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# Some More Results on Root Square Mean Graphs

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**Abstract**

A graph $G=(V,E)$ with $p$ vertices and $q$ edges is called a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,\dots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the resulting edge labels are distinct. In this case $f$ is called a Root Square Mean labeling of $G$. The concept of Root Square Mean labeling was introduced by (S. S. Sandhya, S. Somasundaram and S. Anusa). We investigated the Root Square Mean labeling of several standard graphs such as Path, Cycle, Comb, Ladder, Triangular snake, Quadrilateral snake etc., In this paper, we investigate the Root Square Mean labeling for Double Triangular snake, Alternate Double Triangular snake, Double Quadrilateral snake, Alternate Double Quadrilateral snake, and Polygonal chain.

**Keywords:** Mean graph, Root Square Mean graph, Cycle, Triangular snake, Double Triangular snake, Quadrilateral snake, Double Quadrilateral snake, Polygonal chain.

## 1. Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$.For all detailed survey of graph labeling we refer to Gallian (2010). For all other standard terminology and notations we follow Harary (1988). A Triangular snake $T_n$ is obtained from a path $u_1u_2u_3\cdots u_n$ by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$ for $1\le i\le n-1$. A Double Triangular Snake $D(T_n)$ consists of two Triangular snakes that have a common path. An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining $u_i$ and $u_{i+1}$ (Alternatively) to new vertex $v_i$ .An Alternate Double Triangular Snake $A(D(T_n))$ consists of two Alternate Triangular snakes that have a common path. A Quadrilateral snake $Q_n$ is obtained from a path $u_1u_2\dots u_n$ by joining $u_i$ and $u_{i+1}$ to new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$. A Double Quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path. An Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining $u_i$ and $u_{i+1}$ (Alternatively) to new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$.An Alternate Double Quadrilateral snake $A(D(Q_n))$ consists of two Alternate Quadrilateral snakes that have a common path.A Polygonal chain $G_{m,n}$ is a connected graph all of whose $m$ blocks are polygons on $n$ sides.

S. Somasundaram and R. Ponraj introduced the concept of mean labeling of graphs and investigated the mean labeling of some standard graphs. S. Somasundaram and S. S. Sandhya introduced the concept of Harmonic mean labeling of graphs. S. Somasundaram and P. Vidhya Rani introduced the concept of Geometric mean labeling of graphs. In this paper we prove that Double Triangular snake, Alternate Double Triangular snake, Double Quadrilateral snake, Alternate Double Quadrilateral snake and Polygonal Chains are Root Square mean graphs.

We make frequent reference to the following results.

**Theorem 1.1:** (S. Somasundaram & R. Ponraj) Triangular snakes and Quadrilateral snakes are mean graphs.

**Theorem 1.2:** (S. S. Sandhya, S. Somasundaram & S. Anusa) Double Triangular and Double Quadrilateral snakes are mean graphs.

**Theorem 1.3:** (S. S. Sandhya & S. Somasundaram) Triangular snakes and Quadrilateral snakes are Harmonic mean graphs.

**Theorem 1.4:** (C. Jaya Sekaran, S. S. Sandhya & C. David Raj) Double Triangular snakes and Alternate Double Triangular snakes are Harmonic mean graphs.

**Theorem 1.5:** (C. David Raj, C. Jaya Sekaran & S. S. Sandhya) Double Quadrilateral and Alternate Double Quadrilateral snakes are Harmonic mean graphs.

**Theorem 1.6:** (S. S. Sandhya & S. Somasundaram) Double Triangular snakes are Geometric mean graphs.

**Theorem 1.7:** (S. S. Sandhya & S. Somasundaram) Double Quadrilateral snakes are Geometric mean graphs.

## 2. Root Square Mean Labeling

**Definition 2.1:** A graph $G=(V,E)$ with $p$ vertices and $q$ edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $0,1,2,\dots,q$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\begin{cases}\frac{f(u)+f(v)}{2} & if\ \ f(u)+f(v)\ is\ even\\ \frac{f(u)+f(v)+1}{2} & if\ \ f(u)+f(v)\ is\ odd\end{cases}$ , then the edge labels are distinct.

In this case $f$ is called Mean labeling of $G$.

**Definition 2.2:** A graph with $p$ vertices and $q$ edges is called as Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\dots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil\ or\ \left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$, then the edge labels are distinct. In this case $f$ is called Harmonic mean labeling of $G$.

**Definition 2.3:** A graph with $p$ vertices and $q$ edges is called as Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\dots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\left\lceil\sqrt{f(u)f(v)}\ \right\rceil or \left\lfloor\sqrt{f(u)f(v)}\ \right\rfloor$ , then the edge labels are distinct. In this case $f$ is called Geometric mean labeling of $G$.

**Definition 2.4:** A graph with $p$ vertices and $q$ edges is called a Root Square mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\dots,q+1$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil\ 0r\ \left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$ , then the edge labels are distinct. In this case $f$ is called Root Square mean labeling of $G$.

**Theorem 2.5:** Double Triangular snakes $D(T_n)$ are Root square mean graphs.

**Proof:** Consider a path $u_1u_2u_3\cdots u_n$ .Join $u_i$ and $u_{i+1}$ $,1\le i\le n-1$ to two new vertices $v_i$ , $w_i$, $1\le i\le n-1$.Define a function $f\colon V(D(T_n))\to\{1,2,\dots q+1\}$ by

$f(u_1)=1\ ,\ f(u_2)=5,\ f(u_i)=5i-4, 3\le i\le n,$

$f(v_i)=5i-3, 1\le i\le n-1,$

$f(w_i)=5i-2, 1\le i\le n-1.$

The edges are labeled as

$f(u_1u_2)=4,\qquad f(u_iu_{i+1})=5i-2, 2\le i\le n-1$

$f(u_iv_i)=5i-4,\ 1\le i\le n-1$

$f(u_2v_1)=3,\qquad f(u_{i+1}v_i)=5i-1, 2\le i\le n-1$

$f(u_iw_i)=5i-3, 1\le i\le n-1$

$f(u_{i+1}w_i)=5i, 1\le i\le n-1.$

Then the edge labels are distinct. Hence Double Triangular snakes are Root Square mean graphs.

**Example 2.6:** Root Square mean labeling of $D(T_5)$ is given below.

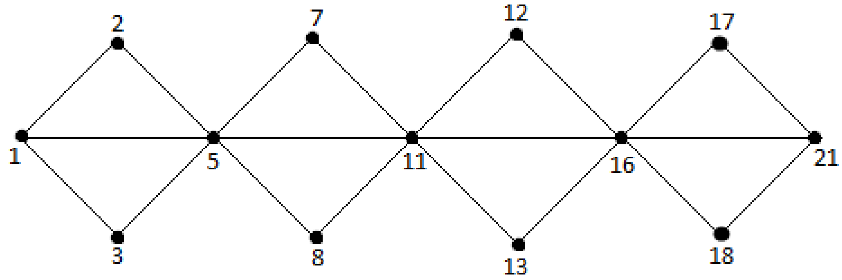


Figure 1

**Theorem 2.7:** Alternate Double Triangular snakes $A(D(T_n))$ is a Root Square mean graph.

**Proof:** Let $G$ be the graph $A(D(T_n))$. Consider a path $u_1u_2u_3 \cdots u_n$ .To construct $G$, join $u_i$ and $u_{i+1}$ (Alternatively) with two new vertices $v_i$ and $w_i$, $1 \le i \le n-1$.There are two different cases to be considered.

**Case 1:** If the Double Triangle starts from $u_1$, then we consider two sub cases.

**Sub Case 1(a):** If $n$ is even, then

Define a function $f: V(G) \to \{1,2, \ldots q+1\}$ by

$f(u_1) = 1$, $f(u_i) = 3i - 1, 2 \le i \le n,$

$f(v_1) = 2,\quad f(v_i) = 6(i-1), 2 \le i \le \frac{n}{2},$

$f(w_1) = 3,\ f(w_i) = 6i - 2,\ 2 \le i \le \frac{n}{2}.$

The edges are labeled as

$f(u_iu_{i+1}) = 3i,\quad 1 \le i \le n-1$

$f(u_{2i-1}v_i) = 6i - 5,\ 1 \le i \le \frac{n}{2}$

$f(u_2v_1) = 4,\quad f(u_{2i}v_i) = 6i - 4, 2 \le i \le \frac{n}{2}$

$f(u_1w_1) = 2,\quad f(u_{2i-1}w_i) = 6i - 2, 2 \le i \le \frac{n}{2}$

$f(u_{2i}w_i) = 6i - 1, 1 \le i \le \frac{n}{2}.$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.

The labeling pattern is shown below.

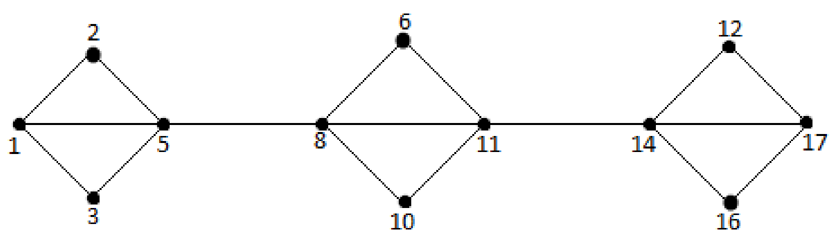


Figure 2

**Sub Case 1(b):** If $n$ is odd then

Define a function $f: V(G) \to \{1,2,\ldots q+1\}$ by

$f(u_1) = 1$, $f(u_i) = 3i - 1, 2 \le i \le n-1, f(u_n) = 3n-2$ ,

$f(v_1) = 2, \quad f(v_i) = 6(i-1), 2 \le i \le \dfrac{n-1}{2},$

$f(w_1) = 3, \; f(w_i) = 6i - 2, \; 2 \le i \le \frac{n-1}{2}.$

The edges are labeled as

$f(u_i u_{i+1}) = 3i, \quad 1 \le i \le n-1$

$f(u_{2i-1} v_i) = 6i - 5, \; 1 \le i \le \frac{n-1}{2}$

$f(u_2 v_1) = 4, \quad f(u_{2i} v_i) = 6i - 4, 2 \le i \le \dfrac{n-1}{2}$

$f(u_1 w_1) = 2, \quad f(u_{2i-1} w_i) = 6i - 2, 2 \le i \le \dfrac{n-1}{2}$

$f(u_{2i} w_i) = 6i - 1, 1 \le i \le \dfrac{n-1}{2}.$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.
The labeling pattern is shown below.

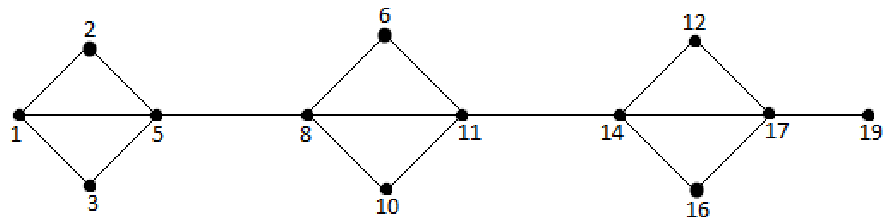


Figure 3

**Case 2:** If the triangle starts from $u_2$ , then we have to consider two sub cases.

**Sub case 2(a):** If $n$ is even , then

Define a function $f: V(G) \to \{1,2,\ldots q+1\}$ by

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_i) = 3i - 3, 3 \le i \le n-1, f(u_n) = 3n-4$ ,

$f(v_1) = 3, \quad f(v_i) = 6i - 5, 2 \le i \le \dfrac{n-2}{2},$

$f(w_i) = 6i - 1, \; 1 \le i \le \frac{n-2}{2}.$

The edges are labeled as

$f(u_i u_{i+1}) = 3i - 2, \quad 1 \le i \le n-1$

$f(u_{2i} v_i) = 6i - 4, \; 1 \le i \le \frac{n-2}{2}$

$f(u_3 v_1) = 5, \quad f(u_{2i+1} v_i) = 6i - 3, 2 \le i \le \dfrac{n-2}{2}$

$f(u_2 w_1) = 3, \quad f(u_{2i} w_i) = 6i - 1, 2 \le i \le \dfrac{n-2}{2}$

$f(u_{2i+1}w_i) = 6i, 1 \leq i \leq \frac{n-2}{2}$.

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$ .

The labeling pattern is shown below.

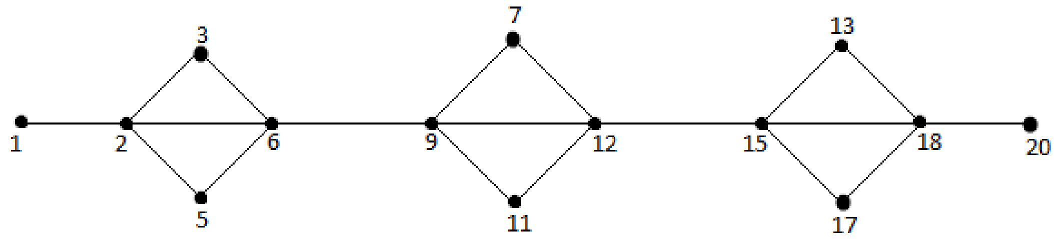


Figure 4

**Sub Case 2(b):** If $n$ is odd

Define a function $f: V(G) \to \{1,2, \dots q+1\}$ by

$f(u_1) = 1$ , $f(u_2) = 2$, $f(u_i) = 3i - 3, 3 \leq i \leq n$ ,

$f(v_1) = 3$, $\quad f(v_i) = 6i - 5, 2 \leq i \leq \frac{n-1}{2}$,

$f(w_i) = 6i - 1,\ 1 \leq i \leq \frac{n-1}{2}$ .

The edges are labeled as

$f(u_i u_{i+1}) = 3i - 2$ , $\quad 1 \leq i \leq n - 1$

$f(u_{2i}v_i) = 6i - 4,\ 1 \leq i \leq \frac{n-1}{2}$

$f(u_3v_1) = 5$, $\quad f(u_{2i+1}v_i) = 6i - 3, 2 \leq i \leq \frac{n-1}{2}$

$f(u_2w_1) = 3$, $\quad f(u_{2i}w_i) = 6i - 1, 2 \leq i \leq \frac{n-1}{2}$

$f(u_{2i+1}w_i) = 6i, 1 \leq i \leq \frac{n-1}{2}$.

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.

The labeling pattern is shown below.

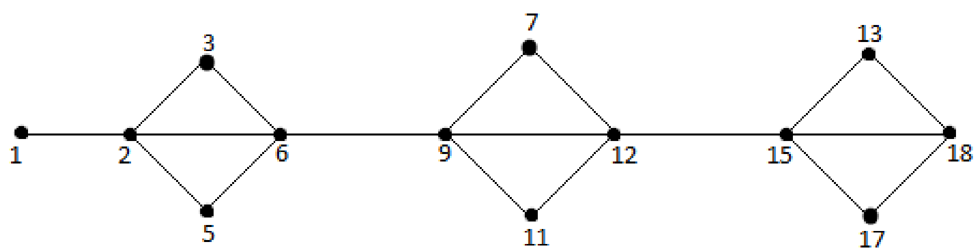


Figure 5

From all the above cases, we conclude that Alternate Double Triangular Snakes $A(D(T_n))$ are Root Square mean graphs.

**Theorem 2.8:** Double Quadrilateral snake graph $D(Q_n)$ is a Root Square mean graphs.

**Proof:** Let $P_n$ be the path $u_1u_2u_3\cdots u_n$ .To construct $D(Q_n)$, join $u_i$ and $u_{i+1}$ to four new vertices $v_i, w_i, v_i'$ and $w_i'$ by the edges $u_iv_i$, $u_{i+1}w_i$, $v_iw_i$, $u_iv_i'$, $u_{i+1}w_i'$ and $v_i'w_i'$ , for $1 \le i \le n-1$.

Define a function $f: V(D(Q_n)) \to \{1,2, \dots q+1\}$ by

$f(u_1) = 1$ , $f(u_i) = 7(i-1), 2 \le i \le n$ ,

$f(v_1) = 2$, $f(v_i) = 7i - 2, 2 \le i \le n-1$,

$f(w_1) = 5$, $f(w_i) = 7i - 1$, $2 \le i \le n-1$ ,

$f(v_1') = 3$, $f(v_i') = 7i - 5, 2 \le i \le n-1$,

$f(w_1') = 4$, $f(w_i') = 7i - 3, 2 \le i \le n-1$.

The edges are labeled as

$f(u_1u_2) = 5$, $f(u_iu_{i+1}) = 7i - 3 , 2 \le i \le n-1$

$f(u_1v_1) = 1$, $f(u_iv_i) = 7i - 5$, $2 \le i \le n-1$

$f(u_{i+1}w_i) = 7i, 1 \le i \le n-1$,

$f(v_1w_1) = 4$, $f(v_iw_i) = 7i - 2, 2 \le i \le n-1$

$f(u_1v_1') = 2$, $f(u_iv_i') = 7i - 6 , 2 \le i \le n-1$

$f(u_{i+1}w_i') = 7i - 1, 1 \le i \le n-1$,

$f(v_i'w_i') = 7i - 4, 1 \le i \le n-1$.

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.

**Example 2.9:** The Root Square mean labeling of $D(Q_5)$ is given below.

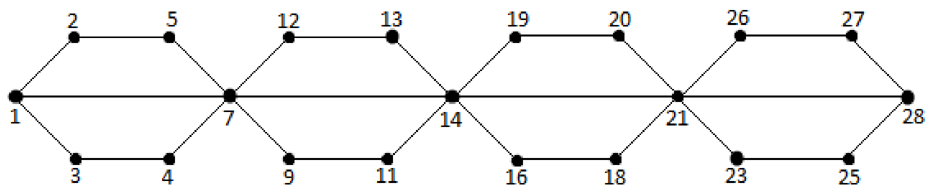


Figure 6

**Theorem 2.10:** Alternate Double Quadrilateral snake graphs $A(D(Q_n))$ are Root Square mean graphs.

**Proof:** Let $G$ be the Alternate Double Quadrilateral snake $A(D(Q_n))$ .Consider a path $u_1u_2u_3\cdots u_n$ .Join $u_i$ and $u_{i+1}$ (Alternatively) with to four new vertices $v_i, v_{i+1}, w_i$ and $w_{i+1}$.Here we consider two different cases.

**Case 1:** If the Double Quadrilateral starts from $u_1$ , then we consider two sub cases.

**Sub Case 1(a):** If $n$ is even then

Define a function $f: V(G) \to \{1,2, \dots q+1\}$ by

$f(u_1) = 1$ , $f(u_i) = 4i - 2, 2 \le i \le n$,

$f(v_1) = 2$, $f(v_i) = 4i - 3, 2 \le i \le n-1$,

$f(w_1) = 3$, $f(w_i) = 4i - 1$, $2 \le i \le n-1$ .

The edges are labeled as

$f(u_iu_{i+1}) = 4i$ , $1 \le i \le n$

$f(u_iv_i) = 4i - 3$, $i = 1,3,5, \dots, n-1$

$f(u_iv_i) = 4i - 2$, $i = 2,4,6, \dots, n$

$f(u_iw_i) = 4i - 2, i = 1,3,5, \dots, n-1$

$f(u_iw_i) = 4i - 1, i = 2,4,6, \dots, n$

$$f(v_iv_{i+1}) = 8i - 5, 1 \le i \le \frac{n-1}{2}$$

$$f(w_iw_{i+1}) = 8i - 3, 1 \le i \le \frac{n-1}{2}$$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.

The labeling pattern is shown below.

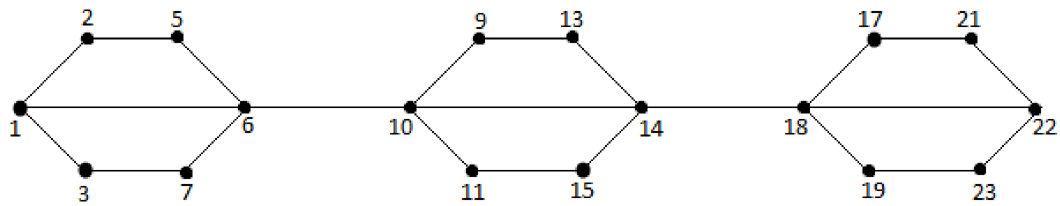


Figure 7

**Sub Case 1(b):** If $n$ is odd then

Define a function $f: V(G) \to \{1,2, \dots q + 1\}$ by

$f(u_1) = 1$, $f(u_i) = 4i - 2, 2 \le i \le n - 1, f(u_n) = 4n - 3$ ,

$f(v_1) = 2$, $f(v_i) = 4i - 3, 2 \le i \le n - 1$,

$f(w_1) = 3$, $f(w_i) = 4i - 1,\ 2 \le i \le n - 1$ .

The edges are labeled as

$f(u_iu_{i+1}) = 4i$, $1 \le i \le n$

$f(u_iv_i) = 4i - 3,\ i = 1,3,5, \dots, n$

$f(u_iv_i) = 4i - 2,\ i = 2,4,6, \dots, n - 1$

$f(u_iw_i) = 4i - 2, i = 1,3,5, \dots, n$

$f(u_iw_i) = 4i - 1, i = 2,4,6, \dots, n - 1$

$$f(v_iv_{i+1}) = 8i - 5, 1 \le i \le \frac{n-1}{2}$$

$$f(w_iw_{i+1}) = 8i - 3, 1 \le i \le \frac{n-1}{2}$$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.

The labeling pattern is shown below.

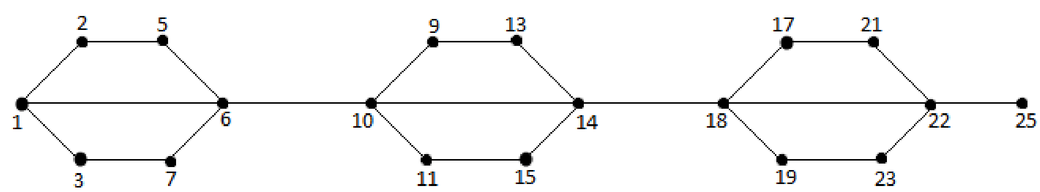


Figure 8

**Case 2:** If the Double Quadrilateral starts from $u_2$ , then we have to consider two sub cases.

**Sub case 2(a):** If $n$ is even , then

Define a function $f: V(G) \to \{1,2, \dots q + 1\}$ by

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_i) = 4i - 5, 3 \le i \le n - 1, f(u_n) = 4n - 6$ ,

$f(v_1) = 3$, $f(v_i) = 4i - 2, 2 \le i \le n - 2$,

$f(w_i) = 4i$ , $1 \le i \le n - 2$.

The edges are labeled as

$f(u_iu_{i+1}) = 4i - 3, \quad 1 \le i \le n$

$f(u_{i+1}v_i) = 4i - 2, \; i = 1,3,5, \dots, n-1$

$f(u_{i+1}v_i) = 4i - 1, i = 2,4,6, \dots, n$

$f(u_{i+1}w_i) = 4i - 1, \quad i = 1,3,5, \dots, n-1$

$f(u_{i+1}w_i) = 4i, i = 2,4,6, \dots, n$

$f(v_iv_{i+1}) = 8i - 4, 1 \le i \le \frac{n-2}{2}$

$f(w_iw_{i+1}) = 8i - 2, 1 \le i \le \frac{n-2}{2}$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.
The labeling pattern is shown below.

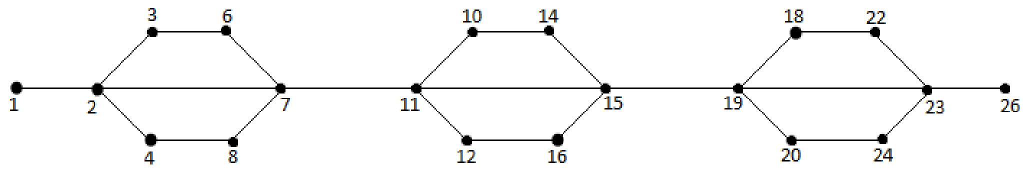


Figure 9

**Sub Case 2(b):** If $n$ is odd , then

Define a function $f: V(G) \to \{1,2, \dots q+1\}$ by

$f(u_1) = 1, \quad f(u_2) = 2, \quad f(u_i) = 4i - 5, 3 \le i \le n,$

$f(v_1) = 3, \quad f(v_i) = 4i - 2, 2 \le i \le n-1,$

$f(w_i) = 4i, \; 1 \le i \le n-1.$

The edges are labeled as

$f(u_iu_{i+1}) = 4i - 3, \quad 1 \le i \le n$

$f(u_{i+1}v_i) = 4i - 2, \; i = 1,3,5, \dots, n$

$f(u_{i+1}v_i) = 4i - 1, i = 2,4,6, \dots, n-1$

$f(u_{i+1}w_i) = 4i - 1, \quad i = 1,3,5, \dots, n$

$f(u_{i+1}w_i) = 4i, i = 2,4,6, \dots, n-1$

$f(v_iv_{i+1}) = 8i - 4, 1 \le i \le \frac{n-1}{2}$

$f(w_iw_{i+1}) = 8i - 2, 1 \le i \le \frac{n-1}{2}$

Then the edge labels are distinct. Hence in this case $f$ is a Root Square mean labeling of $G$.
The labeling pattern is shown below.

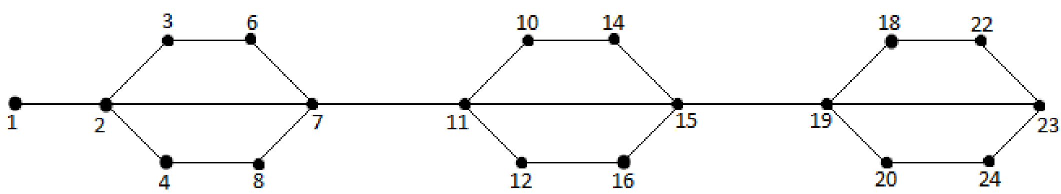


Figure 10

**Theorem 2.11:** Polygonal chain $G_{m,n}$ are Root Square mean graphs for all $m$ and $n$.

**Proof:** In $G_{m,n}$ , let $u_1u_2u_4u_6 \cdots u_{n-6}u_{n-4}u_{n-2}u_{n+1}u_{n-1}u_{n-3}u_{n-5} \cdots u_9u_7u_5u_3u_1$ be the first cycle. The second cycle is connected to the first cycle at the vertex $u_{n+1}$ .

Let $u_{n+1}u_{n+2}u_{n+4} \cdots u_{2n+1}u_{2n-1}u_{2n-3} \cdots u_{n+7}u_{n+5}u_{n+3}u_{n+1}$ be the second cycle.In general the $r^{th}$ cycle is connected to the (r-1)$^{th}$ cycle at the vertex $u_{rn+1}$.Let the $r^{th}$ cycle be $u_{rn+1}u_{rn+2}u_{rn+4}u_{rn+6} \cdots u_{(r+1)n-4}u_{(r+1)n-2}u_{(r+1)n+1}u_{(r+1)n-1}u_{(r+1)n-3}$ $u_{(r+1)n-5} \cdots u_{rn+5}u_{rn+3}u_{rn+1}$.The figure of the $r^{th}$ cycle is given below.

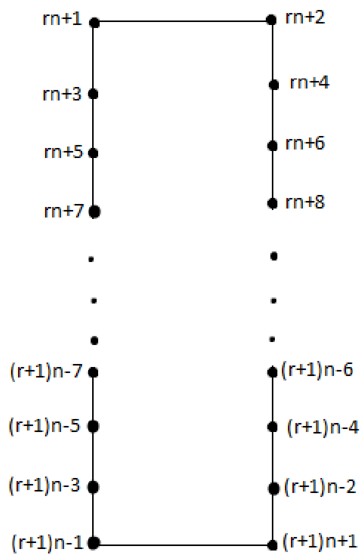


Figure 11

Let the graph has $m$ cycles . Define a function $f: V(G) \to \{1,2, \dots q+1\}$ by $f(v_i) = i , 1 \le i \le mn-1$ , $f(v_n) = mn+1$. Then the label of the edges is given below.$f(u_{mn+1}u_{mn+2}) = mn+1$ , $f(u_{mn+i}u_{mn+i+2}) = mn+i+1$, $f\left(u_{(m+1)n-2}u_{(m+1)n+1}\right) = (m+1)n-1$, $f\left(u_{(m+1)n+1}u_{(m+1)n-1}\right) = (m+1)n$.

Hence the graph $G_{m,n}$ has distinct edge labels, hence $G_{m,n}$ is a Root Square mean graph.

**Example 2.12:** Root Square mean labeling of $G_{4,6}$ chain is given below.

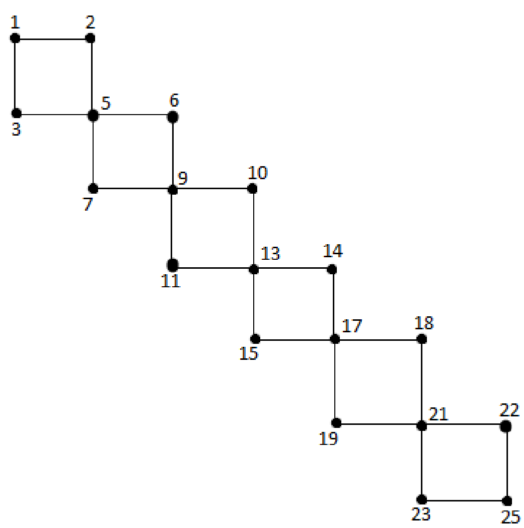


Figure 12

## Reference

David Raj, C., Jayasekaran, C., & Sandhya, S. S. (2013). Harmonic Mean Labeling on Double Quadrilateral Snake Graph. *Global Journal of Theoretical and Applied Mathematics Sciences, 3*(2), 67-72.

Gallian, J. A. (2010). A dynamic Survey of graph labeling. *The electronic Journal of Combinatories, 17#DS6*.

Harary, F. (1988). Graph Theory, Narosa Publishing House Reading, New Delhi.

Jayasekaran, C., Sandhya, S. S., & David Raj, C. (2013). Harmonic Mean Labeling on Double Triangular Snakes. *International Journal of Mathematics Research, 5*(2), 251-256.

Sandhya, S. S., & Anusa, S. Mean Labeling on Double Triangular and Double Quadrilateral Snake Graphs. *Proceedings of the Kanyakumari Academy of Arts and Sciences*, ISBN 978-93-81658-10-9 Volume-3.

Sandhya, S. S., Somasundaram, S., & Anusa, S. Root Square Mean Labeling Of Graphs. *International Journal of Contemporary Mathematical Sciences*.

Sandhya, S. S., Somasundaram, S., & Anusa, S. Some Results On Root Square Mean Graphs. *Journal of Scientific Research.*

Sandhya, S. S., Somasundaram, S., & Ponraj, R. (2012). Some More Results on Harmonic Mean Graphs. *Journal of Mathematics Research, 4*(1), 2012. http://dx.doi.org/10.5539/jmr.v4n1p21

Somasundaram, S., & Ponraj, R. (2003). Mean labeling of graphs. *National Academy of Science Letters, 26*, 210-213.

Somasundaram, S., & Ponraj, R. (2003). Some results on mean graphs. *Pure and Applied Mathematika Sciences, 58*, 29-35.

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