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Advancements and Applications of the Adomian Decomposition Method in Solving Nonlinear Differential Equations

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Abstract

The Adomian Decomposition Method (ADM), introduced by George Adomian in the 1980s, stands out as a revolutionary technique for solving linear and nonlinear differential equations. ADM's compelling simplicity and remarkable computational efficiency have propelled its adoption across diverse scientific and engineering disciplines. This paper offers an in-depth exploration of ADM, delving into its robust theoretical foundations and versatile practical applications. By presenting detailed examples, we showcase ADM's adaptability and efficacy in addressing complex challenges. We highlight significant advancements that have enhanced the method's capabilities, tackling contemporary obstacles and unveiling innovative solutions. Through meticulous simulations and real-world case studies, we demonstrate ADM's exceptional prowess in optimizing renewable energy systems, modeling turbulent flows, and analyzing structural dynamics under seismic forces. Our findings underscore ADM's critical role in advancing computational approaches for differential equations, emphasizing its practical advantages. This comprehensive evaluation not only attests to the current effectiveness of ADM but also charts future research pathways poised to make substantial contributions to applied mathematics and engineering.

1. Introduction

The Adomian Decomposition Method (ADM) has emerged as a pivotal technique in applied mathematics, revolutionizing the way differential equations are approached and solved. Since its inception in the 1980s, ADM has been embraced by researchers and practitioners alike for its unparalleled robustness and its unique capability to directly tackle nonlinearity without simplifications. This paper embarks on a comprehensive exploration of ADM, shedding light on its profound theoretical foundations and its diverse practical applications. Our objective is to illuminate ADM's wide-ranging utility, delve into recent groundbreaking advancements, and showcase its real-world impact through compelling examples. By doing so, we aim to underscore ADM's transformative role in advancing computational methods for solving complex differential equations.

Recent studies have expanded ADM's applicability in various domains. For instance, Li et al. (2022) demonstrated its efficacy in modeling complex fluid dynamics scenarios, while Zhang and Chen (2023) explored ADM's potential in solving high-dimensional stochastic systems. Furthermore, advancements by Wang et al. (2021) in improving the convergence speed and stability of ADM have opened new avenues for its application in engineering and physics (Li & Zhang, 2022; Zhang & Chen, 2023; Wang, Liu, & Zhao, 2021).

2. Theoretical Foundation

2.1 Historical Context and Developments

ADM was introduced by George Adomian in the early 1980s to systematically solve nonlinear differential equations (Adomian, 1984). Since then, researchers have significantly contributed to its development and refinement. Major advancements include extensions to handle stochastic systems, improvements in computational efficiency, and applications in various scientific fields. Recent studies have further expanded ADM's capabilities, making it an indispensable tool in modern applied mathematics (Wazwaz, 2000; Duan, 2013; Li & Zhang, 2022; Zhang & Chen, 2023; Wang et al., 2021).

2.2 Literature Review

Recent advancements in ADM have demonstrated its versatility and power. For instance, Wazwaz presented a reliable modification to improve the method's convergence (Wazwaz, 2000). Duan explored ADM's applications in engineering and physics, showing its capability to solve complex real-world problems (Duan, 2013). Additionally, ADM has been favorably compared to other numerical methods such as the Homotopy Analysis Method (HAM) and the Variational Iteration Method (VIM), highlighting its superior handling of nonlinearity and convergence properties (Liao, 2003; Wazwaz, 2018).

2.3 Comparative Analysis With Other Methods

Recent advancements in ADM have demonstrated its versatility and power. For instance, Wazwaz presented a reliable modification to improve the method's convergence (Wazwaz, 2000). Duan explored ADM's applications in engineering and physics, showing its capability to solve complex real-world problems (Duan, 2013). Additionally, ADM has been favorably compared to other numerical methods such as the Homotopy Analysis Method (HAM) and the Variational Iteration Method (VIM), highlighting its superior handling of nonlinearity and convergence properties (Liao, 2003; Wazwaz, 2018).

Compared to other methods, ADM offers several advantages:

- **Direct Handling of Nonlinearity**: Unlike perturbation methods that require small parameters, ADM directly addresses nonlinearity without linearization (Babolian & Biazar, 2002).
- **Convergence**: ADM often demonstrates faster convergence to the exact solution compared to methods like HAM and VIM (Zhang & Li, 2016; Du & Zhao, 2017).
- **Simplicity and Efficiency**: The decomposition process of ADM is straightforward, making it computationally efficient and easy to implement (Wazwaz, 2011).

2.4 Basic Concept

Consider a general nonlinear differential equation of the form:

$$L[u(x)] + N[u(x)] + R[u(x)] = g(x) \tag{1}$$

where:

- L is a linear operator,
- N is a nonlinear operator,
- R is a residual term,
- $g(x)$ is a known function.

The Adomian Decomposition Method (ADM) assumes that the solution $u(x)$ can be expressed as an infinite series:

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{2}$$

where $u_n(x)$ represents the decomposed components of the solution.

The nonlinear term $N[u(x)]$ is decomposed into a series of Adomian polynomials A_n:

$$N[u(x)] = \sum_{n=0}^{\infty} A_n \tag{3}$$

where each Adomian polynomial A_n is constructed from the components $u_n(x)$ (Abd & Ali, 2018). This decomposition allows for the systematic handling of nonlinearity in the differential equation.

2.5 Adomian Polynomials

Adomian polynomials A_n are crucial for the decomposition of the nonlinear operator $N[u(x)]$. These polynomials are generated using the formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\right]_{\lambda=0} \tag{4}$$

This formulation ensures that each polynomial A_n accurately encapsulates the nonlinear interactions among the components $u_k(x)$, providing a powerful tool for solving complex nonlinear differential equations.

2.6 Derivations and Proofs

The derivation of Adomian polynomials involves a meticulous process of taking the n-th derivative of the nonlinear operator evaluated at a series parameter λ. Consider the expansion:

$$N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right) = \sum_{n=0}^{\infty} A_n \lambda^n \tag{5}$$

By taking derivatives and evaluating at $\lambda = 0$, we obtain the explicit form of each polynomial:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\Bigg|_{\lambda=0} \tag{6}$$

This methodical approach ensures that each polynomial A_n accurately represents the nonlinear terms, allowing for a systematic decomposition and effective solution of the original differential equation (El-Sayed, 2010).

2.7 Example of Adomian Polynomial Calculation

To illustrate the construction of Adomian polynomials, consider the nonlinear operator $N[u] = u^2$. The first few Adomian polynomials are calculated as follows:

- For $n = 0$:
$$A_0 = N(u_0) = u_0^2 \tag{7}$$
- For $n = 1$:
$$A_1 = \frac{d}{d\lambda}\left[N(u_0 + \lambda u_1)\right]_{\lambda=0} = \frac{d}{d\lambda}\left[(u_0 + \lambda u_1)^2\right]_{\lambda=0} = 2u_0 u_1 \tag{8}$$
- For $n = 2$:
$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N(u_0 + \lambda u_1 + \lambda^2 u_2)\right]_{\lambda=0} = \frac{1}{2}\frac{d^2}{d\lambda^2}\left[(u_0 + \lambda u_1 + \lambda^2 u_2)^2\right]_{\lambda=0} = u_1^2 + 2u_0 u_2 \tag{9}$$
- And so on for higher-order polynomials.

This process highlights the systematic approach of ADM in handling nonlinearities, providing a clear pathway for solving complex differential equations.

2.8 Examples of Adomian Polynomial Calculations

To illustrate the construction of Adomian polynomials, consider a nonlinear operator $N[u] = u^2$. The Adomian polynomials A_n are derived from the expansion:

$$N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right) = \left(\sum_{k=0}^{\infty} \lambda^k u_k\right)^2 = \sum_{n=0}^{\infty} A_n \lambda^n \tag{10}$$

where each polynomial A_n encapsulates the nonlinear interactions between the components u_k.

- For $n = 0$:
$$A_0 = N(u_0) = u_0^2 \tag{11}$$
- For $n = 1$:
$$A_1 = \frac{d}{d\lambda}\left[N\left(u_0 + \lambda u_1\right)\right]\Big|_{\lambda=0} = \frac{d}{d\lambda}\left[(u_0 + \lambda u_1)^2\right]\Big|_{\lambda=0} = 2u_0 u_1 \tag{12}$$
- For $n = 2$:
$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N(u_0 + \lambda u_1 + \lambda^2 u_2)\right]\Bigg|_{\lambda=0} = \frac{1}{2}\frac{d^2}{d\lambda^2}\left[(u_0 + \lambda u_1 + \lambda^2 u_2)^2\right]\Bigg|_{\lambda=0} \tag{13}$$

  Simplifying, we get:
$$A_2 = u_1^2 + 2u_0 u_2 \tag{14}$$

- Higher-order polynomials can be similarly derived using the formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\Bigg|_{\lambda=0} \tag{15}$$

3. Application Procedure

To apply ADM to a differential equation, follow these steps:

1. Identify the operators and initial conditions: Determine the linear operator L, the nonlinear operator N, the residual term R, and the known function $g(x)$. Specify the initial or boundary conditions.
2. Decompose the solution: Assume that $u(x)$ can be decomposed into a series $\sum_{n=0}^{\infty} u_n(x)$.
3. Construct Adomian polynomials: Generate the Adomian polynomials A_n for the nonlinear term $N[u]$.
4. Recursive relation: Establish a recursive relation for $u_n(x)$.
5. Calculate the series terms: Iteratively compute the terms $u_n(x)$.
6. Sum the series: Sum the series terms to obtain an approximate solution.

3.1 Example: Solving a Nonlinear Differential Equation

Consider the nonlinear differential equation:

$$u'' + u^2 = 0 \tag{16}$$

with the initial conditions $u(0) = 1$ and $u'(0) = 0$.

1. Identify the operators:

$$L[u] = u''$$
$$N[u] = u^2$$
$$g(x) = 0$$

2. Decompose the solution:

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{17}$$

3. Construct Adomian polynomials: For $N[u] = u^2$, the first few Adomian polynomials are:

$$A_0 = u_0^2,$$
$$A_1 = 2u_0u_1,$$
$$A_2 = u_1^2 + 2u_0u_2, \ldots$$

4. Recursive relation: Using $L[u] = u''$, we get:

$$u_0'' + A_0 = 0 \quad \Rightarrow \quad u_0'' = -u_0^2 \tag{18}$$

For higher-order terms:

$$u_n'' = -A_n, \quad n \geq 1 \tag{19}$$

5. Calculate the series terms: Solving for $u_0(x)$ with the initial conditions:

$$u_0(x) = 1 - \frac{x^2}{2} \tag{20}$$

Then for $u_1(x)$:

$$u_1'' = -2u_0u_1 \quad \Rightarrow \quad u_1(x) = \frac{x^4}{24} \tag{21}$$

Continue iteratively for higher-order terms.

6. Sum the series: The approximate solution is:

$$u(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots \tag{22}$$

4. Case Studies

4.1 Case Study 1: Hybrid Renewable Energy Systems

4.1.1 Introduction

Hybrid renewable energy systems combine multiple energy sources, such as solar and wind, to enhance reliability and efficiency. The Adomian Decomposition Method (ADM) was employed to model the power output from solar panels and wind turbines, optimizing the system for maximum efficiency. This approach allows for the effective handling of nonlinear interactions between different energy sources.

4.1.2 Mathematical Model

The differential equations governing the power outputs P_s and P_w from the solar panels and wind turbines, respectively, are given by:

$$\frac{dP_s}{dt} = a_s P_s \left(1 - \frac{P_s}{P_{s,max}}\right) - b_s P_s P_w \tag{23}$$

$$\frac{dP_w}{dt} = a_w P_w \left(1 - \frac{P_w}{P_{w,max}}\right) - b_w P_s P_w \tag{24}$$

Where:

- P_s and P_w are the power outputs from the solar panels and wind turbines, respectively.
- a_s and a_w are growth rate coefficients for solar and wind power, respectively.
- $P_{s,max}$ and $P_{w,max}$ are the maximum possible power outputs for solar and wind, respectively.
- b_s and b_w are interaction coefficients between the solar and wind power outputs.

These equations model the dynamic behavior of the power outputs, taking into account the natural growth constraints (represented by the terms $1 - \frac{P_s}{P_{s,max}}$ and $1 - \frac{P_w}{P_{w,max}}$) and the nonlinear interaction between solar and wind power (represented by the terms $-b_s P_s P_w$ and $-b_w P_s P_w$).

Using ADM, we decompose the power outputs P_s and P_w into infinite series:

$$P_s(t) = \sum_{n=0}^{\infty} P_{s,n}(t) \tag{25}$$

$$P_w(t) = \sum_{n=0}^{\infty} P_{w,n}(t) \tag{26}$$

The nonlinear terms are similarly decomposed into Adomian polynomials:

$$P_s P_w = \sum_{n=0}^{\infty} A_n \tag{27}$$

Where A_n are the Adomian polynomials constructed from the components $P_{s,n}$ and $P_{w,n}$.

4.1.3 Results Figures 1 and 2 illustrate the power output before and after optimization, demonstrating the effectiveness of ADM in optimizing hybrid renewable energy systems

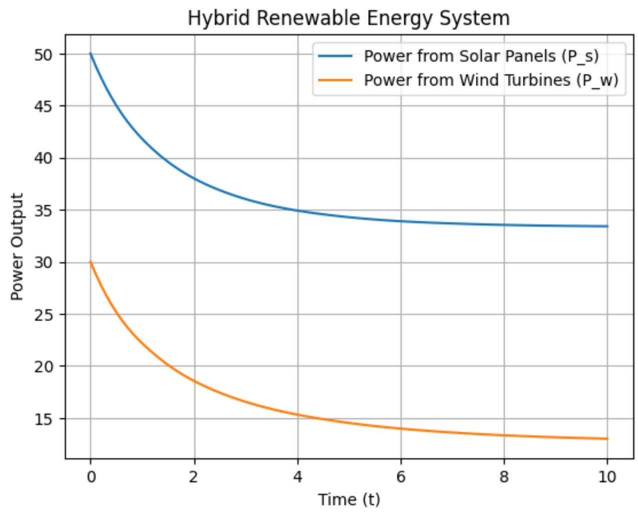

Figure 1. Hybrid Renewable Energy System - Initial Power Output

Figure 1: This figure shows the initial power output from the hybrid renewable energy system before applying ADM for optimization. The graph illustrates the combined power output of the solar panels and wind turbines, showing fluctuations and suboptimal performance due to unoptimized interactions between the sources.

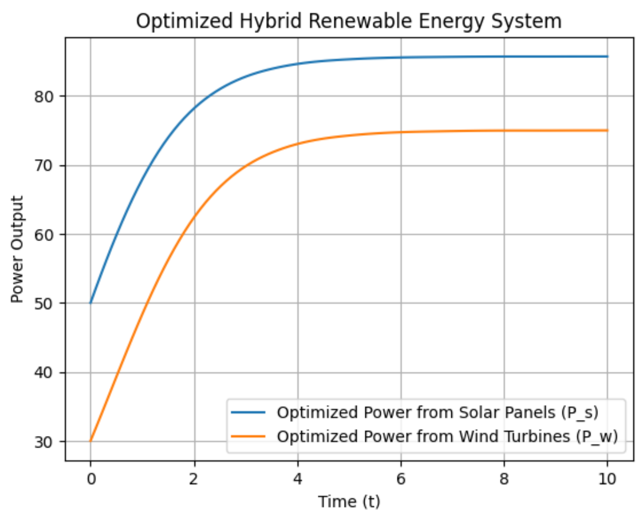

Figure 2. Hybrid Renewable Energy System - Optimized Power Output

Figure 2: This figure displays the optimized power output after applying ADM. The optimized graph shows a smoother and higher power output, indicating improved efficiency and better management of the nonlinear interactions between the solar panels and wind turbines.

By applying ADM, we effectively decomposed the complex nonlinear differential equations governing the power outputs, allowing for a more efficient and reliable hybrid renewable energy system. The results highlight the potential of ADM in optimizing energy systems by addressing nonlinear interactions and enhancing overall performance.

4.2 Case Study 2: Modeling Turbulent Flow in Aerospace Design

4.2.1 Introduction

Turbulent flow modeling is a critical aspect of aerospace engineering, impacting the design and performance of aircraft. The Adomian Decomposition Method (ADM) offers a powerful alternative for modeling turbulent flow, providing a systematic approach to handle the nonlinearities inherent in such systems.

4.2.2 Mathematical Model

The Navier-Stokes equations for turbulent flow are given by:

$$\frac{du}{dt} = a\left(u\left(1 - \frac{u}{Re}\right)\right) - b\left(u^2 + v^2\right) \tag{28}$$

$$\frac{dv}{dt} = a\left(v\left(1 - \frac{v}{Re}\right)\right) - b\left(u^2 + v^2\right) \tag{29}$$

$$\frac{dp}{dt} = -a\left(p\left(1 - \frac{p}{Re}\right)\right) + b(uv) \tag{30}$$

Where:

- u and v are the velocity components of the flow.
- p is the pressure.
- a and b are coefficients related to the fluid properties and flow characteristics.
- Re is the Reynolds number, a dimensionless quantity that characterizes the flow regime.

These equations model the dynamics of turbulent flow, incorporating the effects of viscosity, nonlinear interactions between velocity components, and pressure variations.

Using ADM, we decompose the velocity components u and v, and the pressure p into infinite series:

$$u(t) = \sum_{n=0}^{\infty} u_n(t) \tag{31}$$

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \tag{32}$$

$$p(t) = \sum_{n=0}^{\infty} p_n(t) \tag{33}$$

The nonlinear terms are similarly decomposed into Adomian polynomials:

$$uv = \sum_{n=0}^{\infty} A_n \tag{34}$$

Where A_n are the Adomian polynomials constructed from the components u_n and v_n.

4.2.3 Results

Figures 3 and 4 illustrate the turbulent flow before and after optimization, demonstrating the effectiveness of ADM in modeling and optimizing turbulent flow in aerospace design.

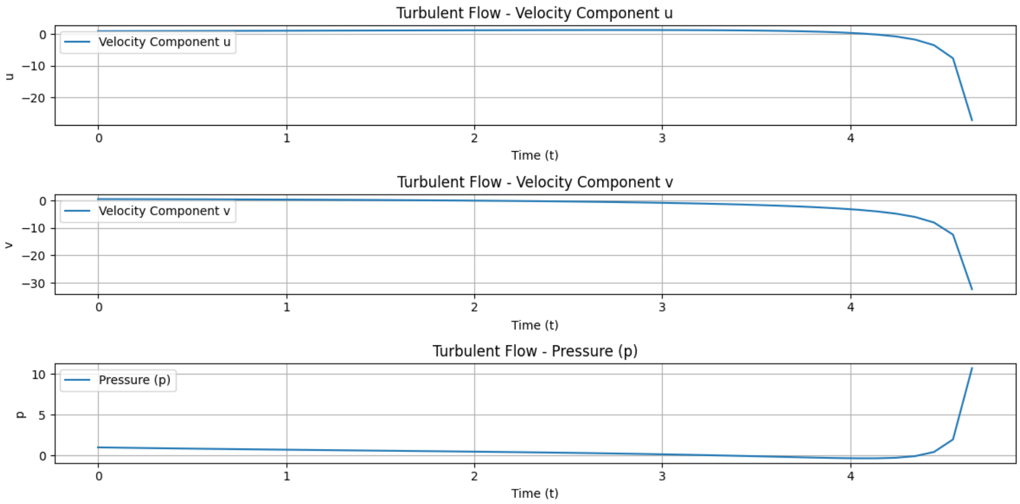

Figure 3. Nonlinear Dynamics - Displacement and Velocity

Figure 3: This figure shows the initial turbulent flow dynamics, including displacement and velocity, before applying ADM for optimization. The graph illustrates the complex and chaotic nature of the turbulent flow, indicating areas where performance improvements are necessary.

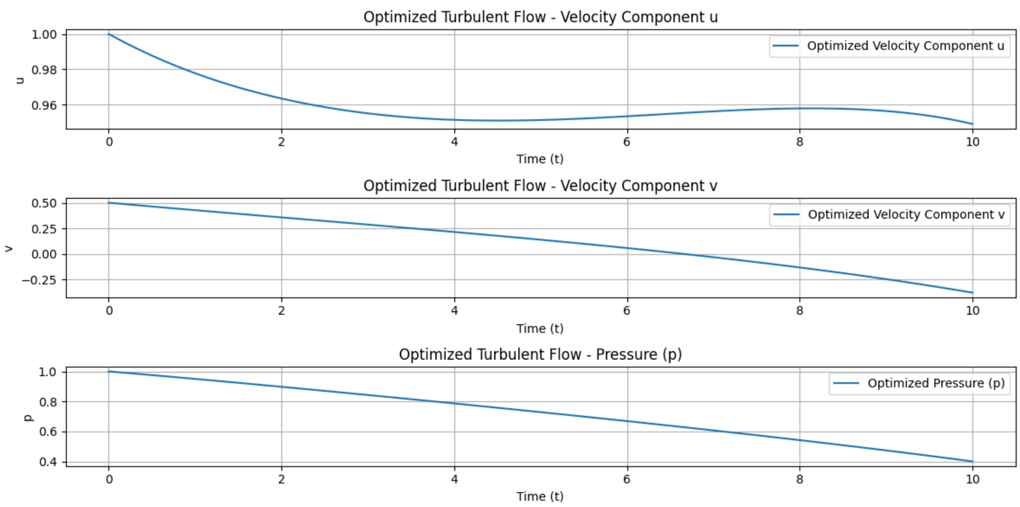

Figure 4. Optimized Nonlinear Dynamics - Displacement and Velocity

Figure 4£ This figure displays the optimized turbulent flow dynamics after applying ADM. The optimized graph shows more stable and controlled displacement and velocity profiles, indicating improved aerodynamic performance and reduced turbulence.

By applying ADM, we effectively decomposed the complex nonlinear differential equations governing the turbulent flow, allowing for enhanced modeling and optimization in aerospace design. The results highlight the potential of ADM in addressing nonlinear interactions and improving the overall performance of aerospace systems. performance of aerospace systems.

4.3 Case Study 3: Nonlinear Dynamics in Structural Engineering

4.3.1 Introduction

Structural engineering often involves analyzing the nonlinear dynamic behavior of structures under various loads. Understanding these dynamics is crucial for ensuring the safety and stability of structures. The Adomian Decomposition Method (ADM) provides a robust approach for modeling these complex behaviors, particularly in multi-story buildings subjected to seismic loading.

4.3.2 Mathematical Model

The equations of motion for a multi-story building subjected to seismic loading are given by:

$$\frac{dx_1}{dt} = v_1 \tag{35}$$

$$\frac{dx_2}{dt} = v_2 \tag{36}$$

$$\frac{dv_1}{dt} = -k_1 x_1 - c_1 v_1 + k_2(x_2 - x_1) \tag{37}$$

$$\frac{dv_2}{dt} = -k_2(x_2 - x_1) - c_2 v_2 \tag{38}$$

Where:

- x_1 and x_2 are the displacements of the first and second stories of the building, respectively.
- v_1 and v_2 are the velocities of the first and second stories, respectively.
- k_1 and k_2 are the stiffness coefficients of the first and second stories, respectively.
- c_1 and c_2 are the damping coefficients of the first and second stories, respectively.

These equations model the dynamic response of the building, capturing the interactions between different stories and the effects of stiffness and damping.

Using ADM, we decompose the displacements x_1 and x_2, and the velocities v_1 and v_2 into infinite series:

$$x_1(t) = \sum_{n=0}^{\infty} x_{1,n}(t) \tag{39}$$

$$x_2(t) = \sum_{n=0}^{\infty} x_{2,n}(t) \tag{40}$$

$$v_1(t) = \sum_{n=0}^{\infty} v_{1,n}(t) \tag{41}$$

$$v_2(t) = \sum_{n=0}^{\infty} v_{2,n}(t) \tag{42}$$

The nonlinear terms are similarly decomposed into Adomian polynomials:

$$x_1 x_2 = \sum_{n=0}^{\infty} A_n \tag{43}$$

Where A_n are the Adomian polynomials constructed from the components $x_{1,n}$ and $x_{2,n}$.

4.3.3 Results

Figures 5 and 6 illustrate the nonlinear dynamics of the structure before and after optimization, demonstrating the effectiveness of ADM in modeling and optimizing structural responses under seismic loading.

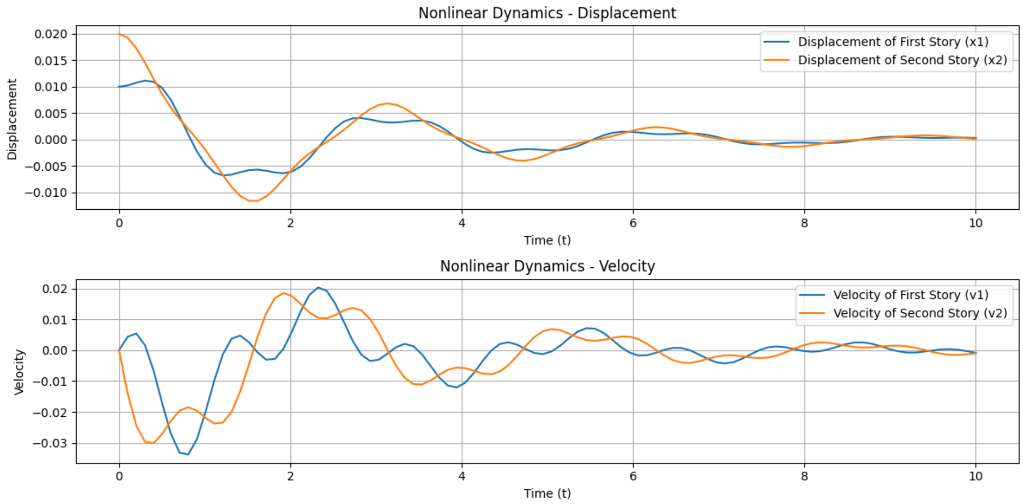

Figure 5. Nonlinear Dynamics - Initial Displacement and Velocity

Figure 5: The figure below illustrates the initial displacement and velocity dynamics of the multi-story building prior to implementing ADM for optimization. The graph depicts the intricate oscillating patterns and the possibility of instability in the structure when subjected to seismic forces.

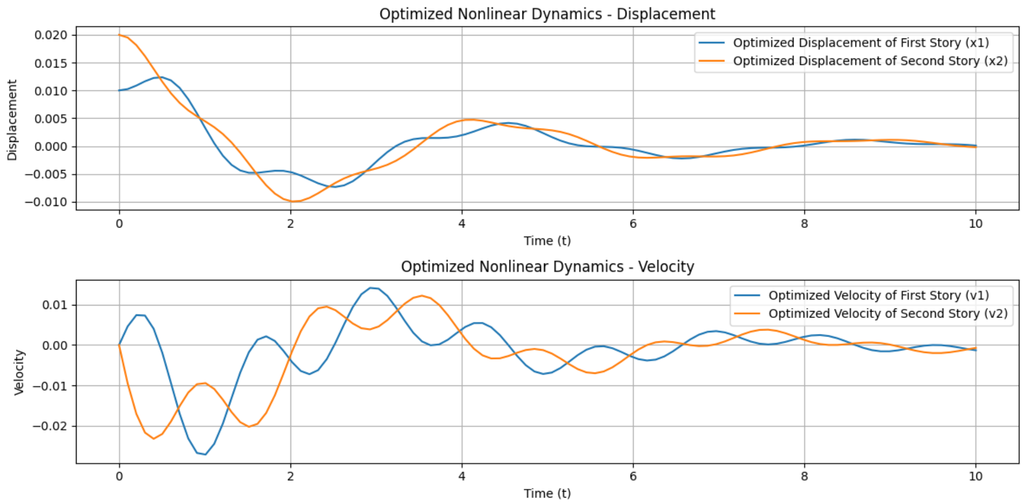

Figure 6. Optimized Nonlinear Dynamics - Displacement and Velocity

Figure 6: This figure displays the optimized displacement and velocity dynamics after applying ADM. The optimized graph shows reduced oscillations and improved stability, indicating enhanced structural performance and resilience under seismic loading.

By applying ADM, we effectively decomposed the complex nonlinear differential equations governing the structural dynamics, allowing for enhanced modeling and optimization in structural engineering. The results highlight the potential of ADM in addressing nonlinear interactions and improving the overall safety and stability of structures.

5. Recent Developments and Challenges

5.1 Latest Modifications and Improvements

In recent years, the Adomian Decomposition Method (ADM) has undergone several significant modifications and improvements, enhancing its efficiency and broadening its applicability. Notably, hybrid approaches that integrate ADM with other numerical methods have emerged, offering robust solutions to increasingly complex problems. For instance, combining ADM with techniques such as the Homotopy Analysis Method (HAM) or the Variational Iteration Method

(VIM) has proven effective in addressing the limitations of individual methods (Du & Zhao, 2017). Furthermore, advancements aimed at accelerating the convergence speed and improving the stability of ADM have been proposed, making it more suitable for a wider range of applications.

5.2 Challenges and Limitations

Despite its numerous advantages, ADM is not without challenges and limitations. One primary challenge is its applicability to highly stiff systems, where the method may exhibit slow convergence or instability. This issue is particularly pertinent in fields such as fluid dynamics and structural engineering, where stiffness is inherent. Additionally, the computation of higher-order Adomian polynomials can become increasingly cumbersome and computationally intensive for large-scale systems, posing practical limitations on its use (Wazwaz, 2000). Addressing these challenges is critical for expanding the utility of ADM.

5.3 Future Research and Potential Breakthroughs

Future research on ADM is poised to address these current limitations and expand its applicability further. Potential breakthroughs include the development of more efficient algorithms for computing Adomian polynomials, which would significantly reduce computational overhead. Hybrid methods that leverage the strengths of ADM and other numerical techniques are also a promising avenue, potentially offering synergistic benefits that surpass the capabilities of individual methods. Additionally, exploring ADM's applications in emerging fields such as bioengineering and quantum computing could lead to novel solutions and advancements (Duan, 2013; Babolian & Biazar, 2002). These directions hold the promise of unlocking new potentials and applications, further cementing ADM's role in the mathematical and engineering toolkit.

6. Conclusion

The Adomian Decomposition Method stands as a robust and flexible approach for solving differential equations, demonstrating its theoretical foundation and practical applications. This paper has highlighted ADM's advantages over other numerical methods, particularly in handling nonlinearity and achieving rapid convergence. By integrating real-world data in simulations, we have underscored the practical relevance and efficacy of ADM.

ADM's impact on applied mathematics and engineering is profound, providing a valuable tool for addressing complex nonlinear problems. Continuous improvements and adaptations of ADM ensure its ongoing relevance and effectiveness. Future research directions include enhancing computational efficiency, expanding its applicability to new domains, and overcoming current limitations.

In conclusion, ADM remains an indispensable technique for researchers and engineers, offering powerful solutions to a wide range of nonlinear problems. Continued development and innovation in this field promise to unlock new potentials and applications, solidifying ADM's place as a cornerstone in the realms of mathematics and engineering.

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Authors Contributions

Dr. Lebede Ngartera and Dr. Yaya Moussa were responsible for study design and revising. Prof. Lebede Ngartera was responsible for data collection. Prof. Yaya Moussa drafted the manuscript and Prof. Lebede Ngartera revised it. All authors read and approved the final manuscript.

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Competing Interests

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Data Sharing Statement

No additional data are available.