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The Construction of a New Kind of Weakening Buffer Operators

Rui Zhou[1] & Jun-jie Li[1]

[1]Lecturer of College of Mathematics and Information, China West Normal University, China

Correspondence: Rui Zhou, master, lecturer of College of Mathematics and Information, China West Normal University, China. E-mail: 85605830@qq.com

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Abstract

Through optimizing the existing weakening buffer operator and introducing then m as parameter, this paper constructs a kind of weakening buffer operator, which improved the prediction accuracy; and verified by an example, the effect tis good

Keywords: shock disturbed system, gray system, weakening, buffer operator

1. Introduction

Established by Mr. Deng Ju-long since the 1980s, the grey system theory has been widely applied in the industry, the agriculture, the economy, the building and so on. The grey system theory through the original data on social, economic, ecological system of mining and finishing to seek its regularity, which is a kind theory system to find the data law from the original data. Both of the experimental data or the statistical data, the data must be analyzed before modeling, otherwise there will be a quantitative analysis of the forecast results and qualitative conclusion does not match, the cause of the problem lies not the model, but because the system behavior data has been shocked. So, find the combine point of quantitative prediction and qualitative analysis, exclude the impulse interference, restore the data true colors, improve the prediction accuracy, that are the most important work. The grey system theory through the original data on social, economic, ecological system of mining and finishing to seek its regularity, which is a kind theory system to find the data law from the data. But using several kinds of weakening operators to process all data which have different change rules, sometimes the effect meets not ideally. When faced the data which have a sharp change from high growth to low-growth, for reaching the high accuracy, it is necessary to process the data by using the high-end weakening buffer operator, the process of treatment is cumbersome. When the prediction data are smaller than the real data by using the first-order weakening buffer operator, and the prediction accuracy is not ideally, if continue to use the high-end weakening buffer operator, the forecasting result will be possibly smaller, the prediction error will further enlarge .

Through optimizing the existing weakening buffer operator and introducing then m as parameter, This paper constructs a kind of buffer operator, which improved the prediction accuracy; and verified by an example, the effect tis good

2. The Existing Buffer Operators

Satisfies the following condition the operator we to call it the buffer operator: Let *X* be a system behavior data series, (1) *D* is the operator affect to *X*, and the operator *D* satisfies $x(n)d=x(n)$. (2) Each data $x(k), k=1, 2, \dots, n$ in *X* should participate fully in the entire process of operator function. (3) Arbitrary $x(k)d, k=1, 2, \dots, n$ can be expressed by a unity primary analytical expression. (The above three conditions are called three axiom of buffer operators).

Definition 1 If $X=(x(1), x(2), \cdots x(n))$ is a system behavior data series, the weight vector of each point is $\omega=(\omega_1, \omega_2, \cdots \omega_n)$, $\omega_k>0, k=1,2,\cdots n$, $XD_i=(x(1)d_i, x(2)d_i, \cdots x(n)d_i), i=1,2$, among them:

$$x(k)d_1 = [\sum_{i=k}^{n} \omega_i (x(k))^2] / \sum_{i=k}^{n} \omega_i (x(i)), \quad x(k)d_2 = (x(k))^2 / [\prod_{i=k}^{n} x^{\omega_i}(i)]^{1/\sum_{i=k}^{n}\omega_i}, k = 1, 2, \cdots n$$

Then D_1 is the weighted average strengthening buffer operator (WASBO). D_2 is the weighted geometric average strengthening buffer operator (WGASBO).

Especially, when $\omega_k = 1, k = 1, 2, \cdots n$, let $XD_3 = XD_1, XD_4 = XD_2$, then D_3 is the average strengthening buffer operator (ASBO). D_4 is the geometric average strengthening buffer operator (GASBO).

Definition 2 If $X = (x(1), x(2), \cdots x(n))$ is a system behavior data series, the weight vector of each point is $\omega = (\omega_1, \omega_2, \cdots \omega_n), \omega_k > 0, k = 1, 2, \cdots n$. $XD_i = (x(1)d_i, x(2)d_i, \cdots x(n)d_i), i = 5, 6$, among them:

$$x(k)d_5 = [\sum_{i=k}^{n} \omega_i x(i)] / \sum_{i=k}^{n} \omega_i, x(k)d_6 = [\prod_{i=k}^{n} x^{\omega_i}(i)]^{1/\sum_{i=k}^{n}\omega_i}, k = 1, 2, \cdots n$$

Then D_5 is the weighted average weakening buffer operator (WAWBO), D_6 is the weighted geometric average weakening buffer operator (WGAWBO). Especially when $\omega_k = 1, k = 1, 2, \cdots n$, let $XD_7 = XD_5, XD_8 = XD_6$, then D_7 is the average weakening buffer operator (AWBO). D_8 is the geometric average weakening buffer operator (GAWBO).

3. The Construction of Weakening Buffer Series and Their Characteristics

If $X = (x(1), x(2), \cdots x(n))$ is a system behavior data series, and $x(k)d = \sqrt[m-1]{\frac{x^m(k) + x^m(k+1) + \cdots x^m(n)}{x(k) + x(k+1) + \cdots x(n)}}$, let $XD = (x(1)d, x(2)d, \cdots x(n)d)$, X is a monotone increasing(decreasing) series, then D is a weakening buffer operator.

Proof: (1) If $X = (x(1), x(2), \cdots x(n))$ is a non-negative monotone increasing series, that means $0 < x(1) \le \cdots \le x(n)$, then $0 < x^m(1) \le \cdots \le x^m(n), m = 1, 2 \cdots n$

$$0 < (x(k) + \cdots + x(n)) x^{m-1}(k) \le x^m(k) + \cdots + x^m(n)$$

And $x^{m-1}(k) \le \frac{x^m(k) + \cdots + x^m(n)}{x(k) + \cdots + x(n)}$, that is $x(k)d = \sqrt[m-1]{\frac{x^m(k) + \cdots + x^m(n)}{x(k) + \cdots + x(n)}} \ge x(k)$

So D is a weakening buffer operator.

(2) If $X = (x(1), x(2), \cdots x(n))$ is a non-negative monotone decreasing series, that means $x(1) \ge \cdots \ge x(n) > 0$, then $x^m(1) \ge \cdots \ge x^m(n) > 0, m = 1, 2 \cdots n$

$$(x(k) + \cdots + x(n)) x^{m-1}(k) \ge x^m(k) + \cdots + x^m(n) > 0$$

And $x^{m-1}(k) \ge \frac{x^m(k) + \cdots + x^m(n)}{x(k) + \cdots + x(n)}$, $x(k)d = \sqrt[m-1]{\frac{x^m(k) + \cdots + x^m(n)}{x(k) + \cdots + x(n)}} \le x(k)$

So D is a weakening buffer operator.

(3) If $X = (x(1), x(2), \cdots x(n))$ is a non-negative oscillatory series, let $x(k) = \max x(i), 1 \le i \le n$, $x(h) = \min x(h), 1 \le h \le n$, for arbitrary $i \in \{1, 2, \cdots n\}$

$$(x(k) + \cdots + x(n)) x^{m-1}(k) \ge x^m(k) + \cdots + x^m(n) > 0$$

And $x^{m-1}(k) \ge \frac{x^m(k)+\cdots+x^m(n)}{x(k)+\cdots+x(n)}$, that means $x(k)d = \sqrt[m-1]{\frac{x^m(k)+\cdots+x^m(n)}{x(k)+\cdots+x(n)}} \le x(k)$

$$0 < (x(h)+\cdots+x(n))x^{m-1}(h) \le x^m(h)+\cdots+x^m(n)$$

And $x^{m-1}(h) \le \frac{x^m(h)+\cdots+x^m(n)}{x(h)+\cdots+x(n)}$, that means $x(h)d = \sqrt[m-1]{\frac{x^m(h)+\cdots+x^m(n)}{x(h)+\cdots+x(n)}} \ge x(k)$

$$\max x(i) \ge \max x(i)d, i = 1, 2\cdots n$$
$$\min x(i) \le \min x(i)d, i = 1, 2\cdots n$$

So D is a weakening buffer operator.

Characteristic Weakening buffer series XD and system behavior data series X have the same monotonicity.

Proof: (1) If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a non-negative monotone increasing series,

$$\begin{aligned}
&\left(x^m(k+1)+\cdots x^m(n)\right)\cdot\left(x(k)+x(k+1)+\cdots x(n)\right)\\
&=\left(x^m(k+1)+\cdots x^m(n)\right)\cdot x(k)+\left(x^m(k+1)+\cdots x^m(n)\right)\cdot\left(x(k+1)+\cdots x(n)\right)\\
&>\left(x^{m-1}(k)x(k+1)+\cdots x^{m-1}(k)x(n)\right)\cdot x(k)+\left(x^m(k+1)+\cdots x^m(n)\right)\cdot\left(x(k+1)+\cdots x(n)\right) \text{ that is}\\
&=\left(x(k+1)+\cdots x(n)\right)\cdot x^m(k)+\left(x^m(k+1)+\cdots x^m(n)\right)\cdot\left(x(k+1)+\cdots x(n)\right)\\
&=\left(x(k+1)+\cdots x(n)\right)\cdot\left(x^m(k)+x^m(k+1)+\cdots x^m(n)\right)
\end{aligned}$$

$$\left(x^m(k+1)+\cdots x^m(n)\right)\cdot\left(x(k)+x(k+1)+\cdots x(n)\right) > \left(x(k+1)+\cdots x(n)\right)\cdot\left(x^m(k)+x^m(k+1)+\cdots x^m(n)\right)$$

$$\frac{x^m(k+1)+\cdots x^m(n)}{x(k+1)+\cdots x(n)} > \frac{x^m(k)+x^m(k+1)+\cdots x^m(n)}{x(k)+x(k+1)+\cdots x(n)}$$

And X is a non-negative monotone increasing series,

So $\sqrt[m-1]{\frac{x^m(k+1)+\cdots x^m(n)}{x(k+1)+\cdots x(n)}} > \sqrt[m-1]{\frac{x^m(k)+x^m(k+1)+\cdots x^m(n)}{x(k)+x(k+1)+\cdots x(n)}}$

That is $x(k)d < x(k+1)d$

(2) By the same methods show, if X is a non-negative monotone decreasing series, XD is also a non-negative monotone decreasing series.

4. Example Analysis

Some city industry total output value (1984-1991) (Unit: billion) is (1786.3 2585 2993.8 3574 4590.3 5448.4 5888.4 6632.3)

Takes the modeling data by 1984-1989 years data, and takes the simulation examination data by 1990 and 1991 data. From the raw data, we can find that the tertiary industry total output value average growth rate is 25.4% from 1984 to 1989. Analyzes it, the main reason is in the initial period of reform and opening, our government has made a series of preferential policy to support the development of industries. After ten remaining years' development, our country economic obtains the big development. As the support from country become weakening and the influence of domestic and foreign political configuration, economic situation, this kind of development speed is unable maintains. If carrying on the forecast with this data, the result is difficult to be believed. To make the reasonable forecast to the tertiary industry total output value, the raw data series must be weakened.

Using the average weakening buffer operator (AWBO) D_7 and its second order buffer operator, the geometric weakening buffer operator (GAWBO) D_8 and its second order buffer operator and the method of this paper (Let $m = 0.2$) process the raw data, and get the weakening series data, which are:

XD_7: (3496.3 3838.3 4151.6 4537.5 5019.3 5448.4)

XD_7D_7: (4415.2 4599 4789.2 5001.7 5233.9 5448.4)

XD_8: (3275.8 3698.2 4044.5 4471.1 5001 5448.4)

XD_8D_8: (4259.1 4488.7 4711.4 4957.3 5219.9 5448.4)

XD: (3541.8 3867.0 4173.3 4550.9 5023.0 5448.4)

Establish GM (1, 1) model by the raw data and the above buffer series in turn, the whitening differential equation as:

Raw data： $\frac{dx^{(1)}}{dt} - 0.1949x^{(1)} = 1905.7$

XD^7: $\frac{dx^{(1)}}{dt} - 0.089x^{(1)} = 3333.6$ XD^7D^7: $\frac{dx^{(1)}}{dt} - 0.043x^{(1)} = 4307.7$

XD^8: $\frac{dx^{(1)}}{dt} - 0.0986x^{(1)} = 3179.7$ XD^8D^8: $\frac{dx^{(1)}}{dt} - 0.049x^{(1)} = 4173.9$

XD: $\frac{dx^{(1)}}{dt} - 0.0837x^{(1)} = 3224.6$

And get the predictive value about 1990 and 1991, the result has shown in Tables:

Table 1. Predictive value and Effects

Year	Raw data(billion)	GM(1,1) Predictive value	***Error %***	XD_7 Predictive value	***Error %***	X D_7 D_7 Predictive value	***Error %***
1990	5888.4	6592.8	0.12	5709.1	0.031	5639.9	0.04
1991	6632.3	8013.1	0.21	6243.7	0.059	5790.8	0.127
Average Error %		0.165		0.045		0.084	

Table 2. Predictive value and Effects

Year	Raw data(billion))	XD_8 Predictive value	***Error %***	XD_8D_8 Predictive value	***Error %***	XD Predictive value	***Error %***
1990	5888.4	5775.6	0.019	5671.5	0.037	5872.2	0.0028
1991	6632.3	6372.6	0.039	5856.3	0.117	6797.7	0.025
Average Error %		0.029		0.077		0.0139	

From the tables, with the raw data series model to predict the effect is not very well, and the average error is nearly 16.5%. Establish models by XD_7 and XD_8, the prediction accuracy has been substantial increased, however, the average errors are 4.5%、2.9%, they are still not very well. Especially, establish models by XD_7D_7 and XD_8D_8, the prediction errors are further enlarge. But using the method in this paper, the average error is only 1.6％ . This confirmed that the weakening buffer series of this paper have a certain practical value.

5. Conclusions

Through optimizing the existing weakening buffer operator and introducing then m as parameter, this paper constructed a kind of weakening buffer operator an confirmed that the weakening buffer operator is exist in theory,. Finally through example, we can find the strengthening buffer operators are effectiveness and have its practical value.

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