bar{D}\}=+\infty$$

so

$$\lim_{t\to T_0^-}sup\{u(x,t),x\in\bar{D}\}=+\infty$$

theorem has been proved.

4. Applied example

In the J. Bear's specialized work (Bcar., 1972), he proposed that the law of fluids in porous medium can be concluded to a quasi linear parabolic equation $u_t=\triangle(u^m)(m>1)$ We can put the $u<0$ situation aside because it does not exist in real life. Unfolding the equation and considering the initial boundary value problem:

$$\begin{cases}\frac{\partial u}{\partial t}=mu^{m-1}\triangle u+m(m-1)u^{m-2}(\nabla_x u)^2, & D\times(0,T)\\ \frac{\partial u}{\partial \nu}+u=g(x,t) & \partial D\times(0,T)\\ u(x,0)=u_0(x) & \bar{D}\end{cases} \tag{17}$$

Let $f(u,\nabla_x u)=m(m-1)u^{m-2}(\nabla_x u)^2$, then $f>0$.

The discussion below is to seek the conditions to the blowing up appearance of the solution of (17).

By the supposed condition A2) it requests,

$$f'_{\nabla_x u} \cdot \frac{\partial(\nabla_x u)}{\partial t} = 2m(m-1)u^{m-2}(\nabla_x u) \cdot \frac{\partial(\nabla_x u)}{\partial t} \geq 0$$

and when $h > 1$,

$$f'_u - (h-1)f = m(m-1)u^{m-3}(\nabla_x u)^2[(m-2) - (h-1)u] \geq 0,$$

and that get

$$(\nabla_x u) \cdot \frac{\partial(\nabla_x u)}{\partial t} \geq 0, \tag{18}$$

and

$$u \leq \frac{m-1}{h-1} \tag{19}$$

By the condition A3), it requests, $g \leq 0, g'_t \geq 0$ and $g'_u + (h-1)u - 1 = (h-1)u - 1 \geq 0$ then we get $u \geq \frac{1}{h-1}$. Considering it with (18) we get

$$\frac{1}{h-1} \leq u \leq \frac{m-2}{h-1} \qquad (m > 3) \tag{20}$$

Finality with the premise of A4), we get

$$m(e^{u_0})^{m-1}\triangle(e^{u_0}) + m(m-1)(e^{u_0})^{m-2}(\nabla_x e^{u_0})^2 = m(e^{u_0})^m[m(\nabla_x u_0)^2 + \triangle u_0] \geq 0$$

obviously $\triangle u_0 \geq 0$ is the only prerequisite. According the discussion above we get the conclusion:

If the initial boundary of problem(4.1) given functions satisfies $g \leq 0, g'_t \geq 0$ and $\triangle u_0 \geq 0$. $u(x,t)$ is the solution of (17) when it satisfies the inequalities of (18), (20). Then there must be definite time T_0 $(0 < T_0 < +\infty)$, it makes blow up at T_0. The blowing up appearance help us to think about that the model of fluids low is reasonable and solvable.

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