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Time Series Analysis of the Exchange Rate of the Ghanaian Cedi to the American Dollar

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Abstract

In this paper, we consider two univariate time series models of predicting the dynamics of the exchange rate (using mid-rate data) of the Ghana cedi to the US dollar over a 10year 2months period from January 2004, to February 2015. We consider out-of-sample forecast for the next three years of the exchange rate. The time series models considered for this objective are the Autoregressive Integrated Moving Average (ARIMA) and the Random walk model. We find modest differences between these two models based on the out-of-sample forecast. Interestingly, both models perform similarly based on forecast values. Forecast values shows that the exchange rate of the Ghana cedi to the American dollar will increase continuously in the next three (3) years.

Keywords: mid-rate data, exchange rate, random walk, ARIMA, Ghana cedi

1. Introduction

It is a fact that the international monetary system has undergone significant changes over the years with many researchers attributing interest in exchange rate volatility to the fact that it is empirically difficult to predict future exchange rate values. The stability of the Ghana cedi visavis major international currencies has continued to pose a challenge to policy makers, with the cedi being described as being in a *free fall* (depreciating) since 2000.

In many developing countries, the exchange rates are determined by the foreign exchange markets. In Ghana, the Ghana cedi has experienced volatile exhange rates against leading currencies (eg. the US dollar, the UK pound) since the year 2000. This is largely due to, some extent, the high import trading volumes and low export trading base in the fundamentals of the economy. In developed countries, a vast range of forecasting tools have been used to predict economic growth and other economic indicators.

The knowledge of the forecasts on exchange rate is important for policy makers to adequately measure economic performance and the Central bank to manage the reserves for future economic shocks.

The influence of high exchange rate on the local currency is an issue addressed by a considerable amount of literature. There have been debates about the volatile nature of exchange rates, a topic which has always been the center of attraction for researches which open up new avenues. In the global economy, a country's survival in the international trade also depends on the structure of its exchange rate. The movements in exchange rate can be associated with unanticipated fluctuations in the level of trade of the specified economy. The importance of the exchange rate risk also augments due to the opening up of world market and the reduction in trade barriers (Zafar and Ahmad, 2011).

Appiah and Adetunde in their 2011 paper used time series analysis to obtain a forecast model, which they then used to produce a forecast plot for 2011 and 2012, using the exchange rate data from 1994 to 2010 (for Ghana). They did not however, produce the exact forecast values , and the model they chose for their forecast was the ARIMA(1, 1, 1) model, which did not account for seasonality.

The aim of this paper is not to compare the forecast capabilities of the random walk model and the ARIMA model chosen, but to present both forecast values. There have been debates about whether exchange rates are really random walks. Barbara Rossi, in her PhD thesis attempted to answer that question. She noted that , it was well known that the proportion of exchange rate fluctuations that current economic models could predict was essentially zero and that a random walk model forecasts exchange rates better than economic models, a fact first noticed by Meese and Rogoff (1983a,b and 1988). Her subsequent paper addressed the problem of model selection between economic models of exchange rate determination and the random walk in the presence of parameter instability (Rossi, 2005).

2. Method

The generics of the ARIMA model are Autoregressive model(AR), the Moving Average(MA) and the process being integrated of order d being the "differencing".

For observed data that are non-stationary, we use differencing to transform to stationary time series. The classical Box-Jenkins model would be used to analyze stationary time series.

We use the sample autocorrelation function (SACF) and sample partial autocorrelation function (SPACF) to identify a Box-Jenkins model which adequately describes the stationary time series, achieved by implementing the Box-Jenkins model selection methods.

Essentially, our aim is to analyze the times series data of the exchange rate of the Ghana Cedi to the American Dollar to elicit the underlying relationships/trend between the two currencies and then based on the analysis find the most appropriate model to forecast future figures or trend.

For the analysis be using the ARIMA$(p, d, q)(P, D, Q)^s$ model, generally given by;

$$w_t = \sum_{i=1}^{p} \phi_i w_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \sum_{i=1}^{P} \phi_{is} Z_{t-is} + \sum_{j=1}^{Q} \theta_{js} Z_{t-js} + \mu + \varepsilon_t;$$

where

$$w_t = (1 - B)^d Y_t; \qquad Z_t = (1 - B)^D Y_{t-s}.$$

2.1 Box-Jenkins Model Selection

Box-Jenkins promoted a three-stage procedure aimed at selecting an appropriate ARIMA model from a collection of probable models for the purpose of estimating and forecasting a univariate time series. The three stages are: (i) the identification stage, (ii) the estimation stage, and (iii) the diagnostic checking stage, which we briefly describe.

2.1.1 Identification

A comparison of the sample ACF and PACF to those of various theoretical ARIMA processes may suggest several probable models. A common stationarity-inducing transformation is to take logarithms and then first differences of the series (or any number of differencing required). Once stationarity is achieved, the next step is to identify the p and q orders of the ARIMA model.

2.1.2 Estimation

At this second stage, the estimated models are compared using their AIC (Akaike Information Criteria) and BIC (Bayesian Information Criterion). The parameters of the model are estimated using least square method. Importance of each model parameter can be tested using the hypothesis that $H_o : \theta_i = 0, i > q$ against $H_a : \theta_i \neq 0, i > q$ and similarly for the AR part. Using the t-test we can reject the null hypothesis when the corresponding *p-value* of a parameter estimate is less than the preset significance level α.

2.1.3 Diagnostic Checking

In the diagnostic checking stage we examine the goodness of fit of the model. The special statistic that we use here are the Box-Piece statistic (BP) and the Ljung-Box (LB) Q statistic, which serve to test for autocorrelations of the residuals.

2.2 Random Walk(RW)

A random walk process consists of a sequence of discrete steps of fixed length. It can also be considered as the mathematical formalization of a path that consists of a succession of random steps.

A simple random walk model with a drift, δ, is given by;

$$Y_t = \delta + Y_{t-1} + \varepsilon_t \qquad \text{where} \qquad \varepsilon_t \sim WN(0, \sigma_e^2).$$

Here, we model the present midrate against the previous midrate series.

3. Results and Discussions

This section entails an analysis of the monthly exchange rate performance of the Ghana Cedi to the American Dollar($), using the mid-rate with data available from January, 2004 to Febuary, 2015. The statistical computing tool employed for this work is R software.

3.1 Statistics and Data Analysis

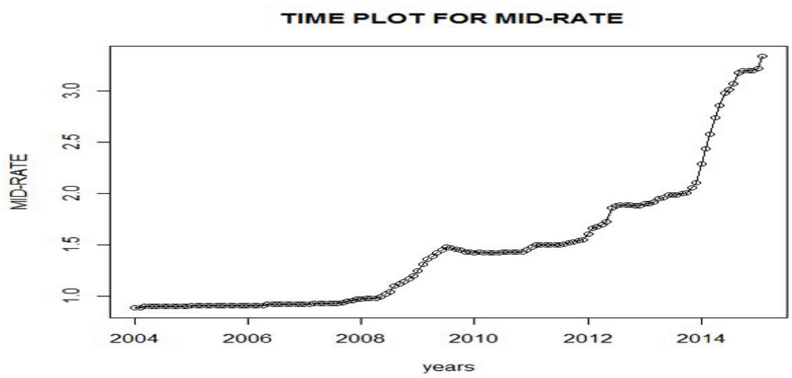

Figure 3.1. Time series plot of monthly mid-rate against time from January 2004 to February 2015

There is a sometimes gradual, and sometimes steep increasing underlying trend with maximum peak in February 2015, which recorded a figure of 3.728 Ghana cedis. The minimum amount recorded, occurred in January 2004, with a figure of 0.8810 Ghana cedis. It is clear from the figure 3.1, that there was a somewhat stable trend from 2004 to 2008 and this could be attributed to the economic stability experienced in Ghana during that period among other factors. The time series plot exhibits non-stationary variations based on the KPSS test ($p - value = 0.981$), and hence there is a need to apply differencing to achieve stationarity.

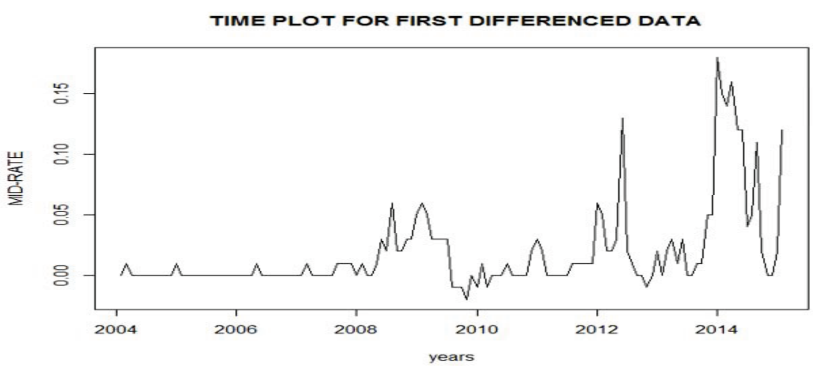

Figure 3.2. Time plot of first differenced data from January 2004 to February 2015

After carrying out the KPSS test for level stationarity (at 5% significance level) , it was observed that the first differenced data was still non-stationary, thus a second differencing was carried out.

Upon carrying out the KPSS test for level stationarity on the second differenced, the results indicated that the second differenced data exhibited no significant variability, thus the series stationary.

Inspecting the sample ACF and the sample PACF of the second differenced data (given by figure 3.4), the following seasonal autoregressive integrated moving average models were suggested:

- ARIMA(2,2,1)(0,0,2)[12]
- ARIMA(3,2,1)(0,0,2)[12]

- ARIMA(1,2,1)(0,0,2)[12]
- ARIMA(1,2,3)(0,0,2)[12]

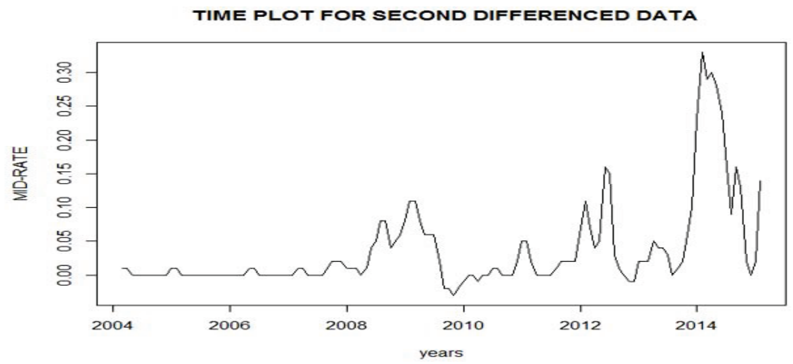

Figure 3.3. Time plot of second differenced data from January 2004 to February 2015

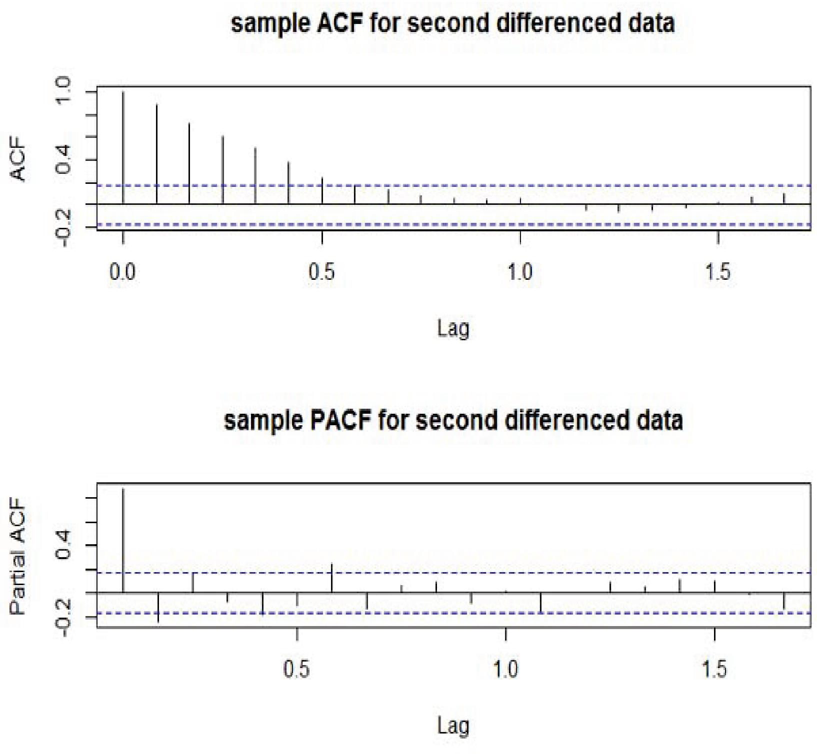

Figure 3.4. sample ACF and sample PACF for the second differenced data

In order to select the best model for forecasting, each model is assessed based on its parameter estimates, the corresponding diagnostics of the residuals and the AIC, AICc and BIC.

The final model was selected based on the one with the minimum AIC, AICc and BIC. It is the one that achieves the best compromise between the various values. Table 1 shows the corresponding values for the four ARIMA models under consideration.

Table 1. AIC, AICc and BIC for the ARIMA models

Model	AIC	AICc	BIC
ARIMA(2,2,1)(0,0,2)[12]	-599.16	-598.49	-581.86
ARIMA(3,2,1)(0,0,2)[12]	-597.20	-596.30	-577.03
ARIMA(1,2,1)(0,0,2)[12]	-599.81	-599.33	-585.40
ARIMA(1,2,3)(0,0,2)[12]	-597.26	-596.36	-577.08

From table 1 above, it is clear that the ARIMA(1,2,1)(0,0,2)[12] model is the best model since its AIC and BIC values are the minimum among the models under consideration. Thus, for the forecasts will be made using ARIMA(1,2,1)(0,0,2)[12,] which has the its parameters given by table 2.

Table 2. ARIMA(1,2,1)(0,0,2)[12] σ^2 estimated as 0.0005673

Coefficients	Estimate	Standard Error
ar1	0.6655	0.0868
ma1	-0.9479	0.0390
sma1	0.1189	0.0994
sma2	0.2452	0.1102

log likelihood= 304.9

3.2 Residual Analysis

Residuals analysis performed on the chosen models produced the following outputs;

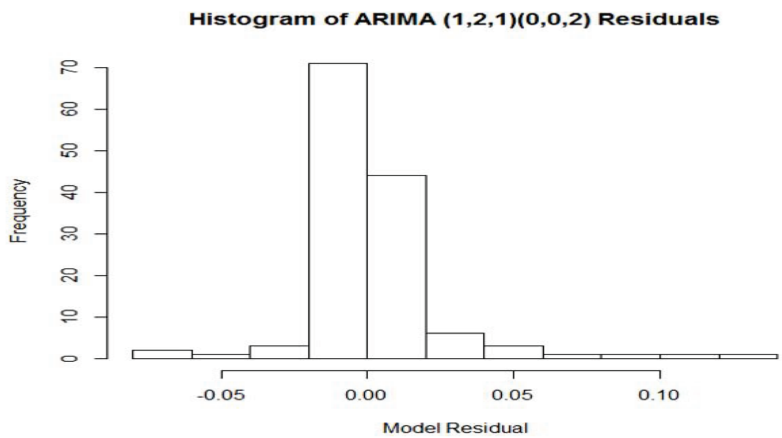

Figure 3.5. Histogram of ARIMA(1,2,1)(0,0,2) residuals

From the plot in figure 3.5, the histogram of the residuals displayed above gives an indication of an approximate symmetric distribution, thus it shape looks bell-like. The QQ-normal plot (figure 3.6) for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and its distribution looks approximately linear (Unimodal).

From figure 3.7, it can be observed from the ACF of residuals that none of the sample autocorrelations for the lags exceed the significant bounds. Thus we can conclude that the residuals exhibits no autocorrelation and conclude that there is no evidence for non-zero autocorrelations in the residuals at all lags (i.e. the residuals are independently distributed).

Thus, the chosen model is given by;

$$\text{thus, } y_t = 2.6655y_{t-1} - 2.331y_{t-2} + 0.6655y_{t-3} + 0.9479\varepsilon_{t-1} + 0.1189y_{t-24} + 0.2452y_{t-36} + \varepsilon_t + \mu$$

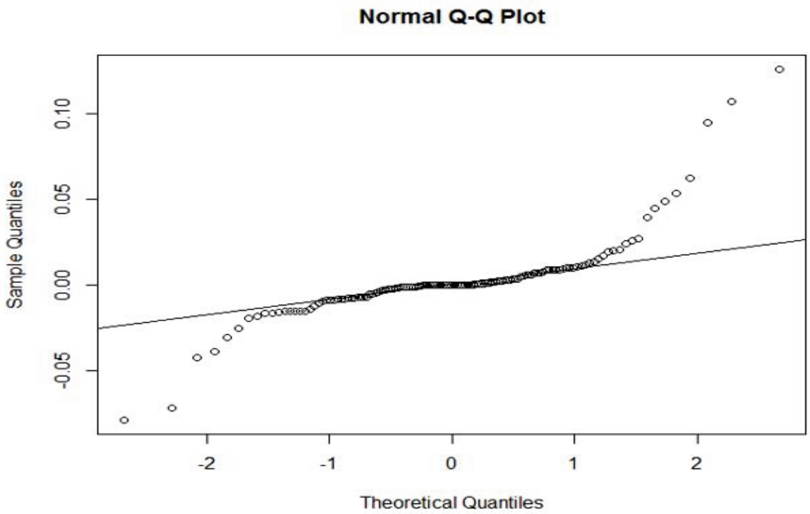

Figure 3.6. Normality plot of ARIMA(1,2,1)(0,0,2) residuals

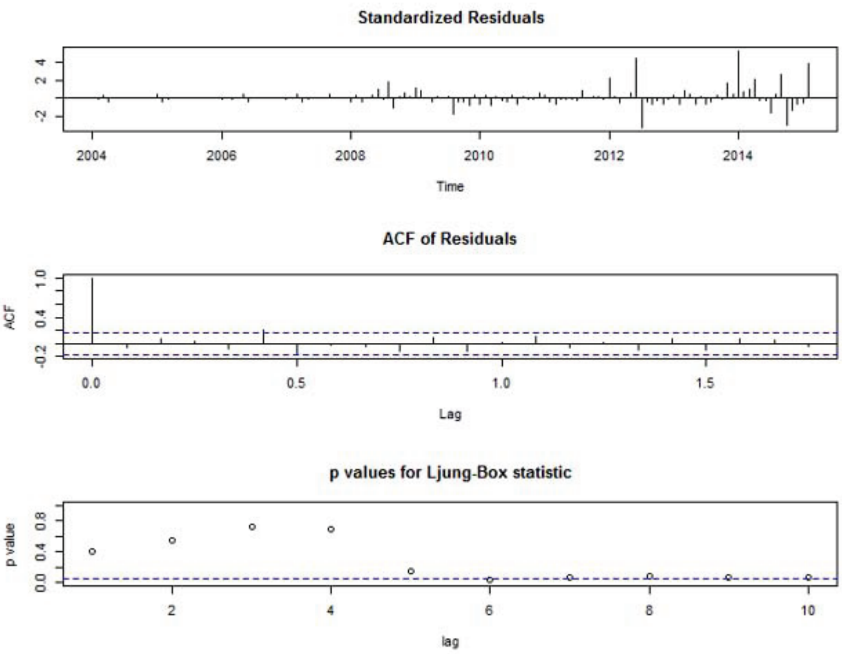

Figure 3.7. Diagnostic plots of ARIMA(1,2,1)(0,0,2) residuals

3.3 Forecasting

Forecasts using ARIMA (1,2,1)(0,0,2) for the next three years:

Table 3. 3-year forecast with ARIMA(1,2,1)(0,0,2) model

	2015	2016	2017	2018
Jan		4.164108	5.059343	5.920344
Feb		4.261208	5.141307	5.988842
Mar	3.447233	4.354207	5.218732	6.057305
Apr	3.546904	4.450472	5.293137	
May	3.628423	4.535893	5.365531	
Jun	3.703329	4.614326	5.436588	
Jul	3.764663	4.679940	5.506755	
Aug	3.824010	4.748382	5.576329	
Sep	3.891989	4.831719	5.645508	
Oct	3.948203	4.892855	5.714426	
Nov	4.010980	4.948422	5.783168	
Dec	4.073312	5.003511	5.851795	

Table 4. 3-year point-forecasts with the Random walk model

2015		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	Point			3.358	3.377	3.395	3.414	3.432	3.451	3.469	3.487	3.506	3.524
	Lo 90			3.299	3.292	3.291	3.293	3.297	3.302	3.308	3.314	3.321	3.329
	Hi 90			3.418	3.462	3.499	3.534	3.567	3.599	3.630	3.660	3.690	3.719
	Lo 95			3.287	3.276	3.271	3.270	3.271	3.273	3.277	3.281	3.286	3.292
	Hi 95			3.430	3.478	3.519	3.557	3.593	3.628	3.661	3.694	3.725	3.756
2016													
	Point	3.543	3.561	3.579	3.598	3.616	3.635	3.653	3.672	3.690	3.708	3.727	3.745
	Lo 90	3.338	3.346	3.355	3.364	3.373	3.383	3.393	3.403	3.413	3.423	3.434	3.444
	Hi 90	3.748	3.776	3.804	3.832	3.859	3.886	3.913	3.940	3.967	3.994	4.020	4.046
	Lo 95	3.298	3.305	3.312	3.319	3.327	3.335	3.343	3.351	3.360	3.369	3.378	3.387
	Hi 95	3.787	3.817	3.847	3.876	3.906	3.935	3.963	3.991	4.020	4.048	4.076	4.104
2017													
	Point	3.764	3.782	3.801	3.819	3.837	3.856	3.874	3.893	3.911	3.929	3.948	3.966
	Lo 90	3.455	3.466	3.477	3.488	3.499	3.510	3.521	3.532	3.544	3.555	3.566	3.578
	Hi 90	4.072	4.098	4.124	4.150	4.176	4.202	4.227	4.253	4.279	4.304	4.329	4.355
	Lo 95	3.396	3.405	3.415	3.424	3.434	3.443	3.453	3.463	3.473	3.483	3.493	3.504
	Hi 95	4.132	4.159	4.187	4.214	4.241	4.268	4.295	4.322	4.349	4.376	4.402	4.429
2018													
	Point	3.985	4.003	4.022									
	Lo 90	3.590	3.601	3.613									
	Hi 90	4.380	4.405	4.430									
	Lo 95	3.514	3.524	3.535									
	Hi 95	4.456	4.482	4.509									

The values in table 4 have been rounded up to 3 significant figures.

4. Conclusion

By the problem description and research conducted, we conclude that; the exchange rate between the Ghana Cedi and the US dollar was found to be non-stationary; hence the probability law that governs the behavior of the process changes over time. This means the process was not in statistical stability.

The forecasts from the ARIMA and RW models suggest that, in the absence of any structural changes, the upward trend against the US dollar might continue in the near future.

The following interventions are recommended: The fiscal, the monetary and international trade policies in place should be revised or modified to address the continuous depreciation of the Ghana Cedi. The Ghanaian populace should be educated on their insatiable taste for foreign goods and its negative impact on the value of the local currency.

Further studies are recommended to be conducted with the inclusion of covariates (economic indicators) such as interest rate parity, purchasing power parity, money aggregate and business cycle. Also, additional model features popular in financial modeling (like ARCH and GARCH) could also be employed in the analysis of the data since the variability in figures 3.2 and 3.3 would actually promote and idea of heteroscedasticity.

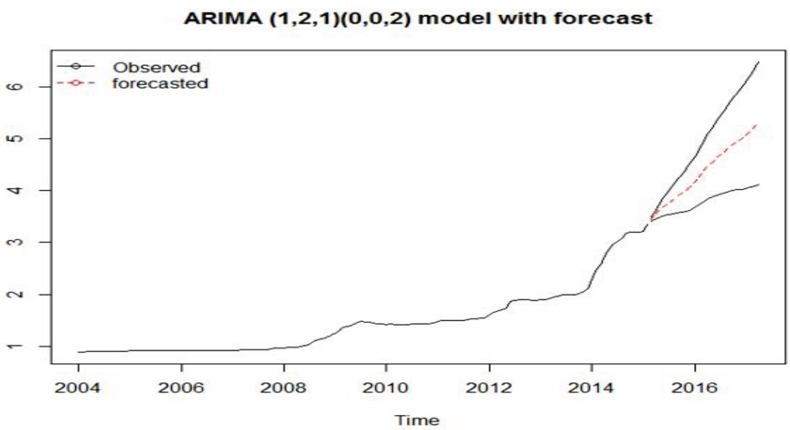

Figure 3.8. 3-year forecast with ARIMA(1,2,1)(0,0,2) model

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