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The Grey Modeling Method of Wave Development Coefficient

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Abstract

In this paper, through analyzing the value trend of the data sequence development coefficient, to classify the data sequence $X^{(0)}$ and putting forward a new modeling method of fluctuating development coefficient sequence with the original GM(1, 1), through examples, this method has good simulation accuracy, and has certain practical value.

Keywords: fluctuation, development coefficient, modeling, method

1. Introduction

Since the establishment, the grey system models have been used in all aspects of national life (Deng, 2002). But the scope of traditional grey model application is narrow, which not only required the original data sequence is close to exponential sequence, but also required the class ratio should be sufficiently close to 1 (Liu & Deng, 2000; LV & Wu, 2001; Luo, Liu, & Dang, 2003). So many scholars optimized the grey model from different angles, which makes the strict index series have repeatability albino index (Zhou & Wei, 2006; Xue & Wei, 2008; Y. N. Wang, Li, B. N. Wang, & Chen, 2002; Tan, 2000), However, the existing model fitting accuracy is poor for the high-growth data and which can't reflect the true fluctuations of development index. Through analyzing the development index of original sequence, this paper reflected the fluctuations of original data development index, which combining the original GM(1,1) proposed the new modeling method of Fluctuation development index, further improve the dynamic model. By an example, this modeling method is better, there is a certain theoretical and practical value.

2. Analysis of Development Trend Coefficient

For the practical original series $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(n)\}$, The development coefficient of adjacent data $\ln\left(\frac{x^{(0)}(i+1)}{x^{(0)}(i)}\right)$ isn't a constant. Then, the original series $X^{(0)}$ will become a geometric sequence, and which lost the value of modeling. Then, for the adjacent data of original sequence $X^{(0)}$, whose development coefficient is a series $A = \{a_1, a_2, \cdots a_{n-1}\}$, and $a_i = \ln\left(\frac{x^{(0)}(i+1)}{x^{(0)}(i)}\right), k = 1, 2, \cdots n-1$ (Deng, 2002). In the coordinate plane, the distribution of this series generally have three situations: 1) $\{a_i\}$ is waving in a certain range (as show in Figure 1); 2) $\{a_i\}$ is a sustained growth series (as show in Figure 2); 3) $\{a_i\}$ is a sustained decrease series (as show in Figure 3). The development coefficient a is a average of all data in $\{a_i\}$. If the data of $\{a_i\}$ traded in a tight range, let $y = a$, because the a and $\{a_i\}$ gap, the model simulation error can be controlled in a small range. But if the data of $\{a_i\}$ traded in a wide range, let $y = a$, the model simulation error will be large, and It will cause the intermediate data have small error, the other data have lager error. Because this error is caused by the model itself, it can't be avoided unless change the modeling method.

3. The Modeling Method of Wave Development Coefficient

Definition let $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(n)\right)$ is the original series, $a_k = \ln\left(\frac{x^{(0)}(k+1)}{x^{(0)}(k)}\right), k = 1, 2, 3, \cdots n-1,$

1) If $X^{(0)}$ a sustained growth series (or a sustained decrease series) and $0 < a_k \le a_{k+1}$ (or $0 > a_k \ge a_{k+1}$), $k = 1, 2, \cdots n-2$, then $X^{(0)}$ is a acceleration growth (attenuation) dynamic series.

2) If $X^{(0)}$ a sustained growth series (or a sustained decrease series) and $a_k \ge a_{k+1} > 0$ (or $0 < a_{k+1} \le a_k$),

$k = 1, 2, \cdots n-2$, then $X^{(0)}$ is a decelerate growth (attenuation) dynamic series.

3) If $X^{(0)}$ a sustained growth series (or a sustained decrease series) and There exists a positive integer i, j, which make $a_i \geq a_{i+1}, a_j \leq a_{j+1}$, then $X^{(0)}$ is a wave growth (attenuation) dynamic series.

Now we discuss the modeling method of wave growth (attenuation) dynamic series.

Set the original sequence is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(n)\}$, then the $1 - AGO$ (First-order Accumulated generating operation, Deng, 2002) sequence of $X^{(0)}$ is $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots x^{(1)}(n)\}$, and $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$.

(1) Test the original series, determine the series type of $X^{(0)}$.

(2) If $X^{(0)}$ is a Wave growth (attenuation) dynamic series. Find the series $\left\{a^{(0)}(i)\right\}$,

$$a^{(0)}(i) = \ln\left(\frac{x^{(0)}(i+1)}{x^{(0)}(i)}\right), i = 1, 2, \cdots n-1.$$

(3) Transform $A^{(0)} = \{a^{(0)}(1), a^{(0)}(2), \cdots a^{(0)}(n-1)\}$, getting

$$A'^{(0)} = \{a'^{(0)}(1), a'^{(0)}(2), \cdots a'^{(0)}(n-1)\},$$

In which, $a'^{(0)}(k) = a^{(0)}(k)T^{k1}, k = 1, 2, \cdots, n$. And

$$T = \max\left\{a^{(0)}(k)/k \in \{1, 2, \cdots n\}\right\} \Big/ \min\left\{a^{(0)}(k)/k \in \{1, 2, \cdots n\}\right\}.$$

(4) Getting $A'^{(0)} = \{a'^{(0)}(1), a'^{(0)}(2), \cdots a'^{(0)}(n-1)\}$ into the original equation GM(1,1), then

$$\hat{A}'^{(0)} = \{\hat{a}'^{(0)}(1), \hat{a}'^{(0)}(2), \cdots \hat{a}'^{(0)}(n-1)\}.$$

(5) Restore the value $\hat{x}^{(0)}(i) = \hat{x}^{(0)}(i-1)e^{\hat{a}'^{(0)}(i-1)/T^{(i-1)}}, i = 2, 3, \cdots n$, and $\hat{x}^{(0)}(1) = x^{(0)}(1)$.

4. Instance Analysis

$X^{(0)} = \{1.0000\ \ 3.6693\ \ 12.1825\ \ 49.4024\ \ 181.2722\ \ 601.8450\}$ is a group of high-growth sequence, whose development index sequence is $A = \{1.2\ 1.3\ 1.2\ 1.4\ 1.3\ 1.2\}$. Let original GM(1,1) model is Model 1, the model in reference (Xue & Wei, 2008) is Model 2, the modeling method in this Model 3.

Table 1. Comparison of the Simulation Precision

Real value	Model 1		Model 2		Model 3	
	Simulated value	Relative error	Simulated value	Relative error	Simulated value	Relative error
3.6693	7.6887	109.5415	−−	−−	3.6693	0
12.1825	22.9814	88.6428	15.54231	27.5790	13.6969	12.4307
49.4024	68.6910	39.0440	52.59938	6.4713	49.7438	0.6910
181.2722	205.3164	13.2641	178.0106	-1.7993	175.8671	-2.9818
601.8450	613.6872	1.9677	602.4359	0.0982	605.6219	0.6275
Average Relative errors	50.49		8.98		3.3462	

From Table 1, the average relative error of model 1 is 50.49%, which is lost the modeling value. And average relative error of the model 2 is 8.98%, but model 3, modeling with the method of this paper, whose average relative error is only 3.3462%, the modeling accuracy is the best. Therefore, the modeling effect of this paper is better, and it has practical value.

5. Discussion

Through analyzing the development index of original sequence, this paper reflected the fluctuations of original data development index, which combining the original GM(1,1) proposed the new modeling method of Fluctuation development index, further improve the dynamic model. By an example, this modeling method is better, there is a certain theoretical and practical value.

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