Journal of Mathematics Research; Vol. 16, No. 3; June 2024
ISSN 1916-9795 E-ISSN 1916-9809
Published by Canadian Center of Science and Education

# A New Method for Testing Whether a Number Is Prime

Bouchaïb Bahbouhi
Correspondence: Independent Resercher, France. E-mail: bahbouhi.orion.4710@gmail.com

Received: April 30, 2024 Accepted: June 7, 2024 Online Published: June 27, 2024
doi:10.5539/jmr.v16n3p28 URL: https://doi.org/10.5539/jmr.v16n3p28

**Abstract**

The present paper describes a new method for the decomposition of integer numbers into their prime factors. To know if a number is prime, the classic and oldest method is to perform a series of Euclidean divisions by the prime factors in increasing order. In this article, I report a new and alternative method to the classic one. It is based on the fact that an even number that precedes a prime number must in no case have a potential prime factor that lacks a unit. This even number with one unit before the number to be decomposed can then be used to prove its primality or not by looking if it has a factor missing one unit. To achieve this, we divide the even by the prime factors (p) in ascending order and follow the decimal part. If the latter is equal to a ratio of (p - 1)/p (p is any prime factor), then the number to be decomposed is not prime and the prime number giving the ratio is its prime factor. This method has the potential to have applications in computer science and to lead to a new algorithm for decomposing numbers or further improve the performance of those existing.

**Keywords**: decimal number, decomposition, even, factorization, irrational number, odd, primality test, prime factor

## 1. Introduction

The prime number remains a fascinating subject in mathematics. It has intrigued the greatest mathematicians in history since the Greek philosopher Euclid. But if we had to summarize the prime number in a sentence, we would say that its mystery lies in the fact that we cannot predict in advance whether a number is prime or not by a unique and unitary equation (Ndiaye, 2017). Illustrious mathematicians like Pierre de Fermat (died in 1665), Marin Mersenne (1588-1648), Leonhard Euler (1707-1783) tried to find the equation of the prime numbers but these latter do not predict them all.

However, it is the advent of computing, powerful computers that perform calculations at the speed of electrons, which made it possible to circumvent the mystery of prime numbers by creating algorithms for decomposing natural integers into product of prime factors in a few clicks on the keyboard although very huge numbers might be extremely hard to handle (Balasubramanian & Abbas, 2018).

The oldest idea to find prime numbers was developed by Eratosthenes of Cyrene or what we call in mathematics, the sieve of Eratosthenes, which is an ancient algorithm for finding all prime numbers up to any given limit. On the other hand, in arithmetic, the successive division method is the simplest and oldest method for determining whether a natural number is prime or whether it is composite, to find its decomposition into the product of prime factors. This successive division method involves trying to divide an integer n by each prime number less than it, starting with 2, then 3, then 5, and so on. In practice we will only have to try this division up to the first prime number less than $\sqrt{n}$. As soon as a divisibility test passes, a prime factor of n has been found, we can already conclude that n is composite. We can continue the method by successive iterations to obtain the complete factorization of n (Sahin, 2011).

This article proposes a calculation method to both predict whether a number is prime or not and decompose it into prime factors. The method consists of analyzing decimal fractions.

## 2. Objective of the Study

The natural integers succeed one another starting from 3 by reproducing the same arithmetic sequence of 7 terms as shown in table 1 below up to 99 but this is true at infinity. This shows that the prime numbers (P) are always framed by two even numbers and two twin prime numbers always located on the same row of the table. The multiples of prime numbers (M) are also located on the same columns as the prime numbers and are in turn framed by an even before and after except those of 2 and 3 which are located in separate columns. All P or M are formulated by the same equations $6x + 1$ or $6x – 1$ (example $13 = 6 x 2 + 1$; $17 = 6 x 3 – 1$; $25 = 5^2 = 6 x 4 + 1$; $35 = 5 x 7 = 6 x 6 – 1$). How then do we know if P or M is going to follow an even?

The fact that a prime number is preceded by an even number is the guiding idea of this study. This rises the question to know whether the even located before P or M numbers would help us to predict in advance whether the number which will follow is prime or not. To my best knowledge, this article investigates this idea for the first time and materializes it in the form of a practical method.

Table 1. The integer numbers starting with 3 form a sequence of 7 positions to infinity

| 3n | Even (2n) | P or M** | Even (2 x 3n) | P or M** | Even (2n) | 3n |
|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| 45 | 46 | 47 | 48 | 49 | 50 | 51 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 |
| 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 69 | 70 | 71 | 72 | 73 | 74 | 75 |
| 75 | 76 | 77 | 78 | 79 | 80 | 81 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 |
| 87 | 88 | 89 | 90 | 91 | 92 | 93 |
| 93 | 94 | 95 | 96 | 97 | 98 | 99 |

Note. P designates a prime number and M a product of prime factors. 2n and 3n are multiples of 2 and 3, respectively. P are underlined

## 3. Results

### *3.1 Theorem*

"Let there be an odd number $O_n$, it is always preceded by an even number noted $p_r$ (table 1). Let there be two prime factors q and r. If $p_r = qr + (q - 1)$ or $p_r = qr + (r - 1)$, then $O_n$ following it is not prime".

- If $p_r = qr + (q – 1)$ then $O_n = p_r + 1 = q \times (r + 1)$ $\rightarrow$ $O_n$ not prime.. The same reasoning applies for

  $p_r = qr + (r – 1)$.

"This rule is true for all even numbers preceding odd numbers regardless of the number of their prime factors. Let suppose an even $p_r$ number to have as many prime factors as it might be possible: $q_1, q_2, q_3, q_4, q_5...q_n$, if one of these factors is missing one unit such that $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times..q_n + ( q_1 – 1)$ or $p_r = q_1 \times q_2, \times q_3 \times q_4 \times q_5 \times..q_n + (q_2 – 1)$ or $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times..q_n + (q_3 – 1)$...or $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times...q_n + (q_n – 1)$, then the odd number that follows is not prime and the factor missing the one unit in the even $p_r$ is one prime factor of the odd number that follows".

- If $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times..q_n + ( q_1 – 1)$ then $O_n = p_r + 1 = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times..q_n + \mathbf{q_1} = \mathbf{q_1 \times (q_2} \times q_3 \times q_4 \times q_5 \times..q_n \mathbf{+ 1)}$ $\rightarrow$ $O_n$ not prime. The same reasoning applies for $p_r = q_1 \times q_2, \times q_3 \times q_4 \times q_5 \times..q_n + (\mathbf{q_2 – 1})$ or $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times..q_n + (\mathbf{q_3 – 1})$...or $p_r = q_1 \times q_2 \times q_3 \times q_4 \times q_5 \times...q_n + (\mathbf{q_n – 1})$.
- If $p_r$ has no factor missing a unit then $O_n$ has to be prime. To confirm this hypothesis, there is a need for a calculation method that can show whether or not $p_r$ has a factor missing by one unit. This article therefore aims to develop a calculation method to achieve this objective..

Only biprime numbers (q x r) are considered in the whole study for the simple reason that these numbers are the most difficult to decompose, especially when they are larger and their factors are close to each other but far enough from their square roots. Here are some examples with simple numbers to clearly illustrate the Theorem cited above.

The number $48 = 2^4 \times 3 = 7 \times 6 + (7 – 1)$ so it is in equation $p_r = qr + (r – 1)$, it cannot be followed by a prime number, and indeed $49 = 48 + 1 = 7 \times 7$.

The number $120 = (10 \times 11) + (11 – 1)$ so it is in equation $p_r = qr + (r – 1)$, it cannot be followed by a prime number, and indeed $121 = 120 + 1 = 11 \times 11$.

The number $46 = 2 \times 23$ and this number cannot be in equation $p_r = qr + (r – 1)$ whatever the prime factor. For example

46 = (7 x 6) + (7 – 3), 46 = (11 x 4) + (11 – 9), but never in equation $p_r = qr + (r-1)$. The number which follows it 47 = 46 + 1 is therefore prime.

The number 96 = $2^5$ x 3 which cannot be written in any way as an equation $p_r = qr + (r-1)$. 96 = (7 x 13) + (7 – 2) or 96 = (11 x 8) + (11 – 3) but we will never find a prime factor missing by one unit. The number which follows it 97 = 96 + 1 is certainly prime.

*3.2 Locate Prime Numbers Using a New Method Based on the Equation $p_r = qr + (r-1)$:*

"If we want to know if an odd number is prime, we will divide the even number which precedes it by the prime factors in increasing order and only look at the decimal part (at least two digits after the decimal point) of the decimal fraction obtained. If it indicates that it has a factor is missing a unit (decimal part > 0.90) then the odd number following it is not prime". Let's look at small numbers before discussing the limits of the method at infinity. As a reminder, for any biprime number, its square root is always between its smaller and its larger prime factors. Not only this, but the value of the ratio between the large factor and the square root is the same as that between the square root and the small factor. We could look for the prime factor of a number to be decomposed below or beyond the square root, but here, we follow the first procedure starting with 7 (knowing that multiples of 2, 3, and 5 are easily recognized by their digits).

*3.3 Some Specific Examples of Small Numbers to Illustrate the Practical Implementation of the Method*

We start with the number 4 171 as a first example and therefore, we will analyze the even number which precedes it by one unit, that is to say 4 070 (table 2).

Table 2. Primality test of the number 4 171

| Prime factors (P) | 4 170/P | Divisor or not |
|---|---|---|
| 7 | 595.71 | Not |
| 11 | 379.09 | Not |
| 13 | 320.76 | Not |
| 17 | 245.17 | Not |
| 19 | 219.47 | Not |
| 23 | 181.3 | Not |
| 29 | 143.79 | Not |
| 31 | 134.51 | Not |
| 37 | 112.7 | Not |
| 41 | 101.71 | Not |
| 43 | 96.97 | 4171/43 = 97<br>4171 = 43 x 97 |
| 47 | 88.72 | Not |
| 53 | 78.67 | Not |
| 58 | 71.89 | Not |
| 61 | 68.36 | Not |

We see that 4 170/43 = 96.97 and therefore 4 170 = (43 x 96) + (43 – 1), the number 4 170 contains a factor 43 which is missing one unit and this is why the number 4 171 is not a prime number. This factor missing by one unit will give the largest decimal part > 0.90 and which corresponds to 42/43 = 0.97. This method requires the examination of decimal fractions between the even placed one unit before the number to factorize (numerator) and the prime factors in ascending order (denominator); then the search for decimal parts > 90.

Here is another example. Is the number 67 180 097 prime ? Its square root ≈ 8 196 (table 3).

Table 3. Primality Test of the number 67 180 097

| Prime factors (P) | 67 180 096/P | Divisor or not |
|---|---|---|
| 7 | 9 597 156.57 | Not |
| 11 | 6 107 281.45 | Not |
| 13 | 5 167 699.69 | Not |
| 17 | 3 951 770.35 | Not |
| 19 | 3 535 794.52 | Not |
| 23 | 2 920 873.73 | Not |
| 29 | 2 316 555.03 | Not |
| 31 | 2 167 099.87 | Not |
| 37 | 1 815 678.27 | Not |
| 41 | 1 638 538.92 | Not** |
| 43 | 1 562 327.81 | Not |
| 47 | 1 429 363.74 | Not |
| 53 | 1 267 548.98 | 67 180 097/53 = 1 267 549 |
| | | 67 180 097 = 53 x 1 267 549 |

In table 3, the prime factor 41 gives a decimal part 0.92 > 0.9 but when we carry out the Euclidean division we have 67 180 096 : 41 = 1 638 538 and the remainder of the division = 38 so this is not a prime factor missing just one but 3 units (41 - 38 = 3). On the contrary, 67 180 096 : 53 = 1 267 548 with the remainder of the division = 52 and therefore it is the prime factor missing a single unit, we can therefore conclude that 67 180 097 is not prime and has 53 as a prime factor.

Here is an example with the number 8 387 (square root ≈ 91) (table 4).

Table 4. Primality test of the number 8 387

| Prime factors (P) | 8 386/P | Divisor or not |
|---|---|---|
| 7 | 1198 | Not |
| 11 | 762.36 | Not |
| 13 | 645.07 | Not |
| 17 | 487.41 | Not |
| 19 | 441.36 | Not |
| 23 | 364.6 | Not |
| 29 | 289.17 | Not |
| 31 | 270.51 | Not |
| 37 | 226.64 | Not |
| 41 | 204.53 | Not |
| 43 | 195.02 | Not |
| 47 | 178.42 | Not |
| 53 | 158.22 | Not |
| 61 | 137.48 | Not |
| 67 | 125.16 | Not |
| 71 | 118.11 | Not |
| 73 | 114.87 | Not |
| 79 | 106.15 | Not |
| 83 | 101.03 | Not |
| 89 | 94.22 | Not |

In no case do we find a decimal part > 0.9 of the order of 0.96; 0.97; 0.98 or 0.99 which suggests that the number 8 387 is prime because we do not find prime factors missing a unit in the even 8 376 which precedes it. Let's take the number 877 in table 5 (square root ≈ 29) by analyzing the even number that precedes it:

Table 5. Primality test of the number 877

| Prime factor (P) | 876/P ratio | Divisor or not |
|---|---|---|
| 7 | 125.14 | Not |
| 11 | 79.63 | Not |
| 13 | 67.38 | Not |
| 17 | 51.52 | Not |
| 19 | 46.1 | Not |
| 23 | 30.2 | Not |

We therefore conclude that 877 is prime.

Let's choose other examples like the number 22 193 in table 6 (square root ≈ 148) by analyzing the even number 22 192 before it.

Table 6. Primality test of the number 22 193

| Prime factor (P) | 22 192/P | Divisor or not |
|---|---|---|
| 7 | 3170.28 | Not |
| 11 | 2017.45 | Not |
| 13 | 1707.07 | Not |
| 17 | 1305.41 | Not |
| 19 | 1168 | Not |
| 23 | 964.86 | Not |
| 29 | 765.24 | Not |
| 31 | 715.87 | Not |
| 37 | 599.78 | Not |
| 41 | 541.26 | Not |
| 43 | 516.09 | Not |
| 47 | 472.17 | Not |
| 53 | 418.71 | Not |
| 61 | 363.8 | Not |
| 67 | 331.22 | Not |
| 71 | 312.56 | Not |
| 73 | 304 | Not |
| 79 | 280.91 | Not |
| 83 | 267.37 | Not |
| 89 | 249.34 | Not |
| 91 | 243.86 | Not |
| 97 | 228.78 | Not |
| 101 | 219.72 | Not |
| 103 | 215.45 | Not |
| 107 | 207.4 | Not |
| 109 | 203.59 | Not |
| 113 | 196.38 | Not |
| 127 | 174.74 | Not |
| 131 | 169.4 | Not |
| 137 | 161.98 | Not |
| 139 | 159.65 | Not |

For the number 22 193, we have two prime factors which give a decimal part > 0.9. Firstly the factor 79 but the calculation shows that 22 192 = (79 x 280) + 72 or (79 x 280) + (79 – 7), the factor 79 is therefore missing 7 units and not just one. The number 137 produces the highest decimal part of the order of 0.98 but the calculation shows that 22 192 = (137 x 161) + 135 or (137 x 161) + (137 – 2), the factor 137 is missing of two units and not just one. When we have such a high decimal part that gives a factor missing by more than a single unit, we can quickly conclude that the number is prime if it is the highest decimal part. The number 22 193 is therefore prime.

Let's take another example, the number 26 237 in table 7 (square root ≈ 161).

Table 7. Primality test of the number 26 237

| Prime factor (P) | 26 236/P | Divisor or not |
|---|---|---|
| 7 | 3748 | Not |
| 11 | 2 385.09 | Not |
| 13 | 2 018.15 | Not |
| 17 | 1 543.29 | Not |
| 19 | 1 380.84 | Not |
| 23 | 1 140.69 | Not |
| 29 | 904.68 | Not |
| 31 | 846.32 | Not |
| 37 | 709.08 | Not |
| 41 | 639.9 | Not |
| 43 | 610.13 | Not |
| 47 | 558.21 | Not |
| 53 | 495.01 | Not |
| 61 | 430.09 | Not |
| 67 | 391.58 | Not |
| 71 | 369.52 | Not |
| 73 | 359.39 | Not |
| 79 | 332.1 | Not |
| 83 | 316.09 | Not |
| 89 | 294.78 | Not |
| 91 | 288.3 | Not |
| 97 | 270.47 | Not |
| 101 | 259.76 | Not |
| 103 | 254.71 | Not |
| 107 | 245.19 | Not |
| 109 | 240.69 | Not |
| 113 | 232.17 | Not |
| 127 | 206.58 | Not |
| 131 | 200.27 | Not |
| 137 | 191.5 | Not |
| 139 | 188.74 | Not |
| 149 | 176.08 | Not |
| 151 | 173.74 | Not |
| 157 | 167.1 | Not |

The prime factor 41 produces a decimal part of the order of 0.9 but the Euclidean division of26 236: 41 = (41 x 639) + 37 or (41 x 639) + (41 – 4) therefore the factor 41 is missing of 4 units and therefore 26 237 is prime. Here is a final example in table 8 of the number, 25 637 (square root ≈ 160). We analyze the even which precedes it 25 636.

Table 8. Primality test of the number 25 637

| Prime factor (P) | 25 636/P | Divisor or not |
|---|---|---|
| 7 | 3662.28 | Not |
| 11 | 2330.54 | Not |
| 13 | 1972 | Not |
| 17 | 1508 | Not |
| 19 | 1349.26 | Not |
| 23 | 1114.6 | Not |
| 29 | 884 | Not |
| 31 | 826.96 | 25 637 : 31 = 827 |
| 37 | 692.86 | Not |
| 41 | 625.26 | Not |
| 43 | 596.18 | Not |
| 47 | 545.44 | Not |
| 53 | 483.69 | Not |
| 61 | 420.26 | Not |
| 67 | 382.62 | Not |
| 71 | 361.07 | Not |
| 73 | 351.17 | Not |
| 79 | 324.5 | Not |
| 83 | 308.86 | Not |
| 89 | 288.04 | Not |
| 91 | 281.71 | Not |
| 97 | 264.28 | Not |
| 101 | 253.82 | Not |
| 103 | 248.89 | Not |
| 107 | 239.58 | Not |
| 109 | 235.19 | Not |
| 113 | 226.86 | Not |
| 127 | 201.85 | Not |
| 131 | 195.69 | Not |
| 137 | 187.12 | Not |
| 139 | 184.43 | Not |
| 149 | 172.05 | Not |
| 151 | 169.77 | Not |
| 157 | 163.28 | Not |

We see that 31 is the missing factor of a unit in the even 25 636 and therefore it is a prime factor of the number 25 637 which is indeed biprime and therefore 25 637 = 31 x 827. We could stop at 31 but we continued the analysis to show that only prime factors missing a unit or closer produce decimal parts > 0.9 often on the order of 0.96, 0.97, 0.98 or even 0.99.

### *3.4 The Application of the Method With Larger Numbers That Tend Towards Infinity (+∞)*

Let us put an even number $p_r$ preceding a biprime number $b_n$ = p x q. Let us put the even $p_r = b_n – 1$. Therefore, p and q are the prime factors of $b_n$ and so let us say that p' or q' are any other prime numbers that do not divide $b_n$.

During the process of serial divisions, we have two possibilities: either the number $p_r$ is divided by a prime number which is the factor of $b_n$ (i.e. p or q), or by a prime number which is not the factor of bn (p' or q'). In both cases we have a classic

Euclidean division with a remainder. However, the resulting decimal parts do not have the same value. In fact, $p_r$ divided by p or q gives the largest remainder compared to all other prime numbers, and therefore when divided by p' or q', the remainder is lower. In the first case, the remainder is p – 1 or q – 1 while in the latter it is < p' – 1 or q' – 1. In other words, if $p_r = tp + r$ with t the quotient and r the remainder, then $p_r = t'p' + r'$. Thus $p_r/p = t + r/p$ and $p_r/p' = t' + r'/p'$ with r/p and r'/p' the remainders. Because r = p – 1 and r' < p' – 1, $r/p \rightarrow 1$. That is the reason why the decimal fraction with $p_r$ when divided by prime factors p ot q is higher and the closest to 1 therefore containing 9 digits just after the decimal separator.

The other case where the decimal part will vary is when $b_n$ becomes larger and larger. Let us focus on increasingly larger biprime numbers having their two prime factors increasing and exclude the case of larger numbers which are multiples of several small prime factors. The only key element to remember is that all the even numbers preceding by one unit these semi-prime (or biprime) numbers give a remainder = p – 1 (or q – 1) when they are divided by the prime factors of the odd that follows them which we designate by p or q (knowing that in reality each biprime number has its own).

Because $b_n$ is biprime, p or q is missing a unit in the even $p_r$ that precedes it. Therefore, $p_r = pq – 1$ and so if we divide $p_r$ by either p or q we will have only two possible cases :

- $p_r/q = (pq – 1)/q = p\cancel{q}/\cancel{q} - 1/q = p – 1/q$.
- Or following the same reasoning $p_r/p = q - 1/p$.
- Let S = p - 1/q = p + **(0)** – 1/q = p + **( – 1 + 1)** – 1/q = (p - 1) + (1 - 1/q). S is always decimal or irrational and will therefore have an integer part = p - 1 and a decimal part = 1 – 1/q and given that p or q > 5 (we start with 7 during successive divisions) so 0 < 1/q < 1). Likewise let us set S' = q - 1/p = (q - 1) + (1 - 1/p). Like S, S' also has an integer part = q - 1 and a decimal part = 1 – 1 /p.

Therefore, if $q \rightarrow +\infty$, $1/q \rightarrow 0$ and therefore $(1 – 1/q) \rightarrow 1$. Hence $S \rightarrow (p – 1 + 1)$ or $S \rightarrow p$. Reciprocally, if $p \rightarrow +\infty$, $1/p \rightarrow 0$ and therefore $(1 – 1/p) \rightarrow 1$. Hence $S' \rightarrow q$. This means that the more the prime factors are larger, the more the decimal fraction will tend to 1 and thus there would be more and more 9 digits after the decimal separator. The frequency of the digit 9 in the decimal parts of both S and S' would be $\rightarrow +\infty$ as the prime factors p or $q \rightarrow +\infty$.

Let two biprime numbers be: $b_n$ = p x q and $b_n'$ = t x t'. Let us name the evens which precede them by a single unit $p_r$ and $p_r'$, respectively. Let us assume for simplicity that $b_n' > b_n$ and that t > p, t > q and t' > p and t' > q. Then $p_r/p$ or $p_r/q$ give decimal parts containing fewer digits 9 just after the decimal point than $p_r'/t$ or $p_r'/t'$.

In Table 9 below, we have increasingly larger biprime numbers = p x p'. In agreement with the demonstration detailed above, when we divide the evens placed one unit before the biprime numbers with their increasingly larger prime factors p or p', we see clearly that we obtain decimal numbers whose integer part is close to p - 1 or p' - 1 then which decimal part tends towards 1 as digits 9 continues to increase in the decimal part (as we tend towards infinity).

The number of digits 9 in the decimal part increases with larger and larger biprime numbers

| p | p' | N = p x p' | N – 1 | (N – 1)/p | (N – 1)/ p' | (p – 1)/p | (p'– 1)/p' |
|---|---|---|---|---|---|---|---|
| 2591 | 4391 | 11377081 | 11377080 | 4390.**999** | 2590.**999** | 0**.999**6140486 | 0**.999**7722614 |
| 5813 | 7669 | 44579897 | 44579896 | 7668.**999** | 5812.**999** | 0**.999**8279718 | 0**.999**8696049 |
| 58979 | 63361 | 3736968419 | 3736968418 | 63360.**9999** | 58978.**9999** | 0**.9999**830448 | 0**.9999**842174 |
| 74869 | 98419 | 7368532111 | 7368532110 | 98418.**9999** | 74868.**99999** | 0**.9999**866433 | 0**.9999**898394 |
| 100411 | 788261 | 79150075271 | 79150075270 | 788260.**99999** | 100410.**999999** | 0**.99999**00409 | 0**.99999**87314 |
| 653707 | 856813 | 560104655791 | 560104655790 | 856812.**99999** | 653706.**999999** | 0**.99999**84703 | 0**.99999**88329 |
| 6112471 | 9745847 | 59571207157937 | 59571207157936 | 9745846.**999999** | 6112470.**999999** | 0**.999999**8364 | 0**.999999**8974 |
| 2337667 | 6543739 | 15297082716913 | 15297082716912 | 6543738.**999999** | 2337666.**999999** | 0**.999999**5722 | 0**.999999**8472 |

Note. p and p' are prime factors of the biprime numbers N = p x p'. The even preceding each number N is N – 1

To better illustrate this with more examples, here are more larger numbers in the following.

q = 597841214791

q' = 687451230299

M = q x q' = 410986678631521666152509

M – 1 = 410986678631521666152508

M – 1/q = 687451230298.**99999999999**

M – 1 /q' = 597841214790.**999999999999**

r = 711257897893

r' = 911457126829

L = r x r' = 648281080047988032871297

L – 1= 648281080047988032871296

L – 1/r = 911457126828.**999999999999**

L – 1/r' = 711257897892.**999999999999**

The method is not restricted to biprime or semiprime numbers which are only chosen out of the need to simplify and be clearer. It is applicable to any number regardless of the number of its prime factors. I give an example below:

Let S = M x L = q x q' x r x r' (same prime factors as above)

S = 266434887908578230020582617285348851925970634173

S – 1 = 266434887908578230020582617285348851925970634172

Then

S – 1/q = 445661626058553875157001976173827802.**99999999999**

S – 1/q' = 387569148421910678147206638235753926.**99999999999**

S – 1/r = 374596737270480307423628792069563960.**99999999999**

S – 1/r' = 292317521105482042195518480487763536.**99999999999**

Here is a number containing 34 digits, the longest that can be used depending on the resources allocated to this study.

5643479997656899909896887654211013 = 17 × 113 × 2937782403777667834407541725253

5643479997656899909896887654211012/17 = 331969411626876465288052214953588.**941176470588235**

(16/17 = **0.941176470588235**).

5643479997656899909896887654211012/113 = 49942300864220353184928209329300.9911504425 (112/113 = **0.9911504425**)

5643479997656899909896887654211012/2937782403777667834407541725253 = 1920.**999999999999999999999999999997**

(2937782403777667834407541725252/2937782403777667834407541725253 = 9.**999999999999999999999999999997**).

We can also see that the larger the number and the larger the prime factor, the number of digits 9 increases proportionally in the decimal part obtained with the fraction whose numerator is the even preceding the number to be decomposed by a unit, and the denominator being prime factor of this number.

We can clearly see that the method is very selective and specific to infinity provided that we can read a sufficient length of the decimal parts of the irrational numbers (p - 1)/p to be able to count all the digits 9. If we divide the even number $p_r$ which precedes a biprime number $b_n$, whether small or very large, by prime numbers which are not prime factors of $b_n$, then the decimal part of the irrational numbers obtained will either be devoid of the digits 9 or contain much less. It is not a question of the size of the numbers that limits the method, the number $p_r$ will always give the most digits 9 when divided by the prime factors of $b_n$. Below two examples where we see that $p_r = b_n – 1$ gives decimal parts with many 9 digits when divided by the *right prime factor.* In contrast, when divided by *prime factors that differ from the right one by few units*, the 9 digits almost disappear from the decimal part.

$b_n$ = 304601108240414762378363 = 678198349291 x 449132777393

$b_n$ – 1/ 4491327773**93** = 678198349290.**99999999999**777348693

$b_n$ –1 / 4491327773**99** = 678198349281.**939895058845**7761824

bn = 776364342355634237292007 = 851885978729 x 911347717583

bn – 1/ 8518859787**29** = 911347717582.**99999999999**882613398

bn – 1/ 8518859787**31** = 911347717580.**860399771009**08188372

*3.5 Decimal Parts of Prime Factors Between 7 and 97*

Now that the principle of the method has been clarified, we might consider its use in a programmable mode, that is to say as soon as a specific decimal part is detected, the system checks whether the number is indeed the prime factor. Let p be a prime factor, we then calculate (p **–** 1)/p (or 1 – 1/p). This ratio is then determined in advance for each prime factor. Each ratio is a specific irrational number < 1. Thus, we could potentially have an algorithm that calculates the decimal fraction between the even placed one unit before the odd to be decomposed and the prime factors in increasing order. As soon as a decimal part of a calculated fraction orresponds to a (p – 1)/p, the system switches to Euclidean division to decompose the number. Here are these decimal parts for prime factors between 7 and 97 (table 10)

Table 10. Decimal parts (p - 1)/p of the smallest prime numbers between 7 and 97

| Prime factors (p) | (p - 1)/p ratio | Prime factors (p) | (p - 1)/p |
|---|---|---|---|
| 7 | 6/7 = 0.8571 | 53 | 52/53 = 0.9811 |
| 11 | 10/11 = 0.9090 | 59 | 58/59 = 0.9830 |
| 13 | 12/13 = 0.9230 | 61 | 60/61 = 0.9836 |
| 17 | 16/17 = 0.9411 | 67 | 66/67 = 0.9850 |
| 19 | 18/19 = 0.9473 | 71 | 70/71 = 0.9859 |
| 23 | 22/23 = 0.9565 | 73 | 72/73 = 0.9863 |
| 29 | 28/29 = 0.9655 | 79 | 78/79 = 0.9873 |
| 31 | 30/31 = 0.9677 | 83 | 82/83 = 0.9879 |
| 37 | 36/37 = 0.9729 | 89 | 88/89 = 0.9887 |
| 41 | 40/41 = 0.9756 | 91 | 90/91 = 0.9890 |
| 43 | 42/43 = 0.9767 | 97 | 96/97 = 0.9896 |
| 47 | 46/47 = 0.9787 | 101 | 100/101 = 0.9900 |

Note. Only four digits are shown after the decimal point. Only the prime factor 7 gives a decimal part < 0.9 of the order of 0.85.

*3.6 An algorithm Based on This Method of Analyzing Even Numbers Located Before the Numbers to be Decomposed*

Here are the calculation steps to prove the primality of an integer number following the method described in this article:

1. First, we exclude the fact that the odd number is a multiple of 3 or 5 by simple examination of its digits. Take

the even number that precedes by one unit the odd number to be tested.

2. Calculate the decimal fraction between the even thus chosen and the prime factors in ascending order (we set 2-4 digits or more after the decimal separator).
3. Determine whether the decimal part of the resulting number is equal to that obtained in the **table 10** above (if between 7 and 97) or to another larger factor stored in a computer system. It would be certainly more accurate to store (p – 1)/p decimal parts of all known prime numbers with as many digits after decimal point as possible in advance. When there is a match, the calculation stops and switches to Euclidean division.
4. Conclude if the tested number is prime or not, and in the latter case, decompose it to prime factors.

How can we then improve the method by combining it with the classic method of successive divisions? ***We can in fact stop dividing by a prime factor as soon as the first digit after the decimal point is < 9*** (except for 7 as a prime factor). Then, we initially limit ourselves to the numbers which have digit 9 after the decimal point; these are the ones which leads to prime factor. Alternatively, the decimal parts collected are instantly checked digit by digit to see if they correspond to a value of a ratio (p -1)/p stored in memory. If it matches, the system will stop immediately all further divisions and factorize the number to be broken down.

### *3.7 Locate Prime Numbers in a Number Box by Excluding Evens That Have a Single Factor Missing by One Unit*

#### 3.7.1 Principle

Even numbers fall into two categories: those that have a prime factor missing one unit and therefore are never followed by a prime number; and those having no factor missing a unit and it is them and only them which are followed by prime numbers. The prime number results from the even number that precedes it. Now we learn that even numbers are not all uniform and this why there are prime numbers. To simplify, let us pose an even $p_r$ with a prime factor which lacks a unit as $p_r = qr + (r – 1)$. For simplicity we restrict ourselves to those ≤ 200 and carry out the calculations to obtain the numbers $p_r = qr + (r – 1)$ between 7 and 200.

#### 3.7.2 Method for Excluding Even Numbers $p_r$ Whose Factor Is Missing a Single Unit

#### 3.7.2 a. Numbers betwween 0 and 200

To better illustrate this method, it is described step by step.

a) First step: Calculation of even numbers $p_r = qr + (r – 1)$ between 0 and 200. For example to calculate a $p_r$ with 7 we carry out the successive operations: 7 x 2 + (7 – 1) then 7 x 4 + (7 – 1); 7 x 6 + (7 – 1)... up to 7 x 28 + (7 – 1). We will do the same for 11, 13, 17 and 19 (table 11).

Table 11. Even numbers missing a factor of one unit up to 200

| Prime factors<br>2n | 7<br>pr = 2n x 7 + (7 – 1) | 11<br>pr = 2n x 11 + (11 – 1) | 13<br>pr = 2n x 13 + (13 – 1) | 17<br>pr = 2n x 17 + (17 – 1) | 19<br>pr = 2n x 19 + (19 – 1) |
|---|---|---|---|---|---|
| 2 | 20 | 32 | 38 | 50 | 56 |
| 4 | 34 | 54 | 64 | 84 | 94 |
| 6 | 48 | 76 | 90 | 118 | 132 |
| 8 | 62 | 98 | 116 | 152 | 170 |
| 10 | 76 | 120 | 142 | 186 | |
| 12 | 90 | 142 | 168 | | |
| 14 | 104 | 164 | 194 | | |
| 16 | 118 | 186 | | | |
| 18 | 132 | | | | |
| 20 | 146 | | | | |
| 22 | 160 | | | | |
| 24 | 174 | | | | |
| 26 | 188 | | | | |
| 28 | | | | | |

With the prime factor 3 which we approach separately, we will exclude all its multiples because the number we calculate is 3 x 2n + (3 – 1) and therefore all the numbers which follow are 3n (table 12). This is the same with 5 which gives 5 x 2n + (5 – 1) and thus all consecutive numbers are multiples of 5 (table 13).

Table 12. Even numbers obtained with prime factor 3 having a missing factor of one unit up to 200

| 2n | 3 x 2n + (3 – 1) | 2n | 3 x 2n + (3 – 1) |
|---|---|---|---|
| | | 34 | 104 |
| 2 | 8 | 36 | 110 |
| 4 | 14 | 38 | 116 |
| 6 | 20 | 40 | 122 |
| 8 | 26 | 42 | 128 |
| 10 | 32 | 44 | 134 |
| 12 | 38 | 46 | 140 |
| 14 | 44 | 48 | 146 |
| 16 | 50 | 50 | 152 |
| 18 | 56 | 52 | 158 |
| 20 | 62 | 54 | 164 |
| 22 | 68 | 56 | 170 |
| 24 | 74 | 58 | 176 |
| 26 | 80 | 60 | 182 |
| 28 | 86 | 62 | 188 |
| 30 | 92 | 64 | 194 |
| 32 | 98 | 66 | 200 |

Table 13. Even numbers obtained with prime factor 5 having a factor missing by one unit up to 200

| 2n | 5 x 2n + (5 – 1) | 2n | 5 x 2n + (5 – 1) |
|---|---|---|---|
| | | 34 | 174 |
| 2 | 14 | 36 | 184 |
| 4 | 24 | 38 | 194 |
| 6 | 34 | 40 | 204 |
| 8 | 44 | 42 | 214 |
| 10 | 54 | 44 | 224 |
| 12 | 64 | 46 | 234 |
| 14 | 74 | 48 | 244 |
| 16 | 84 | 50 | 254 |
| 18 | 94 | 52 | 264 |
| 20 | 104 | 54 | 274 |
| 22 | 114 | 56 | 284 |
| 24 | 124 | 58 | 294 |
| 26 | 134 | 60 | 304 |
| 28 | 144 | 62 | 314 |
| 30 | 154 | 64 | 324 |
| 32 | 164 | 66 | 334 |

b) Second step: exclude the even numbers $p_r = qr + (r – 1)$ calculated in the tables above:

We write the numbers from 0 to 200 then we eliminate (by crossing out) the numbers

$p_r = qr + (r – 1)$ calculated above with 7, 11, 13, 17 and 19, ***and also eliminate the odd numbers which follow them*** as follows:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 ~~20 21~~ 22 23 24 25 26 27 28 29 30 31 ~~32 33~~ ~~34 35~~ 36 37 37 ~~38 39~~ 40 41 42 43 44 45 46 47 ~~48 49~~ ~~50 51~~ 52 53 ~~54 55~~ ~~56 57~~ 58 59 60 61~~62 63~~ ~~64 65~~ 66 67 68 69 70 71 72 73 ~~74 75~~ ~~76 77~~ 78 79 80 871 82 83 ~~84 85~~ 86 87 88 89 ~~90 91~~ 92 93 94 95 96 97 ~~98 99~~ 100 101 102 103 ~~104 105~~ 106 107 108 109 110 111

112 113 114 115 ~~116 117~~ ~~118 119 120 121~~ 122 123 124 125 126 127 128 129 130 131 ~~132 133~~ 134 135 136 137 138 139 140 141 ~~142 143~~ 144 145 ~~146 147~~ 148 149 150 151 ~~152 153~~ 154 155 156 157 158 159 ~~160 161~~ 162 163 ~~164 165~~ 166 167 ~~168 169~~ ~~170 171~~ 172 173 ~~174 175~~ 176 177 178 179 180 181 182 183 184 185 ~~186 187~~ ~~188 189~~ 190 191 192 193 ~~194 195~~ 196 197 198 199 200

c) Third step: we exclude the odd multiples of 3 and 5, and ***also the evens which precede them*** (highlighted) since the latter have a prime factor missing by one unit:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 ~~20 21~~ 22 23 24 25 26 27 28 29 30 31 ~~32 33~~ ~~34 35~~ 36 37 ~~38 39~~ 40 41 42 43 44 45 46 47 ~~48 49~~ ~~50 51~~ 52 53 ~~54 55~~ ~~56 57~~ 58 59 60 61 ~~62 63~~ ~~64 65~~ 66 67 68 69 70 71 72 73 ~~74 75~~ ~~76 77~~ 78 79 80 81 82 83 ~~84 85~~ 86 87 88 89 ~~90 91~~ 92 93 94 95 96 97 ~~98 99~~ 100 101 102 103 ~~104 105~~ 106 107 108 109 110 111 112 113 114 115 ~~116 117~~ ~~118 119 120 121~~ 122 123 124 125 126 127 128 129 130 131 ~~132 133~~ 134 135 136 137 138 139 140 141 ~~142 143~~ 144 145 ~~146 147~~ 148 149 150 151 ~~152 153~~ 154 155 156 157 158 159 ~~160 161~~ 162 163 ~~164 165~~ 166 167 ~~168 169~~ ~~170 171~~ 172 173 ~~174 175~~ 176 177 178 179 180 181 182 183 184 185 ~~186 187~~ ~~188 189~~ 190 191 192 193 ~~194 195~~ 196 197 198 199

d) Last step: locate the couples of prime numbers and the even numbers preceding them (underlined in yellow). ***Note that these evens do not have a prime factor that lacks a single unit and these are the only forms of evens that precede prime numbers***. The only exceptions are couples 2 and 3; 4 and 5, and 6 and 7 whose even numbers cannot be deduced from the equation $p_r = qr + (r - 1)$.

0 1 **2 3 4 5 6 7** 8 9 **10 11 12 13** 14 15 **16 17 18 19** ~~20 21~~ **22 23** 24 25 26 27 **28 29 30 31** ~~32 33~~ ~~34 35~~ **36 37** ~~38 39~~ **40 41 42 43** 44 45 **46 47** ~~48 49~~ ~~50 51~~ **52 53** ~~54 55~~ ~~56 57~~ **58 59 60 61** ~~62 63~~ ~~64 65~~ **66 67** 68 69 **70 71 72 73** ~~74 75~~ ~~76 77~~ **78 79** 80 81 **82 83** ~~84 85~~ 86 87 **88 89** ~~90 91~~ 92 93 94 95 **96 97** ~~98 99~~ **100 101 102 103** ~~104 105~~ **106 107 108 109** 110 111 **112 113** 114 115 ~~116 117~~ ~~118 119 120 121~~ 122 123 124 125 **126 127** 128 129 **130 131** ~~132 133~~ 134 135 **136 137 138 139** 140 141 ~~142 143~~ 144 145 ~~146 147~~ **148 149 150 151** ~~152 153~~ 154 155 **156 157** 158 159 ~~160 161~~ **162 163** ~~164 165~~ **166 167** ~~168 169~~ ~~170 171~~ **172 173** ~~174 175~~ 176 177 **178 179 180 181** 182 183 184 185 ~~186 187~~ ~~188 189~~ **190 191 192 193** 194 195 **196 197 198 199**

We can clearly see that the prime numbers (underlined in yellow) are associated with the even numbers preceding them which must in no case have a factor missing by one unit. This method is therefore able to isolate the prime numbers after few steps. All prime numbers below 200 have thus been found with this method together with the even ones preceding them.

3.7.2 b Larger Numbers Between Two Prime Numbers

Let's take two prime numbers P and P' at random and test the numbers that lie in their area.

P = 433589715233

P' = 433589715253

**433589715232 ; P** ; ~~433589715234~~ ; ~~433589715235~~ ; ~~433589715236~~ ; ~~43358971537~~ ; ~~433589715238~~ ; ~~433589715239~~ ; ~~433589715240~~ ; ~~433589715241~~ ; ~~433589715242~~ ; ~~433589715243~~ : ~~433589715244~~ ; ~~43358971523345~~ ; ~~433589715246~~ ; ~~433589715247~~ ; ~~433589715248~~ ; ~~433589715249~~ ; ~~433589715250~~ ; ~~433589715251~~ ; **433589715252 ; P'** ; 433589715254.

In the number box shown above, the uncolored ones are the odd 5n and the even ones preceding them. The yellow ones are 3n odd and even before them while the blue ones are those having a factor that is missing a unit and therefore cannot be followed by a prime number if my method is applied as shown above. In green are evens followed by a prime numbers P or P'. See more details on the calculation below.

433589715239 = 17 × 29 × 879492323 = 493 x 879492323 and therefore **433589715238** is expected to be = **492 x 879492323 + 879492322** thus following the equation p x r + (r -1). It is therefore crossed out. Note that 433589715238/17 = 25505277366**.94117647058823529412** (corresponding to 16/17 = **0.94117647058823529412**) ; 433589715238/29 = 14951369490**.96551724137931034483** (corresponding to 28/29 = **0.96551724137931034483**) and finally 433589715238/879492323 = 492**.99999999886298041057** (corresponding to 879492322/ 879492323 = 0**.99999999886298041057**). The same argument applies to the following odd numbers 433589715241 = 12457 × 34806913 ; 433589715247 = 13 × 74759 × 446141 and 433589715251 = 73 × 11 × 19 × 239 × 25307. In contrast, 433589715232 and 433589715252 preceding the prime numbers P and P', respectively, do not give (p – 1)/p specific ratios when divided by prime numbers lesser than their square roots. We would have therefore known by the method described in this paper that the odd numbers P and P' that respectively follow them, are indeed prime.

### *3.8 Decompose an Odd Number by Dividing It by 2 or 4*

#### 3.8.1 Principle of the Method

An odd number divided by 2 will give two numbers, each of which contains ½ of one of its prime factors (approximately because one of the halves is greater by 1). By dividing its two halves by this prime factor, we obtain numbers whose addition of the decimal parts must give 1.

#### 3.8.2 Note

This new method is not at all related to the method based on (p – 1)/p numbers from a practical point of view, but both have the same basis. Both are in fact based on the decimal parts of the calculated numbers. We once again limit ourselves to two digits after the decimal point. However, we will see examples with longer decimal parts.

#### 3.8.3 Calculation examples With Division by 2

Let's take the number 5 251 whose square root ≈ 76. We divide it by 2, we have 2 625 and 2 626. We then calculate the ratio between these two halves and prime factors (table 13). In table 13, only data with the smaller half 2 625 are shown and when there is a ratio value approaching 0.5, that when the value for the other half is displayed. We made the sum of both decimal parts to see if it equals 1.

Table 14. Primality test of 5 251 by addition of two decimal fractions

| Prime Factor | N/2 = 2625 | N/2 = 2626 | Prime Factor | N/2 = 2625 | N/2 = 2626 |
|---|---|---|---|---|---|
| 7 | 375 | | 43 | 61.04 | |
| 11 | 238.63 | | 47 | 55.85 | |
| 13 | 201.92 | | <u>53</u> | 49.52 | 49.54 |
| 17 | 154.41 | | <u>59</u> | 44.**49** | 44.**5** |
| 23 | 114.13 | | 61 | 43.03 | |
| <u>29</u> | 90.51 | 90.55 | 67 | 39.17 | |
| 31 | 84.67 | | 71 | 36.97 | |
| 37 | 70.94 | | 73 | 35.95 | |
| 41 | 64.02 | | 79 | 33.22 | |

We see that with the factor 59 the decimal parts 0.49 and 0.5 the addition of which gives 0.99. Hence closest to 1. In contrast, the sum of decimal parts corresponding to 29 or 53 exceeded 1. We deduce that 59 is a factor of the number 2 625 + 2 626 = 5 251 = 59 x 89.

By using the two halves of any odd number we can proceed differently. Let's take an odd number $b_n$ and divide it by 2, we have $b_n/2$ and $(b_n/2) + 1$. We then calculate the ratio $\mathbf{r = (b_n/2 – p/2)/p}$ with p the prime factor. If the ratio gives an integer number with zero decimal part, that is the factor. Let's see this with the same number 5 251 and construct the table corresponding to this calculation. The ratio $\mathbf{r = (b_n/2 – p/2)/p}$ simply means that we will remove half of the prime factor remaining in half of the number and then divide it by the whole prime factor so that we can decompose the original number.

Table 15. Decomposing the number 5 251 by the ratio $r = (b_n/2 – p/2)/p$

| p | p/2 | r | p | p/2 | r |
|---|---|---|---|---|---|
| 7 | 3 | 374.57 | 43 | 21 | 60.55 |
| 11 | 5 | 238.18 | 47 | 23 | 55.36 |
| 13 | 6 | 201.46 | 53 | 26 | 49.03 |
| 17 | 8 | 153.94 | <u>59</u> | 29 | **44** |
| 23 | 11 | 113.65 | 61 | 30 | 42.54 |
| 29 | 14 | 90.03 | 67 | 33 | 38.68 |
| 31 | 15 | 84.19 | 71 | 35 | 36.47 |
| 37 | 18 | 70.45 | 73 | 36 | 35.46 |
| 41 | 20 | 63.53 | 79 | 39 | 32.73 |

We deduce from this that 2 625 – 29 = 2 596 = 44 x 59 and 2 626 + 29 = 2 655 = 45 x 59 and therefore 2 625 + 2 626 = 5 251 = 59 x (44 + 45) = 59 x 89. Note that we subtract 29 from one half and add it to the other and this is how we obtain the prime factor of the number.

#### 3.8.4 Calculation Examples With Division by 4

Let's take the number 13 289 whose square root ≈ 115. We divide it by 4 such that 13 289 : 4 = 3 322.25. We therefore have the number which is divided into 3 322.25 and 3 322.25 x 3 = 9 966.75 to which we then add 0.25 of the quarter and we finally have 3 322 and 9 967. We will then analyze the quarter in the following table as we did with the halves (table 16).

Table 16. Test for primality of the number 13 289 by dividing its quarter 1/4 and ¾ by the prime factors

| Prime Factor | ¼ = 3222 | 3/4 = 9967 | Prime Factor | ¼ = 3322 | 3/4 = 9967 |
|---|---|---|---|---|---|
| 7 | 474.57 | | 67 | 49.58 | |
| 11 | 302 | | 71 | 46.78 | |
| 13 | 255.53 | | 73 | 45.5 | |
| 17 | 195.41 | | 79 | 42.05 | |
| 23 | 144.43 | | 89 | 37.32 | |
| 29 | 114.55 | | 97 | 34.**24** | 102.**75** |
| 31 | 107.16 | | 101 | 32.89 | |
| 37 | 89.78 | | 103 | 32.25 | |
| 41 | 81.02 | | 107 | 31.04 | |
| 43 | 77.25 | 231.79 | 109 | 30.47 | |
| 47 | 70.68 | | 113 | 29.39 | |
| 53 | 62.67 | | | | |
| 59 | 56.3 | | | | |
| 61 | 54.45 | | | | |

We have two decimal fractions which approximate the ratio between ¼ and ¾ and which correspond in the table to the prime factors 43 and 97. However, for 43, we see that the sum of the two decimal fractions 0.25 + 0.79 = 1 .04 > 1 while for 97 we have 0.24 + 0.75 = 0.99 closer to 1. It is 97 the prime factor which divides the number 13 289 = 97 x 137. We see that 3 322 = 34 x 97 **+ 24** and 9 967 = 102 x 97 **+ 73** and therefore 13 289 = 3 322 + 9 967 = 97 (34 + 102) **+ 24 + 73** = 97 x 136 **+ 97** = 97 x 137. Otherwise, we remove 24 from the ¼ and add them back to the ¾ to be able to decompose the number 13 289. The ratio $r = (b_n/4 – p/4)/p$ also allows to do that.

3.8.5 Representative Examples of Longer Decimal Fractions

Here are examples of numbers with 25 digits after the decimal separator.

Decomposition by division by 2 :

p = 544811234663

p' = 622115551859

N = p x p' = 338935541911355394888517

N/2 = 169467770955677697444258.5

Then N = 169467770955677697444258 +169467770955677697444259

169467770955677697444258 / p = 311057775929.**4999999999990822509372273**

169467770955677697444259 / p = 311057775929.**5000000000009177490627727**

0.4999999999990822509372273 + 0.5000000000009177490627727 = 1.

169467770955677697444258 / p' = 272405617331.**4999999999991962907879318**

169467770955677697444259 / p' = 272405617331.**5000000000008037092120682**

0.4999999999991962907879318 + 0.5000000000008037092120682 = 1.

Decomposition by division by 4 :

p = 25612457219

p' = 433589717747

N = p x p' = 11105298096393322565593

N/4 = 2776324524098330641398.25 and so

N = N/4 + (3 x N/4) = 2776324524098330641398**.25** + 8328973572294991924194**.75** or

N = 277632452409833064139**8** + 832897357229499192419**5**

2776324524098330641398/p = 108397429436.**74999999999902391247406538**

8328973572294991924195/p = 325192288310.**25000000000097608752593462**

0.74999999999902391247406538 + 0.25000000000097608752593462 = 1.

2776324524098330641398 / p' = 6403114304.7499999999999423418061436

8328973572294991924195 / p' = 19209342914.2500000000000576581938564

0.7499999999999423418061436 + 0.2500000000000576581938564 = 1.

## 4. Conclusion

The problem of factoring integers into primes is central to computational number theory. It has been studied since the ancient times, and many methods have been developed. Not all numbers of a given length are equally hard to factor. The hardest odd numbers to decompose are biprimes, the product of two prime numbers. When their prime factors are both large and randomly chosen, and about the same size but enough distant from the square root value, even the fastest prime factorization algorithms on the fastest computers can take enough time to break them down. This is the main reason why this study focuses on biprime numbers. As the number of digits of the integer being factored increases, the number of operations required to perform the factorization on any computer increases drastically. This study was conducted to find a new perspective in the field of the decomposition of natural numbers and their primality.

The method described in this article is new and the data collected in this paper strongly suggest that it has a good potential for decomposing natural numbers. However, it is derived from or close to the classic method called (trial division) or Euclidean divisions in series. This last method is based on a simple observation: is the remainder of the Euclidean division 0 or not? If it is 0 the number is factorized otherwise we move on to the next division. This classic method is known to be the least efficient because it requires a long time, especially for large numbers, compared to the most recent algorithms such as Pollard's (Hegde & Deepthi, 2015). The latter in turn depends on the square root of the smallest factor of the number to be decomposed. To summarize, today it is mainly a question of calculation speed and economy, that is to say fewer operations to achieve the result (Duta, Gheorghe & Tapus, 2016; Decker, Greuel, & Pfister, 1999).

The method described in this paper is primarily based on the calculation of decimal fractions between the even which precedes the odd number to be decomposed and the prime factors in increasing order. It shows that these fractions have a decimal part = (p – 1)/p with p the prime factor. When it is the factor that divides the even, the decimal part begins with 9 and can contain a series of digits 9 that is longer as the factor and the number are larger. The great advantage it offers is the fact that we can stop any calculation as soon as the fraction does not contain a digit 9 after the decimal point (except for 7). The other advantage is that as the number to decompose is larger the calculated fraction contains a longer sequence of 9 digits that seems to happen only with the right prime factor. Finally, it allows us to calculate (p - 1)/p in advance and store them in memory and do match tests between the decimal part of the calculated fractions and the (p - 1)/p precalculated. As soon as there is a mtach, the calculation can fade and decompose the number.

This paper further raises important questions for fundamental mathematics: could we find the equation that allows us to spot even numbers that have a factor missing a single unit and that are before an odd number ? This would mean that we would be able to identify and predict in advance any prime number from 0 to infinity. To put it better, an even number has its own prime factors and can be written as a product of prime factors, but even numbers that are before non-prime odd numbers are written another way. For example, an even number can be written with two factors as q x r + (r - 1) which means that the odd number after it is not prime and that it has r as a prime factor. In addition, this method uses irrational numbers, another hot topic of mathematics, the most famous number of which is pi (Agarwal & Agarwal, 2021; Shiver & Klosterman, 2022). These complex numbers can still have practical use and serve as tools to decompose a number into its prime factors. Each irrational number is unique in itself and despite this, they can obey classical rules of calculation and give a predictable result. This is also the case for pi which remains essential for calculating the surface area of a circle.

All these elements mean that the method described in this article will undoubtedly stimulate future research and reflection. First, see if the method can be programmed and if it can be as efficient as existing algorithms in performance and speed. Calculating decimal fractions by this method is likely more useful than simply looking at remainders because the result can be expected since the first digit after the decimal separator and even predicted by a more specific calculation. For larger numbers, the method is even more effective because the ratio (p - 1)/p generates a sequence of digits 9 which can take an unlimited length, which proves that irrational numbers have decimal parts which can go to infinity. But it is certain that this requires lengthy calculations and the method does not escape the obstacles faced by existing algorithms. However, we can predict these ratios in advance and stop the calculation as soon as we have sufficient concordance between a (p - 1)/p and a decimal part of the calculated fraction. If it is possible by a more subtle calculation to know in advance which prime factor could generate digits 9 in the decimal part of the fraction with the even; we will therefore be able to find the prime factor of the odd to decompose without going through serial divisions. Let's not forget that the even number before the biprime number generates digits 9 when divided by its prime factors while the even number which is after the same biprime number all the opposite generates as many zeros (data not shown). This even after the biprime number can serve to confirm that we have the right prime factor.

These data deserve our future consideration.

Globally, this method is a potential new approach to decomposing numbers which instead uses the decimal fraction and therefore goes beyond a simple examination of division remainder. But it is certain that it will bring new knowledge about prime numbers because it would seem that irrational numbers are inevitable in order to understand the problem of prime numbers. *Isn't it true that we say that a number is prime because it produces decimal or irrational numbers when divided by a prime factor (except when it is divided by 1 and itself) ?*

Since the (p - 1)/p can be calculated in advance, and since the good prime factor automatically gives one or more digits 9 after the decimal point of the fraction, this method would undoubtedly be programmable and promises to bring new shed light on the mysteries of prime numbers as well as on the decomposition of integers into prime factors, in particular biprime numbers of high value.

**References**

Agarwal, R. P., & Agarwal, H. (2021). Origin of irrational numbers and their approximations. *Computation*, *9*(3), 29. https://doi.org/10.3390/computation9030029

Balasubramanian, K., & Abbas, A. M. (2018). Integer factoring algorithms. In *Algorithmic Strategies for Solving Complex Problems in Cryptography.* 228-240. IGI Global. https://doi.org/10.4018/978-1-5225-2915-6.ch017

Decker, W., Greuel, G. M., Pfister, G. (1999). Primary Decomposition: Algorithms and Comparisons. In: Matzat, B.H., Greuel, GM., Hiss, G. (eds) Algorithmic Algebra and Number Theory. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-59932-3_10

Duta, C. L., Gheorghe, L., Tapus, N. (2016). Framework for evaluation and comparison of integer factorization algorithms. *SAI Computing Conference (SAI)*, London, UK, 1047-1053. https://doi.org/10.1109/SAI.2016.7556107

Hegde, N., & Deepthi, P. (2015). Pollard RHO algorithm for integer factorization and discrete logarithm problem. *International Journal of Computer Applications*, *121*(18), 14-17. https://doi.org/10.5120/21639-4969

Ndiaye, M. (2017). The Prime Numbers. *International Journal for Innovation Education and Research*, *5*(6), 41-66. https://doi.org/10.31686/ijier.vol5.iss6.672

Sahin, M. (2011). Generalized Trial Division. *Int. J. Contemp. Math. Sciences*, *6*(2), 59-64.

Shiver, J., & Klosterman, P. (2022). Making irrational numbers real. *Middle School Journal*, *53*(1), 36-42. https://doi.org/10.1080/00940771.2021.1997534

**Acknowledgments**

Not applicable.

**Authors contributions**

Not applicable.

**Funding**

Not applicable.

**Competing interests**

Not applicable.

**Informed consent**

Obtained.

**Ethics approval**

The Publication Ethics Committee of the Canadian Center of Science and Education.

The journal’s policies adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

**Provenance and peer review**

Not commissioned; externally double-blind peer reviewed.

**Data availability statement**

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

**Data sharing statement**

No additional data are available.