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Existence of Positive Periodic Solutions of a Lotka-Volterra System with Multiple Time Delays

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Abstract

In this paper, a class of Lotka-Volterra system with multiple time delays is considered. By using the continuation theorem of coincidence degree theory, we derive a set of easily verifiable sufficient conditions that guarantees the existence of at least a positive periodic solution.

Keywords: Lotka-Volterra system, Periodic solution, Multiple time delay, Continuation theorem, Topological degree

1. Introduction

Lotka-Volterra system is an important population system and has been studied by many authors, see (Chen Shihua, et.al, 2004, Fan Meng, et.al,1999, Li Yongkun, et.al, 2001) and the reference therein. but most of the previous results focused on the stability, attractiveness, persistence and periodicity of solution to the ordinary differential systems or time delay systems with constant delays. Rare work has been done for the systems with varying delays and varying coefficients. In 1991, Weng di Wang et. al (Wang Wending,et. al, 1991) had considered a two-dimensional predator-prey system with a finite constant number discrete delays

$$\begin{cases} \dot{x}(t) = x(t)\left[r_1 - \sum_{j=1}^{m} a_{1j}x(t-\tau_{1j}) - \sum_{j=1}^{m} b_{1j}y(t-\rho_{1j})\right], \\ \dot{y}(t) = y(t)\left[r_2 + \sum_{j=1}^{m} a_{2j}x(t-\tau_{2j}) - \sum_{j=1}^{m} b_{2j}y(t-\rho_{2j})\right], \end{cases} \tag{1}$$

with initial conditions

$$x(s) = \varphi(s) \geq 0, s \in [-\tau, 0]; \varphi(0) > 0,$$

$$y(s) = \psi(s) \geq 0, s \in [-\tau, 0]; \psi(0) > 0,$$

where r_1, r_2 are real constants with $r_1 > 0$; a_{ij}, b_{ij}, τ_{ij}, ρ_{ij} $(i = 1, 2; j = 1, 2, ..., m)$ are non-negative constants. Not all of a_{1j} and not all b_{1j} $(j = 1, 2, ..., m)$ are zero; Both $\varphi(s)$ and $\psi(s)$ are continuous on the interval $[-\tau, 0]$ in which $\tau = max\{\tau_{ij}, \rho_{ij} : i = 1, 2; j = 1, 2, ..., m\}$. And obtained the conclusion that the time delays are harmless for uniform persistence of the solutions to the system.

We note that any biological or environmental parameters are naturally subject to fluctuation in time. It is necessary and important to consider models with periodic ecological parameters or perturbations which might be naturally exposed (for example, those due to seasonal effects of weather, food supply, mating habits, hunting or harvesting seasons, etc.). Thus, the assumption of periodicity of the parameters is a way of incorporating the periodicity of the environment.

In this paper, we are concerned with the effects of periodicity of ecological and environmental parameters and time delays. Then system (1) can be modified as the form:

$$\begin{cases} \dot{x}(t) = x(t)\left[r_1(t) - \sum_{j=1}^{m} a_{1j}(t)x(t-\tau_{1j}(t)) - \sum_{j=1}^{m} b_{1j}(t)y(t-\rho_{1j}(t))\right], \\ \dot{y}(t) = y(t)\left[r_2(t) + \sum_{j=1}^{m} a_{2j}(t)x(t-\tau_{2j}(t)) - \sum_{j=1}^{m} b_{2j}(t)y(t-\rho_{2j}(t))\right], \end{cases} \tag{2}$$

with initial conditions

$$\begin{cases} x(s) = \varphi(s) \geq 0, s \in [-\tau, 0]; \varphi(0) > 0, \\ y(s) = \psi(s) \geq 0, s \in [-\tau, 0]; \psi(0) > 0, \end{cases} \tag{3}$$

where $r_1(t), r_2(t)$ are real functions with $r_i(t) > 0, (i = 1, 2)$; $a_{ij}(t), b_{ij}(t), \tau_{ij}(t), \rho_{ij}(t) (i = 1, 2; j = 1, 2, ..., m)$ are non-negative functions. Not all of $a_{1j}(t)$ and not all $b_{1j}(t) (j = 1, 2, ..., m)$ are zero; Both $\varphi(s)$ and $\psi(s)$ are continuous on the interval $[-\tau, 0]$ in which $\tau = \max_{t\in R} \max\{\tau_{ij}(t), \rho_{ij}(t) : i = 1, 2; j = 1, 2, ..., m\}$.

Throughout the paper, we always assume that
(H_1) $r_i(t),\ a_{ij}(t),\ b_{ij}(t),\ \tau_{ij}(t),\ \rho_{ij}(t) (i = 1, 2; j = 1, 2, ..., m)$ are ω periodic, i.e.,

$$r_i(t+\omega) = r_i(t),\ a_{ij}(t+\omega) = a_{ij}(t),\ b_{ij}(t+\omega) = b_{ij}(t),$$

$$\tau_{ij}(t+\omega) = \tau_{ij}(t),\ \rho_{ij}(t+\omega) = \rho_{ij}(t)$$

for any $t \in R$.
(H_2) $r_i(t),\ a_{ij}(t),\ b_{ij}(t),\ \tau_{ij}(t),\ \rho_{ij}(t)\ (i = 1, 2; j = 1, 2, ..., m)$ are all positive, i.e.,

$$r_i(t),\ a_{ij}(t),\ b_{ij}(t),\ \tau_{ij}(t),\ \rho_{ij}(t)\ (i = 1, 2; j = 1, 2, ..., m) > 0.$$

The principle object of this article is to find a set of sufficient conditions that guarantees the existence of at least a positive periodic solution for system (2) (3).

2. Basic lemma

In order to explore the existence of positive periodic solutions of (2) (3) and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing from (Yang Zhihui, et. al, 2007).

Let X, Y be normed vector spaces, $L : DomL \subset X \to Y$ is a linear mapping, $N : X \to Y$ is a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dimKerL = codimImL < +\infty$ and ImL is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $ImP = KerL, ImL = KerQ = Im(I - Q)$, it follows that $L \mid DomL \cap KerP : (I - P)X \to ImL$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since ImQ is isomorphic to $KerL$, there exist isomorphisms $J : ImQ \to KerL$.

Lemma 2.1. (Robert E. Gaines et. al, 1991)(Continuation Theorem) *Let L be a fredholm mapping of index zero and let N be $L-$compact on $\bar{\Omega}$. Suppose*
(a) for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;
(b) $QNx \neq 0$ for each $x \in KerL \bigcap \partial\Omega$, and $deg\{JQN, \Omega \bigcap \partial KerL, 0\} \neq 0$;
Then the equation $Lx = Nx$ has at least one solution lying in $DomL \bigcap \bar{\Omega}$.

Lemma 2.2. $R_+^2 = \{((x(t), y(t))^T \in R^2 \mid x(t) > 0, y(t) > 0\}$ *is positive invariant with respect to system (2) (3).*

Proof. In fact,

$$x(t) = \varphi(0)exp \int_0^t \left[r_1(s) - \sum_{j=1}^{m} a_{1j}(s)x(s-\tau_{1j}(s)) - \sum_{j=1}^{m} b_{1j}(s)y(s-\rho_{1j}(s))\right] ds,$$

$$y(t) = \psi(0)exp \int_0^t \left[r_2(s) + \sum_{j=1}^{m} a_{2j}(s)x(s-\tau_{2j}(s)) - \sum_{j=1}^{m} b_{2j}(s)y(s-\rho_{2j}(s))\right] ds.$$

In view of $\varphi(0) > 0, \psi(0) > 0, (i = 1, 2)$, obviously, the conclusion follows.

3. Existence of positive periodic solutions

For convenience and simplicity in the following discussion ,we always use the notations below throughout the paper:

$$\bar{g} = \frac{1}{\omega}\int_0^{\omega} g(t)dt,\ g^L = \min_{t\in[0,\omega]} g(t),\ g^M = \max_{t\in[0,\omega]} g(t),$$

where $g(t)$ is a ω continuous periodic function. Let $\sigma_{ij}(t) = t - \tau_{ij}(t),\ \theta_{ij}(t) = t - \rho_{ij}(t),\ t \in R,\ i = 1, 2;\ j = 1, 2, ..., m.$ Assume that

$$(H_3)\ \tau'_{ij}(t) < 1,\ \rho'_{ij}(t) < 1,\ (i = 1, 2; j = 1, 2, ..., m)$$

Then $\sigma_{ij}(t)$ and $\theta_{ij}(t)$ have inverse functions denoted by $\mu_{ij}(t), \varepsilon_{ij}(t), (i = 1, 2; j = 1, 2, ..., m)$, respectively. Obviously, $\mu_{ij}(t + \omega) = \mu_{ij}(t) + \omega, \varepsilon_{ij}(t + \omega) = \varepsilon_{ij}(t) + \omega$.

In the following, we will ready to state and prove our result.

Theorem 3.1. *Suppose that* $(H_1), (H_2), (H_3), (H_4)$ $D^L_{22}\overline{r_1}^L > D^M_{12}\overline{r_2}^M$ *and* (H_5) $\overline{r_1}\sum_{j=1}^m \overline{b_{2j}} > \overline{r_2}\sum_{j=1}^m \overline{b_{1j}}$ *hold, where*

$$D_{22} = \sum_{j=1}^m \frac{b_{2j}(t)\varepsilon_{2j}(t)}{1-\rho'_{2j}(\varepsilon_{2j}(t))},\ D_{12} = \sum_{j=1}^m \frac{b_{1j}(t)\varepsilon_{1j}(t)}{1-\rho'_{1j}(\varepsilon_{1j}(t))},$$

then the system (2) (3) has at least a ω *periodic solution.*

Proof. Since solutions of (2) (3) remained positive for all $t \geq 0$, we let

$$u_1(t) = ln[x(t)],\ u_2(t) = ln[y(t)]. \tag{4}$$

Substituting (4) into (2), we obtain

$$\begin{cases} \dot{u}_1(t) = r_1(t) - \sum_{j=1}^m a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\} - \sum_{j=1}^m b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\}, \\ \dot{u}_2(t) = r_2(t) + \sum_{j=1}^m a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\} - \sum_{j=1}^m b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\}. \end{cases} \tag{5}$$

It is easy to see that if system (5) has one ω periodic solution $(u_1^*(t), u_2^*(t))^T$, then $(x^*(t), y*(t))^T = (exp[u_1^*(t), exp[u_2^*(t)])^T$ is a positive solution of system (2). Therefore, to complete the proof, it suffices to show that system (5) has at least one ω periodic solution.

Let $X = Z = \{u(t)\} = \{(u_1(t), u_2(t))^T \mid u(t) \in C(R, R^2), u(t+\omega) = u(t)\}$, and define $||u|| = ||(u_1(t), u_2(t))^T|| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)|$. Then X and Z are Banach spaces when they are endowed with the norm $||.||$.

Let $L : DomL \subset X \to Z$ and $N : X \to Z$ be the following:

$$Lu = x'(t),$$

$$Nu = \begin{pmatrix} r_1(t) - \sum_{j=1}^m a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\} - \sum_{j=1}^m b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\} \\ r_2(t) + \sum_{j=1}^m a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\} - \sum_{j=1}^m b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\} \end{pmatrix}. \tag{6}$$

Define continuous projective operators P and Q:

$$Pu = \frac{1}{\omega}\int_0^{\omega} u(t)dt, Qu = \frac{1}{\omega}\int_0^{\omega} u(t)dt, u \in X, u \in Z.$$

We can see that $KerL = \{u \in X \mid u = h \in R^2\}, ImL = \{u \in Z \mid \int_0^{\omega} u(t)dt = 0\}$ is closed in X and $dim(KerL) = 2 = codim(ImL)$, then it follows that L is a fredholm mapping of index zero. Moreover, it is easy to check that

$$QNx = \begin{pmatrix} \frac{1}{\omega}\int_0^{\omega}\left[r_1(t) - \sum_{j=1}^m a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\} - \sum_{j=1}^m b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\}\right]dt \\ \frac{1}{\omega}\int_0^{\omega}\left[r_2(t) + \sum_{j=1}^m a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\} - \sum_{j=1}^m b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\}\right]dt \end{pmatrix}.$$

By easily computation, we have

$$K_P(z)=\int_0^{\omega} z(u)du-\frac{1}{\omega}\int_0^{\omega}\left[\int_0^{t} z(u)du\right]dt,$$

$K_P(I-Q)Nu=$

$$\begin{pmatrix}\int_0^t F_1(s)ds\\ \int_0^t F_2(s)ds\end{pmatrix}-\begin{pmatrix}\frac{1}{\omega}\int_0^{\omega}\int_0^t F_1(s)dsdt\\ \frac{1}{\omega}\int_0^{\omega}\int_0^t F_2(s)dsdt\end{pmatrix}-\begin{pmatrix}(\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega} F_1(s)ds\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^{\omega} F_2(s)ds\end{pmatrix}, \tag{7}$$

where

$$\begin{aligned}F_1(s) &= r_1(s)-\sum_{j=1}^{m} a_{1j}(s)exp\{u_1(s-\tau_{1j}(s))\}-\sum_{j=1}^{m} b_{1j}(s)exp\{u_2(s-\rho_{1j}(s))\},\\ F_2(s) &= r_2(s)+\sum_{j=1}^{m} a_{2j}(s)exp\{u_1(s-\tau_{2j}(s))\}-\sum_{j=1}^{m} b_{2j}(s)exp\{u_2(s-\rho_{2j}(s))\}.\end{aligned}$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, using the Ascoli-Arzela theorem, it is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is $L-$compact on $\bar{\Omega}$ with any open bounded set $\Omega\subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Ly=\lambda Ny$, $\lambda\in(0,1)$, we have

$$\begin{cases} u_1(t)=\lambda\left[r_1(t)-\sum_{j=1}^{m} a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\}-\sum_{j=1}^{m} b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\}\right],\\ u_2(t)=\lambda\left[r_2(t)+\sum_{j=1}^{m} a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\}-\sum_{j=1}^{m} b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\}\right].\end{cases} \tag{8}$$

Suppose that $u(t)=(u_1(t),u_2(t))^T\in X$ is an arbitrary solution of system (8) for a certain $\lambda\in(0,1)$, integrating both sides of (8) over the interval $[0,\omega]$ with respect to t , we obtain

$$\begin{cases} \int_0^t\left[\sum_{j=1}^{m} a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\}+\sum_{j=1}^{m} b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\}\right]dt=\overline{r_1}\omega,\\ \int_0^t\left[\sum_{j=1}^{m} a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\}-\sum_{j=1}^{m} b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\}\right]dt=-\overline{r_2}\omega.\end{cases} \tag{9}$$

In view of the following:

$$\begin{aligned}&\int_0^{\omega}\sum_{j=1}^{m} a_{ij}(t)exp\{u_1(t-\tau_{ij}(t))\}dt\\ &= \int_{-\tau_{ij}(0)}^{\omega-\tau_{ij}(\omega)}\sum_{j=1}^{m}\frac{a_{ij}(\mu_{ij}(s))}{1-\tau'_{ij}(\mu_{ij}(s))}exp\{u_1(s)\}ds,\\ &= \int_{-\tau_{ij}(0)}^{\omega-\tau_{ij}(\omega)}\sum_{j=1}^{m}\frac{a_{ij}(\mu_{ij}(s))}{1-\tau'_{ij}(\mu_{ij}(s))}exp\{u_1(s)\}ds\\ &= \int_0^{\omega}\sum_{j=1}^{m}\frac{a_{ij}(\mu_{ij}(s))}{1-\tau'_{ij}(\mu_{ij}(s))}exp\{u_1(s)\}ds,(i=1,2;j=1,2,...,m),(10)\end{aligned}$$

$$\begin{aligned}&\int_0^{\omega}\sum_{j=1}^{m} b_{ij}(t)exp\{u_2(t-\rho_{ij}(t))\}dt\\ &= \int_{-\rho_{ij}(0)}^{\omega-\rho_{ij}(\omega)}\sum_{j=1}^{m}\frac{b_{ij}(\varepsilon_{ij}(s))}{1-\rho'_{ij}(\varepsilon_{ij}(s))}exp\{u_2(s)\}ds\\ &= \int_{-\rho_{ij}(0)}^{\omega-\rho_{ij}(\omega)}\sum_{j=1}^{m}\frac{b_{ij}(\varepsilon_{ij}(s))}{1-\rho'_{ij}(\varepsilon_{ij}(s))}exp\{u_2(s)\}ds\\ &= \int_0^{\omega}\sum_{j=1}^{m}\frac{b_{ij}(\varepsilon_{ij}(s))}{1-\rho'_{ij}(\varepsilon_{ij}(s))}exp\{u_2(s)\}ds,(i=1,2;j=1,2,...,m).(11)\end{aligned}$$

From (9) (10) (11), we can obtain

$$\int_0^{\omega}\sum_{j=1}^{m}\frac{a_{1j}(\mu_{1j}(s))}{1-\tau_{1j}^{'}(\mu_{1j}(s))}exp\{u_1(s)\}ds+\int_0^{t}\sum_{j=1}^{m}\frac{b_{1j}(\varepsilon_{1j}(s))}{1-\rho_{1j}^{'}(\varepsilon_{1j}(s))}exp\{u_2(s)\}ds=\overline{r_1}\omega, \tag{12}$$

$$\int_0^{\omega}\sum_{j=1}^{m}\frac{a_{2j}(\mu_{2j}(s))}{1-\tau_{2j}^{'}(\mu_{2j}(s))}exp\{u_1(s)\}ds-\int_0^{\omega}\sum_{j=1}^{m}\frac{b_{2j}(\varepsilon_{2j}(s))}{1-\rho_{2j}^{'}(\varepsilon_{2j}(s))}exp\{u_2(s)\}ds=-\overline{r_2}\omega. \tag{13}$$

By the mean value theorem for improper integral, there exist $\xi_{ikj}\in[0,\omega]$ $(i=1,2;k=1,2;j=1,2,...,m)$ such that

$$A_{11}\int_0^{\omega}exp\{u_1(s)\}ds+A_{12}\int_0^{\omega}exp\{u_2(s)\}ds=\overline{r_1}\omega, \tag{14}$$

$$A_{21}\int_0^{\omega}exp\{u_1(s)\}ds-A_{22}\int_0^{\omega}exp\{u_2(s)\}ds=-\overline{r_2}\omega, \tag{15}$$

where

$$A_{11}=\sum_{j=1}^{m}\frac{a_{1j}(\xi_{11j})\mu_{1j}(\xi_{11j})}{1-\tau_{1j}^{'}(\mu_{1j}(\xi_{11j}))},$$

$$A_{12}=\sum_{j=1}^{m}\frac{b_{1j}(\xi_{12j})\varepsilon_{1j}(\xi_{12j})}{1-\rho_{1j}^{'}(\varepsilon_{1j}(\xi_{12j}))},$$

$$A_{21}=\sum_{j=1}^{m}\frac{a_{2j}(\xi_{21j})\mu_{2j}(\xi_{21j})}{1-\tau_{2j}^{'}(\mu_{2j}(\xi_{21j}))},$$

$$A_{22}=\sum_{j=1}^{m}\frac{b_{2j}(\xi_{22j})\varepsilon_{2j}(\xi_{22j})}{1-\rho_{2j}^{'}\varepsilon_{2j}((\xi_{22j}))}.$$

Then,

$$\int_0^{\omega}exp\{u_1(s)\}ds=\frac{(A_{22}\overline{r_1}-A_{12}\overline{r_2})\omega}{A_{11}A_{22}+A_{21}A_{12}}, \tag{16}$$

$$\int_0^{\omega}exp\{u_2(s)\}ds=\frac{(A_{21}\overline{r_1}+A_{11}\overline{r_2})\omega}{A_{11}A_{22}+A_{21}A_{12}}. \tag{17}$$

So we have

$$\frac{(D_{22}^{L}\overline{r_1}^{L}-D_{12}^{M}\overline{r_2}^{M})\omega}{D_{11}^{M}D_{22}^{M}+D_{21}^{M}D_{12}^{M}}\le\int_0^{\omega}exp\{u_1(s)\}ds\le\frac{(D_{22}^{M}\overline{r_1}^{M}-D_{12}^{L}\overline{r_2}^{L})\omega}{D_{11}^{L}D_{22}^{L}+D_{21}^{L}D_{12}^{L}}, \tag{18}$$

$$\frac{(D_{21}^{L}\overline{r_1}^{L}+D_{11}^{M}\overline{r_2}^{M})\omega}{D_{11}^{M}D_{22}^{M}+D_{21}^{M}D_{12}^{M}}\le\int_0^{\omega}exp\{u_2(s)\}ds\le\frac{(D_{21}^{M}\overline{r_1}^{M}+D_{11}^{L}\overline{r_2}^{L})\omega}{D_{11}^{L}D_{22}^{L}+D_{21}^{L}D_{12}^{L}}, \tag{19}$$

where

$$D_{11}=\sum_{j=1}^{m}\frac{a_{1j}(t)\mu_{1j}(t)}{1-\tau_{1j}^{'}(\mu_{1j}(t))},$$

$$D_{12}=\sum_{j=1}^{m}\frac{b_{1j}(t)\varepsilon_{1j}(t)}{1-\rho_{1j}^{'}(\varepsilon_{1j}(t))},$$

$$D_{21}=\sum_{j=1}^{m}\frac{a_{2j}(t)\mu_{2j}(t)}{1-\tau_{2j}^{'}(\mu_{2j}(t))},$$

$$D_{22}=\sum_{j=1}^{m}\frac{b_{2j}(t)\varepsilon_{2j}(t)}{1-\rho_{2j}^{'}(\varepsilon_{2j}(t))}.$$

By the condition of theorem 3.1, there exist $t_i\in[0,\omega], i=1,2$ such that

$$u_1(t_1)=\ln\left[\frac{(A_{22}\overline{r_1}-A_{12}\overline{r_2})\omega}{A_{11}A_{22}+A_{21}A_{12}}\right], \tag{20}$$

$$u_2(t_2)=\ln\left[\frac{(A_{21}\overline{r_1}+A_{11}\overline{r_2})\omega}{A_{11}A_{22}+A_{21}A_{12}}\right]. \tag{21}$$

By the condition (H_4) of theorem 3.1, there exist $B_1, B_2 > 0$ such that

$$|u_1(t_1)| \leq B_1, |u_2(t_2)| \leq B_2. \tag{22}$$

In view of the following:

$$\begin{aligned}
\int_0^{\omega} |\dot{u}_1(t)|dt &= \lambda \int_0^{\omega} \left\| \left[r_1(t) - \sum_{j=1}^{m} a_{1j}(t)exp\{u_1(t-\tau_{1j}(t))\} \right.\right. \\
&\quad \left.\left. - \sum_{j=1}^{m} b_{1j}(t)exp\{u_2(t-\rho_{1j}(t))\} \right] \right\| dt \\
&\leq 2\int_0^{\omega} |r_1(t)|dt = 2\int_0^{\omega} r_1(t)dt = 2\overline{r_1}\omega := B_3,
\end{aligned} \tag{23}$$

$$\begin{aligned}
\int_0^{\omega} |\dot{u}_2(t)|dt &= \lambda \int_0^{\omega} \left\| \left[r_2(t) + \sum_{j=1}^{m} a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\} \right.\right. \\
&\quad \left.\left. - \sum_{j=1}^{m} b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\} \right] \right\| dt \\
&\leq \overline{r_2}\omega + \int_0^{\omega} \left| \sum_{j=1}^{m} a_{2j}(t)exp\{u_1(t-\tau_{2j}(t))\} \right| dt \\
&\quad + \int_0^{\omega} \left| \sum_{j=1}^{m} b_{2j}(t)exp\{u_2(t-\rho_{2j}(t))\} \right| dt \\
&\leq \overline{r_2}\omega + \sum_{j=1}^{m} \frac{a_{2j}(\mu_{2j}(s))\omega}{1-\mu_{2j}^{'}(\mu_{2j}(s))} \int_0^{\omega} exp\{u_1(s)\}ds \\
&\quad + \sum_{j=1}^{m} \frac{b_{2j}(\varepsilon_{2j}(s))\omega}{1-\rho_{2j}^{'}(\varepsilon_{2j}(s))} \int_0^{\omega} exp\{u_2(s)\}ds. \\
&\leq \overline{r_2}\omega + \sum_{j=1}^{m} \frac{a_{2j}(\mu_{2j}(s))\omega}{1-\mu_{2j}^{'}(\mu_{2j}(s))} \frac{(A_{22}^{M}\overline{r_1}^{M} - A_{12}^{L}\overline{r_2}^{L})\omega}{A_{11}^{L}A_{22}^{L} + A_{21}^{L}A_{12}^{L}} \\
&\quad + \sum_{j=1}^{m} \frac{b_{2j}(\varepsilon_{2j}(s))\omega}{1-\rho_{2j}^{'}(\varepsilon_{2j}(s))} \frac{(A_{21}^{M}\overline{r_1}^{M} + A_{11}^{L}\overline{r_2}^{L})\omega}{A_{11}^{L}A_{22}^{L} + A_{21}^{L}A_{12}^{L}} := B_4,
\end{aligned} \tag{24}$$

then it follows from (22) (23) (24) that

$$|u_1(t)| \leq |u_1(t_1)| + \int_0^{\omega} |\dot{u}_1(t)|dt \leq B_1 + B_3 := B_5, \tag{25}$$

$$|u_2(t)| \leq |u_2(t_2)| + \int_0^{\omega} |\dot{u}_2(t)|dt \leq B_2 + B_4 := B_6. \tag{26}$$

Obviously, B_1, B_2, B_3, B_4 are independent of $\lambda \in (0,1)$. By the condition (H_5) of Theorem 3.1, it is easy to show that the algebraic equations

$$\begin{cases} \sum_{i=1}^{m} \overline{a_{1j}}u_1 + \sum_{i=1}^{m} \overline{b_{1j}}u_2 = \overline{r_1}, \\ \sum_{i=1}^{m} \overline{a_{2j}}u_1 - \sum_{i=1}^{m} \overline{b_{2j}}u_2 = -\overline{r_2}, \end{cases} \tag{27}$$

has a unique positive solution $(\tilde{u_1}, \tilde{u_2})^T \in R^2$. Take $M = max\{B_5, B_6\} + B_0$, where B_0 is taken sufficiently large such that the unique positive solution $(\tilde{u}_1, \tilde{u}_2)^T \in R^2$ satisfies $\max_{t\in[0,\omega]} |\tilde{u}_1| + \max_{t\in[0,\omega]} |\tilde{u}_2| < B_0$,

Let $\Omega := \{u = \{u(t)\} \in X : \|u\| < M\}$, then it is easy to see that Ω is an open, bounded set in X and verifies requirement (a) of Lemma 2.1. When $(u_1(t), u_2(t))^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^2, u = \{(u_1, u_2)^T\}$ is a constant vector in R^2 with $\|u\| =$

$\|(u_1(t), u_2(t))^T\| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)| = M$. Then we have

$$QNy = \begin{pmatrix} \overline{r_1} - \sum_{i=1}^{m} \overline{a_{1j}} u_1 - \sum_{i=1}^{m} \overline{b_{1j}} u_2 \\ \overline{r_2} + \sum_{i=1}^{m} \overline{a_{2j}} u_1 - \sum_{i=1}^{m} \overline{b_{2j}} u_2 \end{pmatrix} \neq 0. \tag{28}$$

Letting J be the identity mapping and by direct calculation, we get

$$\begin{aligned} & deg\left\{JQN(u_1, u_2)^T; \partial\Omega \bigcap kerL; 0\right\} \\ = \;& deg\left\{QN(u_1, u_2)^T; \partial\Omega \bigcap kerL; 0\right\} \\ = \;& sign\left\{det\begin{pmatrix} -\sum_{i=1}^{m} \overline{a_{1j}} & -\sum_{i=1}^{m} \overline{b_{1j}} \\ \sum_{i=1}^{m} \overline{a_{2j}} & -\sum_{i=1}^{m} \overline{b_{2j}} \end{pmatrix}\right\} \\ = \;& sign\left\{\sum_{i=1}^{m} \overline{a_{1j}} \sum_{i=1}^{m} \overline{b_{2j}} + \sum_{i=1}^{m} \overline{a_{2j}} \sum_{i=1}^{m} \overline{b_{1j}}\right\} = 1 \neq 0. \end{aligned}$$

This proves that condition (b) in Lemma 2.1 is satisfied. By now, we have proved that Ω verifies all requirements of Lemma 2.1, then it follows that $Lu = Nu$ has at least one solution $(u_1(t), u_2(t))^T$ in $DomL \cap \overline{\Omega}$, that is to say, (5) has at least one ω periodic solution in $DomL \cap \overline{\Omega}$. Then we know that $((x(t), y(t))^T = (exp\{u_1(t)\}, exp\{u_2(t)\})^T$ is an ω periodic solution of system (2) (3) with strictly positive components. This completes the proof.

Remark 3.1. Theorem 3.1 remains valid if some or all terms are replaced by corresponding terms with discrete time delays, distribute delays (finite or infinite), or deviating arguments respectively. At this point, we would like to point out that, when one applies the continuation theorem from the coincidence degree theory to explore the existence of periodic solutions to the system of differential equations or difference equations, time delays of any type or the deviating arguments have no effect on the existence of positive solutions.

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