A Stochastic Model for Reserve Inventory Between Machines in Parallel

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Abstract

Inventory control is the process of deciding what and how much of various items are to be kept in stock. The basic objective of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time. In this article the optimal reserve inventory between machines in parallel is attempted. The output of first machine M_1 is the input for the second and third machines M_2 and M_3. In between the Machines M_1 and M_2, M_3 an inventory is maintained. One of the problems of interest in inventory control theory is the determination of the Optimal size of the buffer between operating systems, namely machines. The necessity for maintaining inventory arises in several situations in a production oriented inventory systems. The study reveals the Optimum policy for maintaining the inventory between machines. The reason for maintaining inventory between machines is, due to ageing of machines and due to some other external reasons, the time taken for receiving the finished goods between parallel system may affect the starting of the next machines. Hence here we study the optimal reserve inventory between machines in parallel. A generalized equation is derived when the number of parallel machines are ‘n’.

Keywords: Serial system, Optimal reserve Inventory

MSC *Subject Classification No:* 90B05

1. Introduction

One of the problem of interest in inventory control theory is the determination of the optimal size of the buffer stock between the systems. The necessity for maintaining inventory arises in several situations in a production oriented inventory system, viz., hydro-electric system, thermal power station etc.

The optimal reserve inventory between the systems, namely, the system that produces the input and the system that consumes the input obtained from the previous system is must.

If various manufacturing processes operate successively, then in the case of breakdown of one at some stage can affect the entire system. Hence stocking points of inventory are created between adjacent stages so as to achieve a certain degree of independence in operating the stages.

The recent applications in a production oriented inventory system, viz., hydro-electric system, thermal power station, etc. In the case of hydro-electric system, the input is the river-flow and the Dam is the reserve to hydro-electric station. The optimal reserve in this case is not exactly the reserve in the dam but it is only the problem of optimal discharge during the different periods of a planning horizon. The question of determination of the optimal reserve is solved.

Similarly in case of the thermal station, Coal or Lignite or Furnance Oil is used as input in the thermal generator and the consumption is at the thermal power station. So the optimal reserve inventory is the reserve between the two systems, namely, the system that produces the input and the system that consumes the input obtained from the first system.

In this paper we consider a model in which the system is conceptualized. We consider ‘3’ machines. It is clear that the output of the first machine is the input for the second and third machine, and let us assume that, the consumption rate is constant.

The optimal size of the reserve inventory between two machines has been discussed by Hanssman (1962), which is the basic model. In that model, he assumed that, the idle time cost of second machine M_2 is very high, so a reserve in between M_1 and M_2 is suggested. If T is a random variable denoting idle time of M_2, it may be noted that

$$T = 0, \text{ if } t \leq \frac{s}{r}$$
$$= (t - \frac{s}{r}), \text{ if } t > \frac{s}{r}$$

The expected total cost due to inventory holding and idle time of M_2 per unit of time is

$$E(C) = hs + \frac{d}{\mu}\int (t - \frac{s}{t})g(t)dt,$$

Where $\frac{1}{\mu}$ denotes the number of breakdowns of M_1 per unit time. To obtain optimal 'S', we find $\frac{dE(C)}{ds} = 0$, i. e., $\frac{d}{ds}(hs + \frac{d}{\mu}\int(t - \frac{s}{t})g(t)dt) = 0$,

This gives the result that $G(\frac{s}{r}) = 1 - \frac{r\mu h}{d}$ on simplification. (1)

In this, the expression for optimal reserve 's' is given, has a limitation that, it should be less than unity, otherwise the solution is not feasible. Hence the new improved solution with no restriction was obtained by Sathyamoorthy. R. & Sachithanandhan, (2007), as follows:

$$E(C) = ht\int_0^{\frac{s}{r}}(\frac{s}{r} - t)g(t)dt + \frac{d}{\mu}\int_{\frac{s}{r}}^{\infty}(t - \frac{s}{r})g(t)dt$$

Then taking $\frac{dE(C)}{ds} = 0$, gives the expression for optimal 's' as

$$G(\frac{s}{r}) = \frac{d}{d + r\mu h} < 1, \text{ for all } r, \mu, h \text{ and } d$$

Where

h - cost of inventory holding per unit

d - idle time cost due to M_2 per unit time

r - constant rate of consumption of M_2

S - reserve inventory between M_1 and M_2

t - continuous random variable denoting the repair time of M_1 with $g(\cdot)$ as p. d. f and $G(\cdot)$ as c. d. f.

This solution is an improved version of (1) with no restrictions.

An extension of the above system is developed into '3' machines, in which second and third machines are parallel.

2. Basic Model

In this model, 3 machines M_1, M_2 and M_3 are considered and the optimal value of the reserve inventory between M_1 and M_2, M_3 for the system is discussed.

Notations: 1. h denote inventory holding costs.

2. r denote consumption rate.

3. d_1, d_2 denote the idle time costs.

4. t denote the repair time duration, which is random variable.

5. U the random variable denoting the inter arrival time between successive breakdown of M_1 which is taken as exponential with parameter μ.

6. S the reserve inventories between M_1 and M_2, M_3.

3. Results

If T, the random variable denoting the idle time of M_2, M_3 then, it may be noted that

$$T = 0, \text{ if } t \leq \frac{S}{r}$$
$$= (t - \frac{S}{r}), \text{ if } t > \frac{S}{r}$$

The average level of inventory is $\int_0^{\frac{s}{r}} (S - rt)g(t)dt$ and assuming that the rate of consumption of M_2 and M_3 is 'r' per unit of time. Where $\frac{1}{\mu}$, denotes the number of breakdowns of M_1 per unit time.

The expected total cost due to inventory holding and idle time per unit time is

$$E(C) = hr\int_0^{\frac{S}{r}} (\frac{S}{r} - t)g(t)dt + \frac{d_1}{\mu}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt + \frac{d_2}{\mu}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt$$

To find optimal 'S', we find $\frac{dE(C)}{ds} = 0$, i. e.,

$$\frac{dE(C)}{ds} = \frac{d}{ds}[hr\int_0^{\frac{S}{r}} (\frac{S}{r} - t)g(t)dt + \frac{d_1}{\mu}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt + \frac{d_2}{\mu}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt]$$

$$\frac{dE(C)}{ds} = hr\int_0^{\frac{S}{r}} \frac{1}{r}g(t)dt + \frac{d_1}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt\} + \frac{d_2}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty} (t - \frac{S}{r})g(t)dt\}$$

$$\frac{dE(C)}{ds} = \frac{hr}{r}g(\frac{S}{r}) + \frac{d_1}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty} [t - \frac{S}{r}]g(t)dt\} + \frac{d_2}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty} [t - \frac{S}{r}]g(t)dt\}$$

Reducing further, we get

$$\frac{dE(C)}{ds} = hg(\frac{S}{r}) - \frac{d_1}{\mu}\{\frac{1}{r}[\int_0^{\infty} g(t)dt - \int_0^{\frac{S}{r}} g(t)dt]\} - \frac{d_2}{\mu}\{\frac{1}{r}[\int_0^{\infty} g(t)dt - \int_0^{\frac{S}{r}} g(t)dt]\}$$

i. e.,

$$hg(\frac{S}{r}) - \frac{d_1}{\mu}(1 + g(\frac{S}{r})) - \frac{d_2}{\mu}(1 + g(\frac{S}{r})) = 0$$

Hence we get,

$$hg(\frac{S}{r}) - \frac{d_1 + d_2}{\mu}(1 + g(\frac{S}{r}))$$

$$G(\frac{S}{r}) = \frac{d_1 + d_2}{d_1 + d_2 + r\mu h} < 1, \text{ for all } r, h, \mu, d_1, \& \ d_2 \tag{2}$$

From the equations 2, we get the optimal reserve inventories for S, between the machines M_1 and M_2, M_3.

Let us prove the result for the system of 'n' machines in parallel by the method of mathematical induction hypothesis. Let us assume that the result is true for k.

Then the equation for the reserve inventory 'S' becomes,

$$G(\frac{S}{r}) = \frac{\Sigma_1^k d_i}{\Sigma_1^k d_i + r\mu h} < 1$$

Assume the result is true for i = 1,

$$G(\frac{S}{r}) = \frac{d_1}{d_1 + r\mu h} < 1$$

It is verified that it is true for $i = 1$.

Let us verify the result for $i + 1$,

$$E(C) = hg(\frac{S}{r}) - \frac{d_{i+1}}{\mu}\{\frac{1}{r}[\int_0^{\infty} g(t)dt - \int_0^{\frac{S}{r}} g(t)dt]\} - \frac{d_{i+2}}{\mu}\{\frac{1}{r}[\int_0^{\infty} g(t)dt - \int_0^{\frac{S}{r}} g(t)dt]\}$$

From this, the optimal reserve inventory $\frac{dE(C)}{ds}$ is obtained as,

$$G(\frac{s_{k+1}}{r_{k+1}}) = \frac{\Sigma_1^k d_{i+1}}{\Sigma_1^k d_{i+1} + r\mu h} < 1, \text{ for all } r, h, \mu \ \& \ d_{i+1}$$

Hence it is true for $i + 1$ also.

Therefore we concluded that this is true for all n. That is,

$$E(C) = \frac{hr}{r}g(\frac{S}{r}) + \frac{d_1}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\} + \frac{d_2}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\} + \frac{d_3}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\}$$

$$+\frac{d_4}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\} + ... + \frac{d_n}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\}$$

When differentiating the above equation,

$$\frac{dE(C)}{dt} = \frac{d}{ds}\{\frac{hr}{r}g(\frac{S}{r}) + \frac{d_1}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\} + \frac{d_2}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\} + ... + \frac{d_n}{\mu}\{\frac{d}{ds}\int_{\frac{S}{r}}^{\infty}[t - \frac{S}{r}]g(t)dt\}\}$$

Applying the same technique of the integral rule, we can conclude that the result obtained as.

$$G(\frac{s}{r}) = \frac{\Sigma_1^n d_{i+1}}{\Sigma_1^n d_{i+1} + r\mu h} \text{ for all values of } r, h, d_i \ (where\ i = 1,\ 2,\ 3,\ ..., n) \text{ and } \mu$$

4. Conclusion

In the above study, the optimal reserve inventory is obtained under general distributions of working and failure times. The solution is an improved version with no restrictions. Previous studies have discussed only the reserve inventory between machines in series only for two machines. But in this study, we discussed the parallel system of machines in addition to that, studied n machines in parallel. A generalized equation is obtained under this study. Further we can extend this study to, limited floor space for the reserve inventories.

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