Fuzzy Shortest Path Problem Based on Index Ranking

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Abstract

The Shortest path problem is a classical network optimisation problem which has wide range of applications in various fields. In this work, we study the task of finding the shortest path in fuzzy weighted graph (network) i.e., vertices remains crisp, but the edge weights will be of fuzzy numbers. It has been proposed to present new algorithms for finding the shortest path in fuzzy sense where illustrative examples are also included to demonstrate our proposed approach.

Keywords: Acyclic Network, Shortest Path Problem (SPP), Fuzzy, π_2 membership function, Acceptability Index (AI), Weighted Average Index (WAI), Total Integral Index (I_T), LR Trapezoidal fuzzy numbers, LR type representation of fuzzy interval, α-cut interval, Convex Index (CoI), Yager Index (I), Index Ranking of fuzzy numbers, Decision Maker (DM)

1. Introduction

In a classical network problem, the weights of the edges in a SPP are supposed to be real numbers, but most practical applications, however, have parameters that are not naturally precise (i.e., costs, demand, time, etc). In such cases it is quite appropriate to use fuzzy numbers for modelling the problem which gives rise to Fuzzy Shortest Path Problem (FSPP). Fuzzy set theory proposed by Zadeh (1978) is an effective tool to tackle the problems of uncertainty.

The FSPP was first introduced by Dubois and Prade (1980). They used Floyd's algorithm and Ford's algorithm to treat the FSPP. In their method the shortest path length can be obtained, but the corresponding path in the network doesn't exist. Klein (1991) proposed a dynamical programming recursion-based fuzzy algorithm. Lin and Chen (1994) found the Fuzzy Shortest Path Length (FSPL) in a network by means of a fuzzy linear programming approach. Okada (2004) defined a new comparison index between the sums of fuzzy numbers by considering interactivity among fuzzy numbers and presented an algorithm to determine the degree of possibility for each arc on a network. Yao and Lin (2003) developed two types of Fuzzy Shortest Path network problems, where the first type of Fuzzy Shortest Path Problem uses triangular fuzzy numbers and the second type uses level $(1-\beta, 1-\alpha)$ interval valued fuzzy numbers, the main result from their study was, the Shortest Path in the fuzzy sense correspond to the actual path in the network, and the Fuzzy Shortest Path Problem is an extension of the crisp case. Kung et al. (2006) proposed a new fuzzy dynamic programming approach to get the Fuzzy Shortest Path Problem where each arc length is assumed to be a triangular fuzzy number. Mahdavi et al. (2007) proposed the Fuzzy Shortest Length method and utilized Fuzzy Similarity Measure to get the Shortest Path. Chuang and Kung (2005) represented each arc length as a triangular fuzzy set and a new algorithm is proposed to deal with the FSPP. First, they proposed a heuristic procedure to find the FSPL among all possible paths in a network. Second, they presented a way to measure the similarity degree between the FSPL and each fuzzy path lengths. The path with the highest similarity degree is the Shortest Path (SP). As an extension of this result, the fuzzy shortest length heuristic procedure was developed by Sujatha and Elizabeth (2011), if the length of the path in the network is trapezoidal fuzzy set where the highest similarlity degree is utilized to identify the shortest path. Chuang and Kung (2006) in their work, represented each arc length, as a discrete fuzzy set and a new algorithm was proposed that can find the Fuzzy Shortest Length and the Fuzzy Similarity Measure was utilized to single out the Shortest Path. Thus numerous papers have been published on the FSPP.

In this paper the fuzzy shortest path analysis is organized in the following way: In section 2, some elementary concepts and definitions in fuzzy set theory used through out this paper are described. Also, some new indices and α-cut interval for LR trapezoidal fuzzy numbers are introduced. In section 3, the algorithms with the classical examples have been proposed for FSPP based on the indices defined in section 2. This paper is concluded in section 4.

2. Pre-Requisities

Definition 1: Acyclic Digraph

A digraph is a graph each of whose edges are directed. Hence an acyclic digraph is a directed graph without cycle.

Crisp graph with fuzzy weights (Type V)

A fifth type of graph fuzziness was proposed by Blue et al. (2002). It occurs when the graph has known vertices and edges, but unknown weights (or capacities) on the edges. Thus only the weights are fuzzy.

Applications: To plan the quickest automobile route from one city to another.

Definition 2: Membership function of the LR triangular fuzzy number

Membership function of the LR triangular fuzzy number $A = (m, \gamma, \delta)_{LR}$ is shown in Figure 1.

$$\mu_A(x) = \begin{cases} 0, & x \leq m - \gamma \\ \dfrac{x-(m-\gamma)}{\gamma}, & m-\gamma < x < m \\ 1, & x = m \\ \dfrac{(m+\delta)-x}{\delta}, & m < x < m+\delta \\ 0, & x \geq m+\delta \end{cases}$$

where m is the center, γ is the left spread and δ is the right spread.

Definition 3: Membership function of the LR trapezoidal fuzzy number

Membership function of the LR trapezoidal fuzzy number $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ is shown in Figure 2.

$$\mu_A(x) = \begin{cases} 0, & x \leq \underline{m} - \gamma \\ \dfrac{x-(\underline{m}-\gamma)}{\gamma}, & \underline{m}-\gamma < x < \underline{m} \\ 1, & \underline{m} \leq x \leq \overline{m} \\ \dfrac{(\overline{m}+\delta)-x}{\delta}, & \overline{m} < x < \overline{m}+\delta \\ 0, & x \geq \overline{m}+\delta \end{cases}$$

The LR trapezoidal fuzzy number A with $\underline{m} = \overline{m} = m$ is called a LR triangular fuzzy number or triangular LR fuzzy number.

Definition 4: Addition operation on LR trapezoidal and LR triangular fuzzy numbers

Let A and B be two LR trapezoidal fuzzy numbers, $A = (\underline{m}_1, \overline{m}_1, \gamma_1, \delta_1)_{LR}$ and $B = (\underline{m}_2, \overline{m}_2, \gamma_2, \delta_2)_{LR}$. Then the fuzzy sum of these two numbers is given by $A + B = (\underline{m}_1, \overline{m}_1, \gamma_1, \delta_1)_{LR} + (\underline{m}_2, \overline{m}_2, \gamma_2, \delta_2)_{LR} = (\underline{m}_1 + \underline{m}_2, \overline{m}_1 + \overline{m}_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)_{LR}$.

Let A and B be two LR triangular fuzzy numbers, $A = (m_1, \gamma_1, \delta_1)_{LR}$ and $B = (m_2, \gamma_2, \delta_2)_{LR}$. Then the fuzzy sum of these two numbers is given by $A + B = (m_1, \gamma_1, \delta_1)_{LR} + (m_2, \gamma_2, \delta_2)_{LR} = (m_1 + m_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)_{LR}$.

Definition 5: LR Type Representation of Fuzzy Interval

If m is not a real number, but is an interval $[\underline{m}, \overline{m}]$ then the Fuzzy set A is not a Fuzzy number but is a Fuzzy Interval. The Fuzzy Interval is denoted by $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ whose membership function is given by

$$\mu_A(x) = \begin{cases} L\left(\dfrac{\underline{m}-x}{\gamma}\right), & \text{if } x < \underline{m}, \\ 1, & \text{if } \underline{m} \leq x \leq \overline{m} \\ R\left(\dfrac{x-\overline{m}}{\delta}\right), & \text{if } x > \overline{m} \end{cases}$$

Definition 6: Yager Index

$I(A) = \int_0^1 \alpha[A_\alpha^L + A_\alpha^U]d\alpha$ where $[A_\alpha^L, A_\alpha^U]$ is the α-cut interval of fuzzy number A. This index is exactly the ranking index developed by Yager (1981).

In this paper the fuzzy number A is taken as $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ for the sake of verification. If A and B are two LR trapezoidal fuzzy numbers, then in Yager's Index, we have $A < B$ iff $I(A) < I(B)$.

Definition 7: π_2 Membership function

The membership function that has the "π_2" shape has four parameters which is shown in Figure 3.

$$\pi_2(x : lw, lp, rp, rw) = \begin{cases} \dfrac{lw}{lp + lw - x}, & x < lp \\ 1, & lp \le x \le rp \\ \dfrac{rw}{x - rp + rw}, & x > rp \end{cases}$$

This membership function was used by Yen and Langari (2006) to define the fuzzy set "Good stopping Accuracy". It is now used in the current research to define the Shortest Path in Fuzzy sense.

The following definitions are introduced in this paper:

Definition 8: Addition Operation on π_2 shaped fuzzy numbers

Assuming that both $A = (lw_1, lp_1, rp_1, rw_1)$ and $B = (lw_2, lp_2, rp_2, rw_2)$ are real numbers, then
$A + B = (lw_1 + lw_2, lp_1 + lp_2, rp_1 + rp_2, rw_1 + rw_2)$.

Definition 9: Minimum Operation on π_2 shaped fuzzy numbers

For two π_2 shaped fuzzy numbers $L_1 = (lw_1, lp_1, rp_1, rw_1)$ and $L_2 = (lw_2, lp_2, rp_2, rw_2)$, we introduce
$L_{min}(L_1, L_2) = (max(lw_1, lw_2), min(lp_1, lp_2), min(rp_1, rp_2), min(rw_1, rw_2))$.

Definition 10: Acceptability Index (AI)

The Acceptability Index (AI) of the proposition $L_{min} = (lw, lp, rp, rw)$ is preferred to $L_i = (lw_i, lp_i, rp_i, rw_i)$ is given by $AI(L_{min} < L_i) = \dfrac{-(rw + lw_i)}{rp - rw - lp_i - lw_i}$. Using this Acceptability Index we define the ranking order based on highest Acceptability Index, i.e., in Acceptability Index, we have $L_1 < L_2$ iff $AI(L_{min} < L_1) > AI(L_{min} < L_2)$.

The above formula is obtained, considering Figure 4, as follows:

Let y_d be the membership function

If $x > rp$, $y_d = \dfrac{rw}{x - rp + rw}$

$\Rightarrow\ rw = y_d(x - rp + rw)$

$rw = xy_d - rpy_d + rwy_d$

$xy_d = rw + rpy_d - rwy_d$

$$x = \frac{rw + rpy_d - rwy_d}{y_d} \tag{1}$$

If $x < lp_i$, $y_d = \dfrac{lw_i}{lp_i + lw_i - x}$

$\Rightarrow\ lw_i = y_d(lp_i + lw_i - x)$

$lw_i = lp_iy_d + lw_iy_d - xy_d$

$xy_d = lp_iy_d + lw_iy_d - lw_i$

$$x = \frac{lp_iy_d + lw_iy_d - lw_i}{y_d} \tag{2}$$

Equating (1) and (2)

$rw + rpy_d - rwy_d = lp_iy_d + lw_iy_d - lw_i$

$rpy_d - rwy_d - lp_iy_d - lw_iy_d = -rw - lw_i$

$y_d(rp - rw - lp_i - lw_i) = -(rw + lw_i)$

$$y_d = \frac{-(rw + lw_i)}{rp - rw - lp_i - lw_i}$$

Definition 11: Weighted Average Index for level λ π_2 Membership function

Let $L_i = (lw_i, lp_i, rp_i, rw_i; \lambda)$ and $L_{min} = (lw, lp, rp, rw; \lambda)$ be two level λ π_2 Membership functions, which is shown in Figure 5. If $lp \leq lp_i$, $rp \leq rp_i$, then the Weighted Average Index between L_i and L_{min} can be calculated as $WAI(L_{min}, L_i) = \dfrac{\lambda A + \lambda A_i}{2\lambda}$, $0 < \lambda \leq 1$, where $A = \dfrac{lp + rp}{2}$ and $A_i = \dfrac{lp_i + rp_i}{2}$, $i = 1$ to n.

In the Weighted Average Index, we have $L_1 < L_2$ iff $WAI(L_{min}, L_1) < WAI(L_{min}, L_2)$.

Definition 12: α-cut interval for LR trapezoidal fuzzy number

α-cut interval for LR trapezoidal fuzzy number $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ is $A_\alpha = [\alpha\gamma + (\underline{m} - \gamma), (\overline{m} + \delta) - \alpha\delta] = [A^L_\alpha, A^U_\alpha]$.

The above α-cut interval is obtained, as follows for all $\alpha \in [0, 1]$

$\dfrac{A^L_\alpha - (\underline{m} - \gamma)}{\gamma} = \alpha \Rightarrow A^L_\alpha = \alpha\gamma + (\underline{m} - \gamma).$

$\dfrac{(\overline{m} + \delta) - A^U_\alpha}{\delta} = \alpha \Rightarrow A^U_\alpha = (\overline{m} + \delta) - \alpha\delta.$

Definition 13: α-cut interval for LR type representation of fuzzy interval

α-cut interval for LR type representation of fuzzy interval $A = [\underline{m}, \overline{m}]$ is given by $A_\alpha = [A^L_\alpha, A^U_\alpha] = [\underline{m} - \alpha\gamma, \alpha\delta + \overline{m}]$.

The above α-cut interval is obtained, as follows for all $\alpha \in [0, 1]$.

$\dfrac{\underline{m} - A^L_\alpha}{\gamma} = \alpha \Rightarrow A^L_\alpha = \underline{m} - \alpha\gamma$

$\dfrac{A^U_\alpha - \overline{m}}{\delta} = \alpha \Rightarrow A^U_\alpha = \alpha\delta + \overline{m}$

Thus, in general the α-cut interval for $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR} = [\underline{m}, \overline{m}]$ is $A_\alpha = [A^L_\alpha, A^U_\alpha] = [\alpha\gamma + (\underline{m} - \gamma), (\overline{m} + \delta) - \alpha\delta] = [\underline{m} - \alpha\gamma, \alpha\delta + \overline{m}]$.

Definition 14: Convex Index (CoI)

Let A be LR trapezoidal fuzzy number, then $CoI(A) = \lambda(A^L_\alpha) + (1 - \lambda)(A^U_\alpha)$ where $[A^L_\alpha, A^U_\alpha]$ is the α-cut interval of $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$, if $\mu_A[\lambda(A^L_\alpha) + (1 - \lambda)(A^U_\alpha)] \geq min\{\mu_A(A^L_\alpha), \mu_A(A^U_\alpha)\}$, for all $\alpha, \lambda \in [0, 1]$. If A and B are two LR trapezoidal fuzzy numbers, then in Convex Index, we have $A < B$ iff $CoI(A) < CoI(B)$.

Definition 15: Total Integral Index

For a fuzzy number $A = (\underline{m}, \overline{m}, \gamma, \delta)_{LR}$, the Total integral index $I_T(A)$ can be constructed from the left integral values $I_L(A)$ and the right integral values $I_R(A)$. The total integral value $I_T(A)$ with index of optimism λ where $0 < \lambda \leq 1$ is then defined as

$$I_T(A) = \lambda I_L(A) + (1 - \lambda)I_R(A). \tag{3}$$

The left integral values $I_L(A)$ and the right integral values $I_R(A)$ of a LR trapezoidal fuzzy number $(\underline{m}, \overline{m}, \gamma, \delta)_{LR}$ can be found as

$$I_L(A) = \left[\left(\underline{m} - \frac{\gamma}{2}\right) + \left(\overline{m} - \frac{\zeta}{2}\right) + \left(c - \frac{\beta}{2}\right)\right] = \left[(\underline{m} + \overline{m} + c) - \left(\frac{\gamma + \zeta + \beta}{2}\right)\right] \tag{4}$$

$$I_R(A) = \left[\left(\underline{m} + \frac{\beta}{2}\right) + \left(\overline{m} + \frac{\delta}{2}\right) + \left(c + \frac{\zeta}{2}\right)\right] = \left[(\underline{m} + \overline{m} + c) + \left(\frac{\beta + \delta + \zeta}{2}\right)\right] \tag{5}$$

where $c = \dfrac{\underline{m} + \overline{m}}{2}$, $\beta = c - \underline{m}$, $\zeta = \overline{m} - c$.

From equations (3) to (5), the total integral value is defined as

$$I_T(A) = \lambda\left[(\underline{m} + \overline{m} + c) - \left(\frac{\gamma + \zeta + \beta}{2}\right)\right] + (1 - \lambda)\left[(\underline{m} + \overline{m} + c) + \left(\frac{\beta + \delta + \zeta}{2}\right)\right]$$

where $c = \dfrac{\underline{m} + \overline{m}}{2}$, $\beta = c - \underline{m}$, $\zeta = \overline{m} - c$. The Total Integral Index is shown in Figure 6.

3. Proposed Algorithm for FSPP

3.1 Algorithm for Fuzzy Shortest Path Problem based on Acceptability Index

Step 1: Construct a Network $G = (V, E)$ where V is the set of vertices and E is the set of edges. Here G is an acyclic digraph.

Step 2: Calculate all possible paths P_i, from source vertex s to the destination vertex d and compute the corresponding path lengths $L_i = (lw_i, lp_i, rp_i, rw_i)$ for $i = 1, 2, \ldots, n$ using Definition 8.

Step 3: Calculate the Fuzzy Shortest Length L_{min} using Definition 9 and set $L_{min} = (lw, lp, rp, rw)$.

Step 4: Calculate the Acceptability Index AI ($L_{min} < L_i$) between L_{min} and L_i for $i = 1, 2, \ldots, n$, using Definition 10 and then Ranking is given to the paths based on Acceptability Index.

Step 5: Identify the Shortest Path with the highest Acceptability Index AI ($L_{min} < L_i$).

Illustrative Example 1

Step 1: Construct a network with 6 vertices and 8 edges as given in Figure 7.

Assume the arc lengths as $A(1-2) = (10, 20, 20, 10)$, $B(1-3) = (10, 62, 65, 5)$, $C(2-3) = (3, 38, 40, 5)$, $D(2-5) = (3, 55, 60, 5)$, $E(3-4) = (3, 13, 17, 3)$, $F(3-5) = (1, 9, 9, 1)$, $G(4-6) = (5, 75, 85, 12)$, $H(5-6) = (20, 70, 80, 20)$.

Step 2: From Figure 7, we get the possible paths and the corresponding path lengths as follows:

Path P_1 1-3-4-6 with $L_1 = B + E + G = (18, 150, 167, 20) = (lw_1, lp_1, rp_1, rw_1)$.

Path P_2 1-3-5-6 with $L_2 = B + F + H = (31, 141, 154, 26) = (lw_2, lp_2, rp_2, rw_2)$.

Path P_3 1-2-3-4-6 with $L_3 = A + C + E + G = (21, 146, 162, 30) = (lw_3, lp_3, rp_3, rw_3)$.

Path P_4 1-2-3-5-6 with $L_4 = A + C + F + H = (34, 137, 149, 36) = (lw_4, lp_4, rp_4, rw_4)$.

Path P_5 1-2-5-6 with $L_5 = A + D + H = (33, 145, 160, 35) = (lw_5, lp_5, rp_5, rw_5)$.

Step 3: $L_{min} = (34, 137, 149, 20) = (lw, lp, rp, rw)$.

Step 4: Results of the network based on Acceptability Index is given in Table 1.

Step 5: The Shortest Path is 1-2-3-5-6 and the Fuzzy Shortest Path Length is (34, 137,149, 36). Since the corresponding path P_4 has the highest Acceptability Index (=1.286).

Results of the network based on Weighted Average Index is given in Table 2.

From Table 2, it is clear that the Shortest Path is 1-2-3-5-6 and the Fuzzy Shortest Path Length is (34, 137,149, 36). Since the corresponding path P_4 has the minimum Weighted Average Index (=143). Here we see the Shortest Path remains the same as in the case of Acceptability Index.

3.2 Algorithm for Fuzzy Shortest Path Problem based on Convex Index

Step 1: Construct a network $G = (V, E)$ where V is the set of vertices and E is the set of edges.

Step 2: Form the possible paths P_i from source vertex s to the destination vertex d and compute the corresponding path lengths L_i, $i = 1, 2, \ldots, m$ for possible m path using Definition 4 and set $L_i = (\underline{m}_i, \overline{m}_i, \gamma_i, \delta_i)_{LR}$.

Step 3: Calculate α-cut interval for LR trapezoidal fuzzy number (or LR type representation of fuzzy interval) for all possible path lengths L_i, $i = 1, 2, \ldots, m$ using Definition 12 (or Definition 13). Set $L_{i(\alpha)} = [L^L_{i(\alpha)}, L^U_{i(\alpha)}]$, $i = 1$ to m.

Step 4: Calculate Convex Index $CoI(L_i) = \lambda(L^L_{i(\alpha)}) + (1-\lambda)(L^U_{i(\alpha)})$, for all possible path lengths L_i, $i = 1$ to m. i.e., using Definition 14.

Step 5: Identify the shortest path with the minimum Convex Index and the corresponding path length L_i is the fuzzy shortest path length.

Illustrative Example 2

Step 1: Construct a network with 6 vertices and 8 edges as given in Figure 7.

Assume the arc lengths as $A[1-2] = (20, 20, 10, 10)_{LR}$, $B[1-3] = (62, 65, 10, 5)_{LR}$, $C[2-3] = (38, 40, 3, 5)_{LR}$, $D[2-5] = (55, 60, 3, 5)_{LR}$, $E[3-4] = (13, 17, 3, 3)_{LR}$, $F[3-5] = (9, 9, 1, 1)_{LR}$, $G[4-6] = (75, 85, 5, 12)_{LR}$, $H[5-6] = (70, 80, 20, 20)_{LR}$.

Step 2: From Figure 7, we get the possible paths and the corresponding path lengths as follows:

Path P_1 1-3-4-6 with $L_1 = B + E + G = (150, 167, 18, 20)_{LR}$.

Path P_2 1-3-5-6 with $L_2 = B + F + H = (141, 154, 31, 26)_{LR}$.

Path P_3 1-2-3-4-6 with $L_3 = A + C + E + G = (146, 162, 21, 30)_{LR}$.

Path P_4 1-2-3-5-6 with $L_4 = A + C + F + H = (137, 149, 34, 36)_{LR}$.

Path P_5 1-2-5-6 with $L_5 = A + D + H = (145, 160, 33, 35)_{LR}$.

Step 3: Let $\alpha = 0.5$ and $\lambda = 0.6$ (since $\alpha, \lambda \in [0, 1]$).

$L_{1\alpha} = [141, 177]$, $L_{2\alpha} = [125.5, 167]$, $L_{3\alpha} = [135.5, 177]$, $L_{4\alpha} = [120, 167]$, $L_{5\alpha} = [128.5, 177.5]$.

Step 4: Results of the network based on Convex Index is given in Table 3.

Step 5: Path P_4 i.e., 1-2-3-5-6 is identified as the Shortest Path since it has the minimum Convex Index (= 138.8) and the corresponding fuzzy shortest path length is $L_4 = (137, 149, 34, 36)_{LR}$.

Verification using Yager's Index

Results of the network based on Yager Index is given in Table 4.

From Table 4, path P_4 i.e., 1-2-3-5-6 is identified as the Shortest Path, since it has the minimum Yager Index (= 143.5) and the corresponding fuzzy shortest path length is $L_4 = (137, 149, 34, 36)_{LR}$.

The main advantage of Convex Index, is whether the arc lengths are represented in the form of LR triangular or LR trapezoidal fuzzy numbers it can be converted in terms of interval numbers using α-cut to maintain the uniqueness of the arc length. It is also verified using the well known Yager's Index and it was found that the shortest path remains the same in both the cases.

Results of the network based on Total Integral Index is given in Table 5 and this table is used to identify path P_4 i.e., 1-2-3-5-6 as the Shortest Path, since it has the minimum Total Integral Index (= 424.8) and the corresponding fuzzy shortest path length is $L_4 = (137, 149, 34, 36)_{LR}$. Here we see the Shortest Path remains the same as in the case of Convex Index and Yager Index.

Results and Discussion

One way to verify the solution obtained is to make an exhaustive comparison

- For Figure 7, Yu and Wei (2007) used linear multiple objective programming and found path $P_4 = \{1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6\}$ as the shortest path with fuzzy shortest path length as $(137, 149, 34, 36)_{LR}$. The solution of the Illustrated Example 2 presented in this paper coincides with the solution obtained by the method of linear multiple objective programming.

- Okada and Soper (2000) developed an algorithm based on the multiple labeling methods for a multicriteria Shortest Path to find a number of non dominated paths. For Figure 7, they obtained the non dominated paths as $P_4 = \{1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6\}$ and $P_2 = \{1 \rightarrow 3 \rightarrow 5 \rightarrow 6\}$. Although P_4 and P_2 are defined to be non dominated by Okada and Soper, the Indices defined in this paper indicates that path P_4 is the shortest path.

4. Conclusion

In this paper we developed two algorithms for solving SPP on a network with fuzzy arc lengths where the shortest path is identified using the concept of ranking function with regard to the fact that the Decision Maker can choose the best path among various alternatives from the list of ranking. Verification is also done with the existing methods, which helps to conclude that the algorithms developed in the current paper is an alternative and improved form of previous methods, to get the Shortest Path in Fuzzy environment.

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Table 1. Results of the network based on Acceptability Index

Paths	AI ($L_{min} < L_i$)	Ranking
P_1 : 1-3-4-6	0.974	5
P_2 : 1-3-5-6	1.186	2
P_3 : 1-2-3-4-6	1.079	4
P_4 : 1-2-3-5-6	1.286	1
P_5 : 1-2-5-6	1.082	3

Table 2. Results of the network based on Weighted Average Index

Paths	$WAI(L_{min}, L_i)$	Ranking
P_1 : 1-3-4-6	150.75	5
P_2 : 1-3-5-6	145.25	2
P_3 : 1-2-3-4-6	148.5	4
P_4 : 1-2-3-5-6	143	1
P_5 : 1-2-5-6	147.75	3

Table 3. Results of the network based on Convex Index

Paths	Convex Index CoI (L_i)	Ranking
P_1 : 1-3-4-6	155.4	5
P_2 : 1-3-5-6	142.1	2
P_3 : 1-2-3-4-6	152.1	4
P_4 : 1-2-3-5-6	138.8	1
P_5 : 1-2-5-6	148.1	3

Table 4. Results of the network based on Yager Index

Paths	Yager Index	Ranking
	$I(L_i) = \int_0^1 0.5[L^L_{i(\alpha)} + L^U_{i(\alpha)}]d\alpha$	
P_1 : 1-3-4-6	159	5
P_2 : 1-3-5-6	146.25	2
P_3 : 1-2-3-4-6	156.25	4
P_4 : 1-2-3-5-6	143.5	1
P_5 : 1-2-5-6	153	3

Table 5. Results of the network based on Total Integral Index

Paths	$I_T(L_i)$	Ranking
P_1 : 1-3-4-6	472.4	5
P_2 : 1-3-5-6	437.1	2
P_3 : 1-2-3-4-6	460.1	4
P_4 : 1-2-3-5-6	424.8	1
P_5 : 1-2-5-6	453.1	3

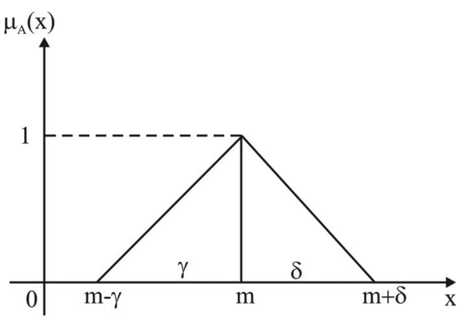

Figure 1. Membership function of the LR triangular fuzzy number A

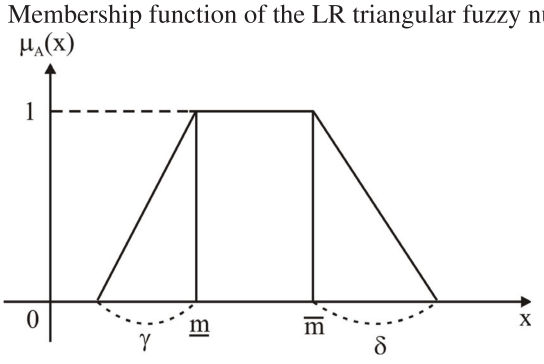

Figure 2. Membership function of the LR trapezoidal fuzzy number A

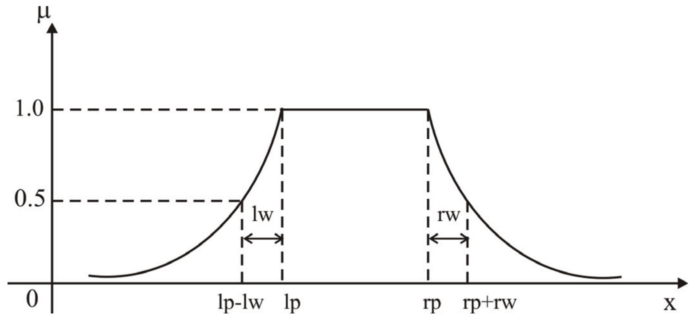

Figure 3. π_2 Membership function

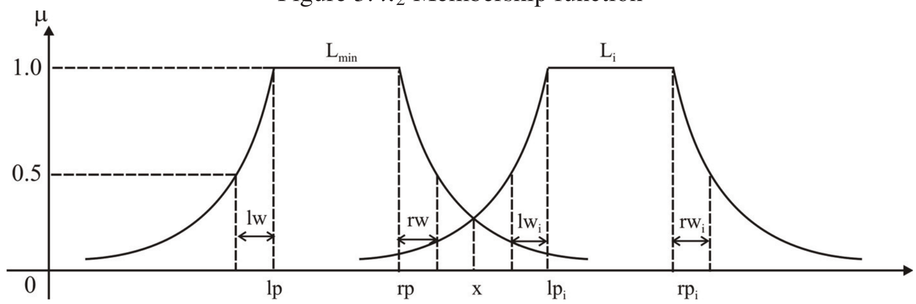

Figure 4. The Acceptability Index diagram for two π_2 shaped fuzzy numbers

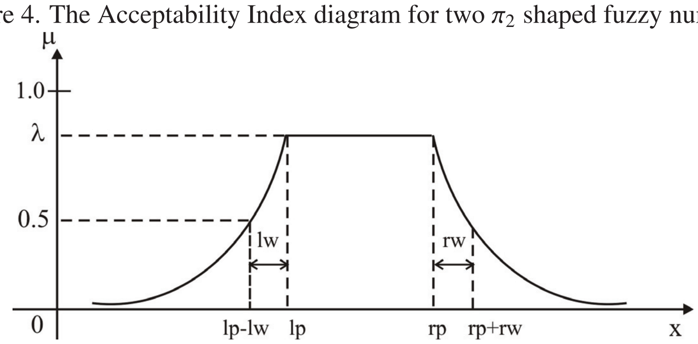

Figure 5. Level λ π_2 Membership function

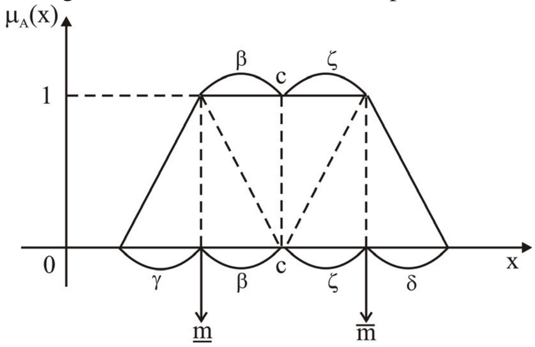

Figure 6. Total Integral Index

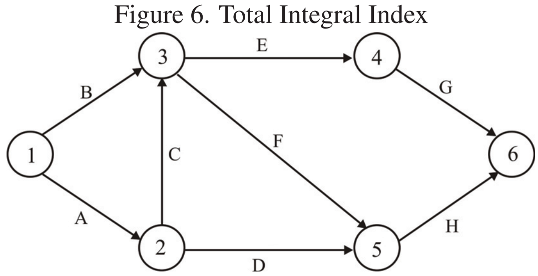

Figure 7. Network